# Debunking the Dot Product of a Magnetic Moment and a Magnetic Field

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**Abstract**

We attempt to calculate the exercise where we show that a dot product of a magnetic moment and a pointwise value of a magnetic field with a minus sign describes an interaction energy. The attempt is not successful due to the divergence of the relevant integrals, and we conclude that the claimed exercise result is wrong. The belief that the dot product with a minus sign would describe an interaction energy most apparently is only a myth.

One of the properties of a classical physical object is a magnetic moment $\mathbf{m} \in \mathbb{R}^3$. According to mainstream theoretical physics, if an object with a magnetic moment $\mathbf{m}$ is very small in size, and if this object is at location $\mathbf{x}$, where there is a magnetic field $\mathbf{B}(\mathbf{x})$ present, there should be an interaction energy $U = -\mathbf{m} \cdot \mathbf{B}(\mathbf{x})$ [1]. This is Equation (5.72) in Jackson's book [3]. For a person who already knows calculus, linear algebra and some basics of electromagnetism, this doesn't look like an extraordinary claim. For example, there is an interesting exercise where we assume that an electric field $\mathbf{E}(\mathbf{x})$ has been produced by two point charges $q_{\mathrm{A}}$ and $q_{\mathrm{B}}$ at locations $\mathbf{x}_{\mathrm{A}}$ and $\mathbf{x}_{\mathrm{B}}$, and where we also assume that an energy density in the electric field is $\frac{\epsilon_0}{2}\|\mathbf{E}(\mathbf{x})\|^2$. Then it turns out that it is possible to recognize an interaction term in the integral of this energy density, and it can be shown with some integration tricks that the integral of that interaction term equals $\frac{q_{\mathrm{A}} q_{\mathrm{B}}}{4\pi\epsilon_0} \frac{1}{\|\mathbf{x}_{\mathrm{B}} - \mathbf{x}_{\mathrm{A}}\|}$ — the well known Coulomb potential energy. When somebody with this kind of background knowledge encounters the claim that $-\mathbf{m} \cdot \mathbf{B}(\mathbf{x})$ is an interaction energy between a magnetic moment and a magnetic field, he or she will probably react by feeling like already roughly knowing what's going on. Surely it must be so that there is an exercise where we assume that an energy density in a magnetic field is $\frac{1}{2\mu_0}\|\mathbf{B}(\mathbf{x})\|^2$, and then, if we substitute into $\mathbf{B}(\mathbf{x})$ a sum of a background field $\mathbf{B}_{\mathrm{b}}(\mathbf{x})$ and the field produced by a point-like magnetic dipole with a magnetic moment $\mathbf{m}$ at a location $\mathbf{x}_0$, it will be possible to recognize an interaction term in the energy density? Surely it must be so that some integration trick can then be used to show that the integral of the interaction term equals $-\mathbf{m} \cdot \mathbf{B}_{\mathrm{b}}(\mathbf{x}_0)$?

It is noteworthy that a derivation of the interaction energy $U = -\mathbf{m} \cdot \mathbf{B}(\mathbf{x})$ as an integral of the energy density $\frac{1}{2\mu_0}\|\mathbf{B}(\mathbf{x})\|^2$ cannot be found anywhere. Consequently, it might be a good idea to turn this into a voluntary exercise for ourselves? Why not take a closer look at where this interaction energy is supposed to come from?

Let's attempt to approach this topic by first studying a small current ring. Suppose a current $I$ flows in a ring

$$\big\{(r\cos(\theta), r\sin(\theta), 0) \;\big|\; 0 \leq \theta < 2\pi\big\}$$

counterclockwise when looked from above. Here $r > 0$ is the radius of the ring. Let's fix some vector $\mathbf{x} = (x_1, 0, x_3)$, and ask that what is the magnetic field $\mathbf{B}(\mathbf{x})$ produced by the current ring at this location. The reason for why we substituted $x_2 = 0$ is that if we find $\mathbf{B}(\mathbf{x})$ at $(x_1, 0, x_3)$, we can then deduce the magnetic field at the other locations from the symmetry of the setting. We can answer the question by using Biot-Savart law [2]. Let's denote that the vector on the ring is

$$\mathbf{x}'(\theta) = (r\cos(\theta),\; r\sin(\theta),\; 0).$$

According to Biot-Savart law the magnetic field is then

$$\mathbf{B}(\mathbf{x}) \;=\; -\frac{\mu_0 I}{4\pi}\int \frac{\mathbf{x}-\mathbf{x}'(\theta)}{\|\mathbf{x}-\mathbf{x}'(\theta)\|^3}\times d\mathbf{x}'(\theta).$$

If we substitute into this the relations

$$\begin{aligned}\mathbf{x}-\mathbf{x}'(\theta) \;&=\; (x_1 - r\cos(\theta),\; -r\sin(\theta),\; x_3),\\ \|\mathbf{x}-\mathbf{x}'(\theta)\|^2 \;&=\; x_1^2 \;+\; x_3^2 \;+\; r^2 \;-\; 2rx_1\cos(\theta)\end{aligned}$$

and

$$d\mathbf{x}'(\theta) \;=\; (-r\sin(\theta)d\theta,\; r\cos(\theta)d\theta,\; 0),$$

we obtain the magnetic field values

$$\begin{aligned}B_1(\mathbf{x}) \;&=\; \frac{\mu_0 I r x_3}{4\pi}\int_0^{2\pi}\frac{\cos(\theta)}{\big(x_1^2+x_3^2+r^2-2rx_1\cos(\theta)\big)^{\frac{3}{2}}}d\theta\\ B_2(\mathbf{x}) \;&=\; \frac{\mu_0 I r x_3}{4\pi}\int_0^{2\pi}\frac{\sin(\theta)}{\big(x_1^2+x_3^2+r^2-2rx_1\cos(\theta)\big)^{\frac{3}{2}}}d\theta\\ B_3(\mathbf{x}) \;&=\; \frac{\mu_0 I r}{4\pi}\int_0^{2\pi}\frac{r - x_1\cos(\theta)}{\big(x_1^2+x_3^2+r^2-2rx_1\cos(\theta)\big)^{\frac{3}{2}}}d\theta.\end{aligned}$$

We can express the same magnetic field values as

$$\begin{aligned}B_1(\mathbf{x}) \;&=\; \frac{\mu_0 I r x_3}{2\pi(x_1^2+x_3^2+r^2)^{\frac{3}{2}}} f_1\Big(\frac{2rx_1}{x_1^2+x_3^2+r^2}\Big)\\ B_2(\mathbf{x}) \;&=\; 0\\ B_3(\mathbf{x}) \;&=\; \frac{\mu_0 I r}{2\pi(x_1^2+x_3^2+r^2)^{\frac{3}{2}}}\Big(r f_0\Big(\frac{2rx_1}{x_1^2+x_3^2+r^2}\Big) \;-\; x_1 f_1\Big(\frac{2rx_1}{x_1^2+x_3^2+r^2}\Big)\Big),\end{aligned}$$

where we denoted

$$f_0(u) \;=\; \int\limits_0^{\pi} \frac{1}{(1-u\cos(\theta))^{\frac{3}{2}}}\,d\theta \qquad \text{and} \qquad f_1(u) \;=\; \int\limits_0^{\pi} \frac{\cos(\theta)}{(1-u\cos(\theta))^{\frac{3}{2}}}\,d\theta.$$

These formulas define some functions

$$f_0 : ]-1,1[ \to \mathbb{R} \qquad \text{and} \qquad f_1 : ]-1,1[ \to \mathbb{R}.$$

By using the Taylor series

$$\frac{1}{(1+z)^{\frac{3}{2}}} \;=\; 1 \;-\; \frac{3}{2}z \;+\; \cdots$$

we find that the functions $f_0$ and $f_1$ have Taylor series approximations

$$f_0(u) \;=\; \pi \;+\; O(u^2) \qquad \text{and} \qquad f_1(u) \;=\; \frac{3\pi}{4}u \;+\; O(u^3).$$

This means that with very small $r$ the magnetic field can be approximated with the formulas

$$B_1(\mathbf{x}) \;=\; \frac{3\mu_0 I r^2 x_1 x_3}{4(x_1^2+x_3^2)^{\frac{5}{2}}}\big(1+O(r^2)\big)$$

and

$$B_3(\mathbf{x}) \;=\; \frac{\mu_0 I r^2(2x_3^2-x_1^2)}{4(x_1^2+x_3^2)^{\frac{5}{2}}}\big(1+O(r^2)\big).$$

What happens, if we take the limit $r \to 0$? If we take this limit in a such way that $I$ remains constant, we get $\mathbf{B}(\mathbf{x}) \to \mathbf{0}$. However, if we take the limits $r \to 0$ and $I \to \infty$ simultaneously in a such way that the quantity $Ir^2$ remains constant, then we get

$$\mathbf{B}(\mathbf{x}) \;\to\; \Big(\frac{3\mu_0 I r^2 x_1 x_3}{4(x_1^2+x_3^2)^{\frac{5}{2}}},\; 0,\; \frac{\mu_0 I r^2(2x_3^2-x_1^2)}{4(x_1^2+x_3^2)^{\frac{5}{2}}}\Big).$$

Next, suppose we want to know $\mathbf{B}(\mathbf{x})$ at some arbitrary location $\mathbf{x} = (x_1, x_2, x_3)$, where possibly $x_2 \neq 0$. It should be possible to obtain the answer by rotating the previous result where $x_2 = 0$. By applying some generic understanding of mathematics we obtain the result that the rotated magnetic field is

$$\mathbf{B}(\mathbf{x}) \;=\; \Big(\frac{3\mu_0 I r^2 x_1 x_3}{4(x_1^2+x_2^2+x_3^2)^{\frac{5}{2}}},\; \frac{3\mu_0 I r^2 x_2 x_3}{4(x_1^2+x_2^2+x_3^2)^{\frac{5}{2}}},\; \frac{\mu_0 I r^2(2x_3^2-x_1^2-x_2^2)}{4(x_1^2+x_2^2+x_3^2)^{\frac{5}{2}}}\Big).$$

Let's define a vector $\mathbf{m}$ by formula

$$\mathbf{m} \;=\; (0,\; 0,\; \pi r^2 I),$$

and call this the magnetic moment of the small current ring. It is then possible to express the just solved magnetic field by a formula

$$\mathbf{B}(\mathbf{x}) \;=\; \frac{\mu_0}{4\pi}\Big(\frac{3(\mathbf{m}\cdot\mathbf{x})\mathbf{x}}{\|\mathbf{x}\|^5} \;-\; \frac{\mathbf{m}}{\|\mathbf{x}\|^3}\Big).$$

This is Equation (5.56) in Jackson's book [3]. Since this way does not use the basis axes of the coordinate set in an explicit way, it can deduced that this formula will give a magnetic field around an arbitrary small magnetic dipole located at the origin, and from now on $\mathbf{m}$ does not need to point in the same direction as $\mathbf{e}_3$.

One question that should be recognized as interesting is that what is the energy contained in the magnetic field around a point-like magnetic dipole? The answer is that $\|\mathbf{B}(\mathbf{x})\|$ diverges to infinity at the limit $\mathbf{x}\to\mathbf{0}$ so fast that the integral of the energy density $\frac{1}{2\mu_0}\|\mathbf{B}(\mathbf{x})\|^2$ is infinite. This can be considered to be a problem, and we'll have to live with it. We probably already knew that the energy in the electric field around a point-like charge is infinite too, so at this point the divergence of the energy of a point-like magnetic dipole isn't alarming news to us.

Suppose $\mathbf{B}(\mathbf{x})$ is the magnetic field component produced by a small magnetic dipole, and that $\mathbf{B}_{\mathrm{b}}(\mathbf{x})$ is some background field component that is present for some outside reason. The energy in the total magnetic field will then be

$$\frac{1}{2\mu_0}\int\limits_{\mathbb{R}^3}\big\|\mathbf{B}_{\mathbf{b}}(\mathbf{x})+\mathbf{B}(\mathbf{x})\big\|^2 d^3x \;=\; \frac{1}{2\mu_0}\int\limits_{\mathbb{R}^3}\big\|\mathbf{B}_{\mathbf{b}}(\mathbf{x})\big\|^2 d^3x$$
$$+\;\frac{1}{2\mu_0}\int\limits_{\mathbb{R}^3}\big\|\mathbf{B}(\mathbf{x})\big\|^2 d^3x \;+\; \frac{1}{\mu_0}\int\limits_{\mathbb{R}^3}\mathbf{B}_{\mathrm{b}}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x})d^3x.$$

The first and the second integrals on the right side don't depend on the location or the orientation of the magnetic dipole, and therefore it is the third integral that is the relevant interaction term. On the face of it, it seems possible that even if the integrals of $\|\mathbf{B}_{\mathrm{b}}(\mathbf{x})\|^2$ and $\|\mathbf{B}(\mathbf{x})\|^2$ diverge, the integral

$$\frac{1}{\mu_0}\int\limits_{\mathbb{R}^3}\mathbf{B}_{\mathrm{b}}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x})d^3x$$

still maybe converges. Suppose the magnetic moment is $\mathbf{m}$ and that the location of the magnetic dipole is $\mathbf{x}_0$. We are interested in the hypothesis that the quantity $-\mathbf{m}\cdot\mathbf{B}_{\mathrm{b}}(\mathbf{x}_0)$ is the interaction energy of the magnetic dipole and the background magnetic field. If this hypothesis was true, it would have to be so because the equation

$$\frac{1}{\mu_0}\int\limits_{\mathbb{R}^3}\mathbf{B}_{\mathrm{b}}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x})d^3x \;=\; -\mathbf{m}\cdot\mathbf{B}_{\mathrm{b}}(\mathbf{x}_0) \tag{1}$$

would be true. There is no other possible explanation for the interaction energy $-\mathbf{m}\cdot\mathbf{B}_{\mathrm{b}}(\mathbf{x}_0)$ in sight. So the big question is that is Equation (1) true or not?

Let's assume that $\|\mathbf{m}\| > 0$, $\|\mathbf{B}_{\mathrm{b}}\| > 0$ and $\varphi \in [0,\pi]$ are some constants, and put $\mathbf{x}_0 = \mathbf{0}$,

$$\mathbf{m} \;=\; \big(0,\;0,\;\|\mathbf{m}\|\big) \qquad\text{and}\qquad \mathbf{B}_{\mathrm{b}}(\mathbf{x}) \;=\; \big(\|\mathbf{B}_{\mathrm{b}}\|\sin(\varphi),\;0,\;\|\mathbf{B}_{\mathrm{b}}\|\cos(\varphi)\big).$$

Now we have

$$\begin{aligned}&\frac{1}{\mu_0}\int\limits_{\mathbb{R}^3}\mathbf{B}_{\mathrm{b}}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x})d^3x\\ &= \frac{\|\mathbf{m}\|\|\mathbf{B}_{\mathrm{b}}\|}{4\pi}\Big(\sin(\varphi)\int\limits_{\mathbb{R}^3}\frac{3x_1x_3}{\|\mathbf{x}\|^5}d^3x \;+\; \cos(\varphi)\int\limits_{\mathbb{R}^3}\Big(\frac{3x_3^2}{\|\mathbf{x}\|^5}-\frac{1}{\|\mathbf{x}\|^3}\Big)d^3x\Big)\end{aligned}$$

and

$$-\mathbf{m}\cdot\mathbf{B}_{\mathrm{b}}(\mathbf{x}_0) \;=\; -\|\mathbf{m}\|\|\mathbf{B}_{\mathrm{b}}\|\cos(\varphi).$$

The only way Equation (1) could be true would be that equations

$$\int\limits_{\mathbb{R}^3}\frac{x_1x_3}{\|\mathbf{x}\|^5}d^3x \;=\; 0 \tag{2}$$

and

$$\int\limits_{\mathbb{R}^3}\Big(\frac{3x_3^2}{\|\mathbf{x}\|^5}-\frac{1}{\|\mathbf{x}\|^3}\Big)d^3x \;=\; -4\pi \tag{3}$$

would be true. Now the big question is that are the equations (2) and (3) true or not? The answer is that the equations (2) and (3) are not true. Some people might say that since the integrand in Equation (2) is antisymmetric with respect to the operation $x_1 \mapsto -x_1$, we see from there that the integral must be zero. The problem with this argument is that the integral should be converging before the application of the antisymmetricity argument. The integral in Equation (2) is diverging, so it's not representing the value zero. We can make an attempt to calculate the integral in Equation (3) with spherical coordinates. The calculation looks like

$$\begin{aligned}\int\limits_{\mathbb{R}^3}\Big(\frac{3x_3^2}{\|\mathbf{x}\|^5}-\frac{1}{\|\mathbf{x}\|^3}\Big)d^3x \;&=\; \int\limits_0^\infty dr\int\limits_0^\pi d\theta\int\limits_0^{2\pi}d\varphi\; r^2\sin(\theta)\Big(\frac{3(\cos(\theta))^2}{r^3}-\frac{1}{r^3}\Big)\\ &=\; \Big(\int\limits_0^\infty\frac{1}{r}dr\Big)\Big(3\cdot\frac{2}{3}-2\Big)\cdot 2\pi.\end{aligned}$$

Mysteriously, also this integral almost vanishes due to cancellation, but the full answer is that the integral is diverging. Since equations (2) and (3) are wrong, we have to conclude that Equation (1) is wrong too. Now we are ready to conclude that the famous interaction energy $U = -\mathbf{m}\cdot\mathbf{B}(\mathbf{x})$ is wrong!

Let's take a look at a force $\mathbf{F}$ experienced by a small magnetic dipole next. Let's assume that $\mathbf{B}_{\mathrm{b}} \in \mathbb{R}^3$, $\partial_1\mathbf{B}_{\mathrm{b}} \in \mathbb{R}^3$, $\partial_2\mathbf{B}_{\mathrm{b}} \in \mathbb{R}^3$ and $\partial_3\mathbf{B}_{\mathrm{b}} \in \mathbb{R}^3$ are some constant vectors, and that a magnetic field

$$\mathbf{B}(\mathbf{x}) \;=\; \mathbf{B}_{\mathrm{b}} \;+\; \sum_{i=1}^{3} x_i\partial_i\mathbf{B}_{\mathrm{b}}$$

fills the space $\mathbb{R}^3$. Let's again use a small ring with a current $I$ and a radius $r > 0$. Let's ignore the magnetic field produced by this current ring, and only take into account the background field that is present for some outside reason. The question that we'll be interested next is that what is the force $\mathbf{F}$ that the magnetic field $\mathbf{B}(\mathbf{x})$ causes on the current ring? One possible technique to accomplish calculating this is that we temporarily discretize the current flow on the ring. Let $N \in \mathbb{N}$ be some large number, and assume that there are $N$ particles, each with a charge $q$, travelling along the ring with speed $v$. Now there's a relation

$$I \;=\; \frac{Nqv}{2\pi r}.$$

Suppose some particle is at the location $(r\cos(\theta), r\sin(\theta), 0)$ at some moment. Then its velocity will be $(-v\sin(\theta), v\cos(\theta), 0)$, and it will experience a Lorentz force

$$\begin{aligned}\Delta\mathbf{F} \;&=\; q\big(-v\sin(\theta),\; v\cos(\theta),\; 0\big)\times\mathbf{B}\big(r\cos(\theta),\; r\sin(\theta),\; 0\big)\\ &=\; \frac{2\pi r I}{N}\big(-\sin(\theta),\; \cos(\theta),\; 0\big)\times\mathbf{B}\big(r\cos(\theta),\; r\sin(\theta),\; 0\big).\end{aligned}$$

The total force on the ring will be the sum of these forces, and it becomes

$$\begin{aligned}\mathbf{F} \;&=\; \sum\frac{2\pi r I}{N}\big(-\sin(\theta),\; \cos(\theta),\; 0\big)\times\mathbf{B}\big(r\cos(\theta),\; r\sin(\theta),\; 0\big)\\ &\underset{N\to\infty}{\to}\; rI\int_0^{2\pi} d\theta\,\big(-\sin(\theta),\; \cos(\theta),\; 0\big)\times\mathbf{B}\big(r\cos(\theta),\; r\sin(\theta),\; 0\big).\end{aligned}$$

We can easily note that

$$\int_0^{2\pi} d\theta\,\big(-\sin(\theta),\; \cos(\theta),\; 0\big)\times\mathbf{B}_{\mathrm{b}} \;=\; 0,$$

so the constant part of the magnetic field is not affecting the possible force at all. The remaining integral is slightly more nontrivial:

$$\begin{aligned}
&\int_0^{2\pi} d\theta\, \big(-\sin(\theta),\ \cos(\theta),\ 0\big) \times \Big(r\cos(\theta)\partial_1\mathbf{B}_{\rm b}\ +\ r\sin(\theta)\partial_2\mathbf{B}_{\rm b}\Big) \\
=\ & -r\Big(\int_0^{2\pi} \sin(\theta)\cos(\theta)d\theta\Big)\big(0,\ -\partial_1(B_{\rm b})_3,\ \partial_1(B_{\rm b})_2\big) \\
&-\ r\Big(\int_0^{2\pi} \big(\sin(\theta)\big)^2 d\theta\Big)\big(0,\ -\partial_2(B_{\rm b})_3,\ \partial_2(B_{\rm b})_2\big) \\
&+\ r\Big(\int_0^{2\pi} \big(\cos(\theta)\big)^2 d\theta\Big)\big(\partial_1(B_{\rm b})_3,\ 0,\ -\partial_1(B_{\rm b})_1\big) \\
&+\ \Big(\int_0^{2\pi} \cos(\theta)\sin(\theta)d\theta\Big)\big(\partial_2(B_{\rm b})_3,\ 0,\ -\partial_2(B_{\rm b})_1\big) \\
=\ & \pi r\big(\partial_1(B_{\rm b})_3,\ \partial_2(B_{\rm b})_3,\ -\partial_1(B_{\rm b})_1 - \partial_2(B_{\rm b})_2\big) \\
=\ & \pi r\big(\nabla(\mathbf{e}_3\cdot\mathbf{B}_{\rm b})\ -\ (\nabla\cdot\mathbf{B}_{\rm b})\mathbf{e}_3\big)
\end{aligned}$$

This means that the force can be expressed as

$$\mathbf{F}\ =\ \nabla\big(\mathbf{m}\cdot\mathbf{B}(\mathbf{x})\big)\ -\ \big(\nabla\cdot\mathbf{B}(\mathbf{x})\big)\mathbf{m}.$$

Since $\nabla\cdot\mathbf{B}(\mathbf{x}) = 0$ according to Maxwell's equations, the formula for force simplifies to

$$\mathbf{F}\ =\ \nabla\big(\mathbf{m}\cdot\mathbf{B}(\mathbf{x})\big).$$

This is Equation (5.69) in Jackson's book [3], Equation (6.3) in Griffiths' book [4], and Equation (11.23) in Purcell's book [5]. Once we have learnt that the force experienced by a small magnetic dipole is $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B}(\mathbf{x}))$, it has become clear why many people believe that there would be an interaction energy $U = -\mathbf{m}\cdot\mathbf{B}(\mathbf{x})$. What's going on is of course that people are believing that the interaction energy $U = -\mathbf{m}\cdot\mathbf{B}(\mathbf{x})$ would be equivalent with the force $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B}(\mathbf{x}))$ that most apparently is valid. Let's think about this. Are these two formulas equivalent? A simple answer that seems reasonable is that these $\mathbf{F}$ and $U$ are equivalent under the assumption that $\mathbf{m}$ remains constant, and that only $\mathbf{x}$ is allowed to vary. In other words, there is mapping of the form $\mathbf{x}\mapsto U(\mathbf{x})$, when $\mathbf{m}$ is assumed to be a constant. This is problematic. If we show the formula $U = -\mathbf{m}\cdot\mathbf{B}(\mathbf{x})$ to people without explaining that here $\mathbf{m}$ must remain as a constant, of course people are going to assume that an energy value $U$ would depend on $\mathbf{m}$ similarly as it depends on $\mathbf{x}$. Now we have a reason to suspect that that is probably wrong. If we allow the vector $\mathbf{m}$ to vary, there is no justification to assume that the corresponding changes in the value of the quantity $-\mathbf{m} \cdot \mathbf{B}(\mathbf{x})$ would represent any changes in any energy. Here we have not derived a valid energy mapping of the form $(\mathbf{m}, \mathbf{x}) \mapsto U(\mathbf{m}, \mathbf{x})$. Similarly, if the mapping $\mathbf{x}' \mapsto \mathbf{B}(\mathbf{x}')$ is allowed to change, there is no justification to assume that the corresponding change in the value of the quantity $-\mathbf{m}\cdot\mathbf{B}(\mathbf{x})$ would represent any change in any energy.

We already know from the basics of electromagnetism that it is not possible to derive the Lorentz force $\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$ out of a one-coordinate potential $U$. We need a four-coordinate vector potential $A^\mu$ for that task. Since the force $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}(\mathbf{x}))$ experienced by a small magnetic dipole comes from a non-trivial Lorentz force, on the face of it attempts to describe this force with a potential $U$ don't look very smart; even if some artificially constructed $U$ eventually can be made somewhat work.

In Chapter 5.7 Jackson [3] gives this explanation:

"*We remark in passing that (5.72) is not the total energy of the magnetic moment in the external field. In bringing the dipole* $\mathbf{m}$ *into its final position in the field, work must be done to keep the current* $\mathbf{J}$*, which produces* $\mathbf{m}$*, constant. Even though the final situation is a steady state, there is a transient period initially in which the relevant fields are time-dependent.*"

If this is correct, doesn't it mean that $U = -\mathbf{m} \cdot \mathbf{B}(\mathbf{x})$ is not working even when only $\mathbf{x}$ is allowed to vary? In other words, there is no situation at all, when this $U$ would function as a valid interaction energy?

## References


[1] https://en.wikipedia.org/wiki/Magnetic_moment

[2] https://en.wikipedia.org/wiki/Biot-Savart_law

[3] *Classical Electrodynamics*, J. D. Jackson, 1962 (1999 3rd edition), John Wiley & Sons, Inc.

[4] *Introduction to Electrodynamics*, D. J. Griffiths, 1989 (2013 4th edition), Pearson Education, Inc.

[5] *Electricity and Magnetism*, E. M. Purcell & D. J. Morin 1963 (2013 3rd edition), Cambridge University Press