Neutrinos and gravitational polarisations

Is there a link?

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This document reworks and deepens a twenty-two-year-old intuition betting that the neutrinos can be indirectly related to gravitational polarisations. Respecting the usual practice, this idea is tested with the variations of the Newtonian fields Γ_N. The quadrupolar tensor [Q] enters the discussion in two different ways: (i) through the calculation of the Jacobian matrix $T_2(o)(\partial_r, \Gamma_N)$ and (ii) in studying the Thirring-Lense effect far from a massive body in rotation. This fact suggests the existence of a link between this Jacobian and the rotations in general. The whirl tensor (syn.: rotational) acting on a generic vector $\mathbf{g} \cong \mathbf{n} \wedge \mathbf{M}$ (where $\mathbf{n}$ is a unitary vector whilst $\mathbf{M}$ is any angular momentum) is then interpreted as a deformation acting on this classical cross product. This approach allows identifying the Jacobian matrix $T_2(o)(\partial_r, \Gamma_N)$ with the kern of decompositions for $\mathbf{rot_r}$ $\mathbf{g}$. It finally extrapolates this idea to any acceleration-field in examining the conditions allowing an identification between any Jacobian matrix $T_2(o)(\partial_r, \Gamma)$ and ordinary Perian matrices. In following this track, some progresses on the road joining the acceleration-fields and the subatomic world are realized.

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Motivations

Despite of numerous efforts, there is still no consensus on the mechanism explaining how the neutrinos gain a mass. As mentioned in [01], the preferred scenario asks about the existence of right-handed neutrinos that cannot be found.

At least three arguments justify this fact: the right-handed neutrinos (i) are sterile (synonym: they don't interact - but why?) or/and (ii) they have a very heavy mass (energy) and it has not yet been possible to detect them in accelerators of particles; (iii) they don't exist at all.

This disappointing situation forces the fundamental research to propose alternative scenarios. They involve the Dirac neutrinos, a variant of the superstring theory, eventually the Majorana neutrinos, a seesaw mechanism, etc.

This document suggests another possibility.

Hypothesis and claim

In increasing the complexity of a discussion, for example in adding more actors (new fields or new particles) and parameters, one does not make it more understandable. Unfortunately, theoretical physics has become accustomed to believing that it can solve puzzles by practicing in this way. Ockham's razor has sometimes been forgotten.

Because (i) the neutrinos masses are extremely small (< 1 eV, [01; p. 3]), and because (ii) the energy of gravitational waves arriving at the surface of the Earth is extremely small too (the LIGO[1] interferometer has measured length differences one thousand times smaller than the size of a proton; i.e.: $\sim 10^{-18}$ m), it ends up being conceivable to think that the fields one is looking for to explain the behaviour of some particles living at this extremely small scale are perhaps only already known fields, for example the gravitational fields or, more precisely, their tiny variations. This is the main intuitive idea assumed in this document.

Caution: *Since scientific teams are yet hunting the right-handed neutrinos, all what follows should only be understood as an alternative and speculative proposition in case the actual quests and alternative scenarios remain unsuccessful.*

Polarisations and rotations

The polarisations are the proof that something happened

It is evident that nothing happens in a uniform context. Conversely, if something did happen somewhere, then it manifests itself through a contrast, a difference, a relief, a wave, etc. A direct application of this philosophical though in physics can be realized in focusing attention on polarisations in general.

The quadrupolar polarisations in electricity

The mathematical tool describing quadrupolar polarisations is well-known [02; § 41, pp. 97-99]. It is a tensor of which the generic formalism can be written like in [02; § 41, p. 98, (41.3)] if it is related to the presence of an electrical charge q :

$$r = \{(x^1)^2 + (x^2)^2 + (x^3)^2\}^{1/2} > 0$$

$$\mathbf{n} = \frac{\mathbf{r}}{r}$$

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[1] Laser Interferometer Gravitational-wave Observatory.

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$$[D_{\alpha\beta}] = q.\,[3.x_\alpha.x_\beta - r^2.\delta_{\alpha\beta}] = -q.\,[r^2.\delta_{\alpha\beta} - 3.x_\alpha.x_\beta] = -q.\,r^2.\,[\delta_{\alpha\beta} - 3.n_\alpha.n_\beta] = -q.\,r^2.\,[Q]$$

The quadrupolar polarisations in presence of a Newtonian field

This tensor can also be related to a distribution of masses [03; § 99, p.354, (99,8)] :

$$[D_{\alpha\beta}] = \int \rho.\,[3.x_\alpha.x_\beta - r^2.\delta_{\alpha\beta}].\,d\tau = -\int \rho.\,r^2.\,[\delta_{\alpha\beta} - 3.n_\alpha.n_\beta].\,d\tau$$

Let consider a Newtonian acceleration field:

$$\boldsymbol{\Gamma}_N = -G.M.\frac{\mathbf{n}}{r^2} \Leftrightarrow \forall\ a = 1, 2, 3:\ \Gamma^a = -G.M.\frac{x^a}{r^3}$$

Then:

$$\forall\ a, b = 1, 2, 3:\ \frac{\partial \Gamma^a}{\partial x^b} = -\frac{G.M}{r^6}.\,(\delta^{ab}.\,r^3 - x^a.\,\frac{\partial r^3}{\partial x^b}) = -\frac{G.M}{r^3}.\,(\delta^{ab} - \frac{3}{r}.\,x^a.\,\frac{\partial r}{\partial x^b}) = -\frac{G.M}{r^3}.\,(\delta^{ab} - \frac{3}{r^2}.\,x^a.\,x^b)$$

Far from the source (which is obviously the case in the empty regions of our universe), the spatial part of the metric is quasi-Galilean (quasi-Euclidean); hence, the subscripts can be lowered without much precaution:

$$T_2(o)(\partial_r, \boldsymbol{\Gamma}_N) \cong [\frac{\partial \Gamma_a}{\partial x_b}] = -\frac{G.M}{r^3}.\,\{Id_3 - 3.\,T_2(\otimes)(\mathbf{n}, \mathbf{n})\} = -\frac{G.M}{r^3}.\,[Q]$$

The quadrupolar polarisations far from a rotating mass

Most interestingly, the quadrupolar tensor also appears in a long exploration calculating the perturbations due to a source (a mass M) in rotation [03; §105, pp. 393-399]. The discussion concludes that a massive source in rotation with angular momentum **M** induces a *Coriolis-like* force at distance r which is characterized by [03; §105, (105,20)]:

$$\boldsymbol{\Omega} \sim \frac{c}{2}.\,\mathbf{rot}_r\,\mathbf{g} = \frac{k}{c^2.r^3}.\,(\mathbf{M} - 3.(\mathbf{M.n}).\mathbf{n})$$

$$\mathbf{g} = \frac{2.k}{c^3.r^2}.\,(\mathbf{n} \wedge \mathbf{M})$$

$$\mathbf{n} = \frac{\mathbf{r}}{r}$$

More than thirty years later, one prefers to describe this result as a "*Thirring-Lense effect*" [04; chapter 30, (30.20) and (30.22)]; or as frame-dragging effect. The existence of this effect has been experimentally verified in 2011 (Gravity Probe B mission).

Proof. Per convenience, let introduce the (3-3) matrix [Q] representing some quadrupolar polarisation:

$$[Q] = [\delta_{\alpha\beta} - 3.n_\alpha.n_\beta] = Id_3 - 3.\,T_2(\otimes)(\mathbf{n}, \mathbf{n})$$

Due to the usual properties of Pythagorean tables built on the classical tensor product:

$$|\Omega> \sim \frac{c}{2}.\,|\mathbf{rot}_r\,\mathbf{g}> = \frac{k}{c^2.r^3}.\,(|\mathbf{M}> - 3.(\mathbf{M.n}).|\mathbf{n}>) = \frac{k}{c^2.r^3}.\,\{Id_3 - \frac{3}{r^2}.T_2(\otimes)(\mathbf{r}, \mathbf{r})\}.|\mathbf{M}> = \frac{k}{c^2.r^3}.\,[Q].|\mathbf{M}>$$

The Jacobian matrix related to an acceleration-field and the rotations

The quadrupolar tensor seems to be useful in a wide variety of circumstances. Moreover, since it appears in a calculation of derivatives of the acceleration-field and in a formula accounting for distant effects of a rotating mass, one is intuitively led to believe in the existence of a link between the Jacobian matrix $T_2(o)(\partial_r, \Gamma)$ and some matrices representing a rotation. Furthermore, the fact that electromagnetic and gravitational phenomenon can manifest themselves with quadrupolar polarisations gives the hope to unify both domains inside a theory.

Accounting for the underlying activity in vacuum

Although one does not expect that empty regions far from the sources of astronomical catastrophes manifest any impressive activities, they exhibit at least one residual activity originating in the initial Big-Bang: the CMB (Cosmological Microwave Background). Therefore, electromagnetic polarisations exist in these regions of the universe. As explained in [a], these polarisations can be described in writing the forces per unit of volume resulting from their existence:

$$|\frac{\partial F}{\partial \tau}>_{Vacuum} = \varepsilon_0.\ T_2(o)(\partial_r, \textbf{E}).\ |\textbf{E}> + (1/\mu_0).\ T_2(o)(\partial_r, \textbf{B}).\ |\textbf{B}>$$

If one is looking for experimental evidence explaining this formalism, one may argue that the electrical part of these forces can be related to a squared Stark-like effect whilst the magnetic part can be related to some Zeeman-like effect. A description of what happens where nothing should exist (i.e.: in vacuum) would be incomplete if it would not include the gravitational polarisations.

A formalism for the gravitational polarisation

"How can the gravitational polarisations be described?"

The universal constant $G = 6,67428.10^{-11}$ has the units: $m^3.kg^{-1}.s^{-1}$. The components of $[\delta_{ij} - 3.n_i.n_j]$ are pure numbers without physical unit. Hence, the Pythagorean table $T_2(o)(\partial_r, \Gamma)$ is proportional to the quadrupolar tensor and it has the units: s^{-1} (per second). In multiplying it on its right side by an acceleration:

$$T_2(o)(\partial_r, \Gamma_N).\ |\ \Gamma_N > = \frac{G^2.M^2}{r^5}.\ [Q].\ |\ \textbf{n} >$$

... one builds a mathematical tool with units: $m.s^{-3}$. Therefore, if a force per unit of volume (with units: $kg.m^{-2}.s^{-2}$) is needed for a discussion involving quadrupolar effects, one must introduce another actor with units $kg.m^{-3}.s$ which is equivalent to introduce a density of matter per unit volume times a time:

$$|\frac{\partial F}{\partial \tau}> = \gamma_0.T_2(o)(\partial_r, \Gamma).\ |\Gamma>$$

$$\gamma_0 \equiv kg.m^{-3}.s$$

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Up to a universal constant related to the time, for example the Planck's time t_P = 5,391247.10^{-44} s, this new actor represents *something which is equivalent to a density of matter per unit of volume*. It will be conventionally denoted ρ^*:

$$|\frac{\partial \boldsymbol{F}}{\partial \tau}> = \gamma_0.\ T_2(o)(\partial_r, \boldsymbol{\Gamma}).\ |\boldsymbol{\Gamma}> = \sqrt{\frac{h.G}{2.\pi.c^5}}.\ \rho^*.\ |\boldsymbol{\Gamma}>$$

This density of matter per unit of volume must be added to the usual ones (if any):

$$|\frac{\partial \boldsymbol{F}}{\partial \tau}>_{\text{Newtonian}} = \rho.\ |\boldsymbol{\Gamma}>$$

This is why it is meaningful to write:

$$\rho^* = \rho_{\text{Total}} - \rho_{\text{Newtonian}} = \delta\rho$$

For reasons that will be explained a little bit later, the writing $\delta\rho$ is preferred.

Variations of the density of matter per unit of volume far from the sources

Following the spirit of Einstein's theory of gravitation, one knows that masses are sources of gravitation fields which modify the geometry around them. These modifications carry a certain amount of energy. There are interesting and ongoing discussions about this topic. One can cite [03; §96, pp. 334-340; in peculiar p.336 (96,8) for a detailed expression of the pseudo-tensor] and [04; chapter 22, pp. 123-124; in peculiar for gravitational waves: chapter 34, p. 191, (34.24)].

In this document:

- ➢ because of the equivalence "mass - energy", one considers that the modifications of gravitational energies are equivalent to variations of the local density of matter per unit of volume $\delta\rho$;
- ➢ because one believes that each kind of sub-atomic particles is characterized by a specific value of $\delta\rho$ which must be the same after any change of frame, one postulates that these densities are correctly described through the roots of the polynomial form:

$$P(\delta\rho) = -|\gamma_0.\ T_2(o)(\partial_r, \boldsymbol{\Gamma}) - t_P.\delta\rho.\ \text{Id}_3|$$

An exhaustive calculation yields (Annex 01):

$$P(\delta\rho) = (t_P.\delta\rho)^3 - (\gamma_0).\ \text{div}\ \boldsymbol{\Gamma}.\ (t_P.\delta\rho)^2 + (\gamma_0)^2.\ £.\ t_P.\delta\rho - (\gamma_0)^3.\ |T_2(o)(\partial_r, \boldsymbol{\Gamma})|$$

This expression opens a debate because a mathematician can always calculate its roots with the Tartaglia-Cardan method [b; § 3, pp. 11-20] and he may eventually find complex values ($\delta\rho \in C$) whilst a physicist is obliged to take care of the experimental reality. As a matter of facts, the density of matter per unit of volume can only be null or real positive ($\rho \in R^+$).

How should the solutions be interpreted?

"How are

- a density of matter per unit of volume,
- a mass
- an energy

... related to each other's?"

The usual relations between these quantities are very simple:

$$\rho \leftrightarrow m = \rho.\tau \leftrightarrow E = \rho.\tau.c^2$$

Since a given non-vanishing volume, τ ($\tau \neq 0$), and the speed of light in vacuum, c ($\sim 3.10^8$ m.s^{-1}), always have real positive values, the sign of a given density ρ gives the sign for a set of masses and for a set of the corresponding energies. Clearly, with this classical and usual understanding of the reality, all ρ, m and E must have either positive or null values. This situation has been recently reenforced by the fact that the anti-hydrogen does not anti-gravitate (e.g.: the results of the ALPHA experiment confirming the weak equivalence principle of general relativity). Hence, all experimental facts impose the logical sentence:

$$\{\tau > 0, m \in R^+, m = \rho.\tau\} \Rightarrow \rho \in R^+$$

The negative and the complex densities of matter ρ cannot exist.

Problem: "How can the negative energies (e.g.: related to anti-particles within the Quantum Chromodynamics Dynamics - alias: QCD) and the complex energies (e.g.: related to transitions between two unstable states like in a Lamb-Rutherford effect) be explained?"

There are fortunately at least two explanations:

1) There is no absolute scale for the energies. A serious discussion about this topic always involves differences between two levels of energy.
2) The volume of a physical object and its density per unit of volume can vary negatively. The Leibniz formula makes this affirmation obvious:

$$\delta m = \delta(\rho.\tau) = \delta\rho.\tau + \rho.\delta\tau$$

The variations of densities far from the source in the Newtonian case

Once more time, the discussion can be illustrated with the help of Newtonian fields. In these circumstances, far from the mass M, at distance r (Annex 02):

$$\text{div}\,\Gamma_N = 0$$

$$\pounds_N = -3.\left(\frac{G.M}{r^3}\right)^2$$

$$|T_2(o)(\partial_r, \Gamma_N)| = 2.(G.M/r^3)^3$$

The polynomial form can be factorized:

$$P(\delta\rho_N) = (t_P.\delta\rho_N - \gamma_0.\frac{G.M}{r^3})^2.\ (t_P.\delta\rho_N + 2.\gamma_0.\frac{G.M}{r^3})$$

There are three solutions (a simple and a double):

$$\sqrt{\frac{h.G}{2.\pi.c^5}}.\ \delta\rho_{N1}(r) = -\gamma_0.\frac{2.G.M}{r^3}$$

$$\sqrt{\frac{h.G}{2.\pi.c^5}}.\ \delta\rho_{N2}(r) = \sqrt{\frac{h.G}{2.\pi.c^5}}.\ \delta\rho_{N3}(r) = \gamma_0.\frac{G.M}{r^3}$$

Within this theory, in vacuum at distance r from a source M:

- five universal constants (h, π, c, γ_0 and G),
- the characteristics of the source (for example its mass M) and
- the distance r to the source

... are fixing the intensity of the three possible variations $\delta\rho_{Na}$ (a = 1, 2, 3).

The sign of γ_0 is unknow. Therefore, either one or two variations may eventually be negative. This fact illustrates the utility of former paragraph titled "How should the roots be interpreted?" Assuming that the equation of state for a simple vacuum is:

$$\rho(r).\ c^2 + p(r) = 0$$

... one obtains:

$$\delta\rho(r).\ c^2 + \delta p(r) = 0$$

This formulation allows negative variations of ρ.

The roots and the cyclic group C6

If, for each distance r, one defines per convention:

$$\gamma_0.\ \sqrt{\frac{2.\pi.c^5.G}{h}}.\ \frac{M}{r^3} = \text{½ unit of variation due to the mass M at distance r}$$

Then, in this theory, two roots have an intensity of ½ unit whilst the third one has an intensity of minus one unit:

$$-2.\gamma_0.\ \sqrt{\frac{2.\pi.c^5.G}{h}}.\ \frac{M}{r^3} = \text{-1 unit}$$

As a matter of facts, the sum of the three roots is null:

$$\delta\rho_{N1}(r) + \delta\rho_{N2}(r) + \delta\rho_{N3}(r) = 0$$

The collective action of the three possible variations is null. This fact vaguely evokes the existence of a link with the complex cubic roots of 1 {1, j, j²} or of -1 {-1, - j, - j²}. "How can this symmetry be explained or justified in a context discussing about Newtonian fields?"

The whirl tensor as tool deforming the classical cross products

This discussion started with a classical Newtonian field of acceleration, like the one which is acting at the surface of the Earth. Although one now knows that our planet is in rotation around one axis, the usual expression of the Newtonian field does neither indicate the existence of this rotation nor give clear information about it. Partial derivations allow the calculation of the matrix $T_2(o)(\partial_r, \Gamma_N)$ and it is proportional to the quadrupolar tensor [Q] whatever the source is doing (spinning or not).

Amazingly, the discussion [03; §105, pp. 393-399] about masses in rotation introduces this tensor too. This fact suggests that partial derivatives act collectively like a rotation does. As a matter of mathematical facts, the presence of the quadrupole tensor [Q] in Ω is resulting from the action of a whirl tensor ($\mathbf{rot}_r$) on the classical cross product (recall):

$$\mathbf{g} = \frac{2.k}{c^2.r^2}. (\mathbf{n} \wedge \mathbf{M}) \to |\mathbf{g}> \to |\mathbf{rot}_r \, \mathbf{g}> = \frac{2.k}{c^3.r^3}. [Q].| \, \mathbf{M} > = \frac{2}{c}. | \, \Omega >$$

Concretely, one gets the sensation that the whirl tensor ($\mathbf{rot}_r$) acts like a tool deforming the classical cross product $\mathbf{g}$. With different words, this tensor has the same effect than a deforming matrix [A]:

$$\mathbf{g} = \frac{2.k}{c^2.r^2}. (\mathbf{n} \wedge \mathbf{M}) \to \frac{2.k}{c^2.r^2}. [\mathbf{n}, \mathbf{M}]_{[A]}$$

This interpretation transports the discussion into the Theory of the (E) Question, alias TEQ [c]. In a three-dimensional space, any deformation due to an antisymmetric cube A is represented through a product of two matrices:

- $[A]^t$ which is the transposed of matrix [A] containing the non-vanishing knots (synonym: components) of some antisymmetric cube A.

- [J] which is representing the generator of the cyclic group C_6. Its presence offers at least a formal link with the underlying symmetries exhibited by the roots of the polynomial form $P(\delta\rho_N)$ in a Newtonian context (see previous paragraph).

A direct application of the inner logic of the TEQ yields:

$$| \, \mathbf{g} > = \frac{2.k}{c^2.r^2}. | \, \mathbf{n} \wedge \mathbf{M} > \xrightarrow{\cdots[A]} \frac{2.k}{c^2.r^2}. |[\mathbf{n}, \mathbf{M}]_{[A]} > = \frac{2.k}{c^2.r^2}. \{[A]^t.[J]\}. | \, \mathbf{n} \wedge \mathbf{M} > = \{[A]^t.[J]\}.| \, \mathbf{g} >$$

Within the classical field theory (CFT), it has been proven that the action of the whirl tensor $\mathbf{rot}_r$ can be more precisely described with:

$$| \, \mathbf{g} > \xrightarrow{\frac{c}{2}. \, \mathbf{rot}_r} | \, \Omega > \sim \frac{c}{2}. | \, \mathbf{rot}_r \, \mathbf{g} > + | \, \varepsilon > = \frac{k}{c^2.r^3}.[Q]. | \, \mathbf{M} > + | \, \varepsilon > \text{ with } \varepsilon \sim \mathbf{0}$$

Both logics can be confronted in writing (hypothesis 1):

$$| \, \mathbf{rot}_r \, (\mathbf{n} \wedge \mathbf{M}) > = |[\mathbf{n}, \mathbf{M}]_{[A]} >$$

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One can go a step further in recalling that the TEQ is also studying the decomposition of deformed cross products:

$$| \Omega > \sim \frac{c}{2}. | \, \text{rot}_r \, \mathbf{g} >= \frac{k}{c.r^2}. | \, [\mathbf{n}, \mathbf{M}]_{[A]} > = \frac{k}{c.r^2}.\{[P]. | \, \mathbf{M} > + | \, \mathbf{z} >\} = \frac{k}{c^2.r^3}.[Q]. | \, \mathbf{M} > + | \, \mathbf{\varepsilon} >$$

The calculations which have been realized inside the classical field theory (short: CFT) [03; §105, pp. 393-399] imposes a pseudo-trivial decomposition ($\mathbf{z} \sim \mathbf{0}$ because $\mathbf{\varepsilon} \sim \mathbf{0}$) for Ω. As immediate consequence, the main part of the decomposition, i.e.: the matrix [P], is pseudo-trivial and coincides (up to a factor) with the quadrupolar tensor [Q]:

$$[P] = \frac{1}{c.r}.[Q]$$

Furthermore, since in general within the TEQ:

$$[P] = \{[A]^t.[J]\}.[N]$$

... if the effective deforming matrix $\{[A]^t.[J]\}$ is not degenerated, the kern of the decomposition writes:

$$[N] = \frac{1}{c.r}. \, [J]^5. \, \{[A]^t\}^{-1}. \, \{Id_3 - T_2(\otimes)(\mathbf{n}, \mathbf{n})\}$$

As explained in [d], the cross products (deformed or not) can have several pseudo-trivial decompositions. They all are accompanied with the obligatory constraint:

$$|_{[J]}\Phi(\mathbf{n}) - [N]| = 0$$

Deepening the analysis

Former paragraphs have:

- suggested the existence of a link between the roots of $P(\delta\rho)$ and the cyclic group C_6 when (i) the acceleration-field is Newtonian and (ii) the roots are in some way the eigenvalues of the quadrupolar tensor associated with this field.
- proven that the cyclic group C_6 can be introduced in a discussion involving the quadrupolar tensor associated to Newtonian fields far from a mass M in rotation if one interprets the whirl tensor as a tool deforming the classical cross products.

"Do these facts, in peculiar does the presence of [J] in the expression of [N], offer a plausible explanation for the presence of C_6 symmetries implicitly contained in the roots of $P(\delta\rho)$ for the Newtonian case?"

Let analyse the theoretical context deeper and state that:

- The calculation of $P(\delta\rho)$ has been done for an observer at distance r of the source of a given Newtonian field; hence, explaining the nullity of $\text{div}_r \, \Gamma$. For an observer who stays inside a spheric distribution of matter, even if its density is very small, one must repeat the calculation with the relation:

$$\text{div}_r \, \Gamma = 4.\pi.G.\rho_N$$

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This constraint impacts the resolution of $P(\delta\rho) = 0$ because the Tartaglia-Cardan method [b; § 3, pp. 11-20] starts in changing the variable:

$$(t_P.\delta\rho) \rightarrow z = (t_P.\delta\rho) - \frac{\gamma_0}{3}. \, div_r \, \Gamma = (t_P.\delta\rho) - \frac{4.\pi.G.\gamma_0}{3}.\rho_N$$

The values of "z" are now the roots one is looking for. One luckily remarks that they coincide with the $\delta\rho$ when $\rho_N = 0$. The variations $\delta\rho$ are now given through the relation:

$$(t_P.\delta\rho) = z + \frac{4.\pi.G.\gamma_0}{3}. \, \rho_N$$

One may ask if the symmetries which have been noted when $\rho_N = 0$ yet exist when $\rho_N \neq 0$. The answer is a little bit subtle because the roots have a specific formalism. One can roughly write them (the details can be verified in [b; § 3, pp. 11-20]):

$$z_0 = Z_- + Z_+$$

$$z_1 = j. \, Z_- + j^2. \, Z_+$$

$$z_2 = j^2. \, Z_- + j. \, Z_+$$

Therefore, one easily states that the sum of the roots of $P(z)$ vanishes:

$$z_0 + z_1 + z_2 = 0$$

But this result only means that:

$$t_P. \, \{\delta\rho_1(r) + \delta\rho_2(r) + \delta\rho_3(r)\} = 4.\pi.G.\gamma_0.\rho_N$$

The sum of the variations $\delta\rho_a(r)$ for a = 1, 2, 3 (i) does no longer systematically vanish and (ii) is only proportional to the density ρ_N. Quite more interesting, one can also rewrite this relation as:

$$\rho_N = \frac{t_P}{4.\pi.G.\gamma_0}. \, \{\delta\rho_1(r) + \delta\rho_2(r) + \delta\rho_3(r)\}$$

Important: Any Newtonian density of matter per unit of volume can always be decomposed in a sum of three terms, each of them being proportional to one of the variations $\delta\rho_a(r)$ for a = 1, 2, 3. This mathematical fact suggests that the Newtonian density can be seen as the result of the simultaneous presence of three inseparable particles.

- The triality doesn't exist because the field is Newtonian. It appears for any acceleration-field Γ. The Tartaglia-Cardan method starts with:

$$(t_P.\delta\rho) \rightarrow z = (t_P.\delta\rho) - \frac{\gamma_0}{3}. \, div_r \, \Gamma$$

The structure of the roots remains unchanged, imposing again:

$$z_0 + z_1 + z_2 = 0$$

This result now means that the sum of the three variations $\delta\rho_a(r)$ for a = 1, 2, 3, are proportional to the divergence of the acceleration field at hand:

$$\delta\rho_1(r) + \delta\rho_2(r) + \delta\rho_3(r) = \frac{\gamma_0}{t_P}.\,\text{div}_r\,\boldsymbol{\Gamma}$$

At a theoretical level, one may now envisage directional variations of the densities in writing:

$$\forall\ a = 1, 2, 3:\quad \delta\rho_a(r) = \frac{\gamma_0}{t_P}.\frac{\partial\Gamma^a}{\partial x^a}$$

One concludes this deepening of the premises of the theory in saying that the triality of the roots is essentially due to the hypothesis interpreting the variations of the densities (of matter per unit of volume) as the eigenvalues of the Pythagorean matrix $T_2(o)(\partial_r, \boldsymbol{\Gamma})$, whatever the acceleration-field $\boldsymbol{\Gamma}$ is (Newtonian, post-Newtonian, anyone).

Perian matrices and masses: a promising introduction

When, for the pedagogy, the discussion focuses only on Newtonian fields in a quasi-Euclidean context (i.e.: [A] = [J]), the constraint resulting from the interpretation of the whirl tensor within the TQE is:

$$\left|-\frac{1}{c.r}.\text{Id}_3 + \frac{3}{c.r}.T_2(\otimes)(\mathbf{n}, \mathbf{n}) + _{[J]}\Phi(\mathbf{n})\right| = 0$$

It introduces the degenerated Perian matrix:

$$[M(\mathbf{a}, \mathbf{n})] = -\frac{1}{c.r}.\text{Id}_3 + \frac{3}{c.r}.T_2(\otimes)(\mathbf{n}, \mathbf{n}) + _{[J]}\Phi(\mathbf{n})$$

Its first argument (i.e.: the vector **a**) has the components:

$$a_1 = -\frac{1}{c.r},\ a_2 = \frac{3}{c.r},\ a_3 = 1$$

The vanishing determinant of this Perian matrix can also be rewritten as:

$$|M(\mathbf{a}, \mathbf{n})| = (a_1 + a_2).(a_1{}^2 + a_3{}^2) = \frac{2}{c.r}.\left(\frac{1}{c^2.r^2} + 1\right) = 0$$

This reformulation confirms that this discussion effectively concerns what occurs far from the source M, at $r \to \infty$.

At this stage, one must recall the existence of a procedure allowing the calculation of masses with the help of Perian matrices (not all; see an introduction and details in [b; § 4.7]) and proving the importance of this kind of matrices.

This statement induces a new question: "Do the masses obtained with the procedure which has been recalled in [b; § 4.7] have any connection with the variations of density one is speaking about in this document?"

Generalization: an essay

The way of thinking

The first part of this discussion lies on the decompositions of the deformations of **g** when the acceleration field is Newtonian. In such circumstances, the Jacobian/Pythagorean matrix $T_2(o)(\partial_r, \Gamma)$ is proportional to the quadrupolar tensor.

$$\Gamma = - G.M.\frac{\mathbf{n}}{r^2} \Rightarrow T_2(o)(\partial_r, \Gamma) = - \frac{G.M}{r^3}.[Q]$$

This result is quasi-certainly false when the acceleration field is no longer Newtonian.

Nevertheless, one may imagine or bet that the matrix $T_2(o)(\partial_r, \Gamma)$ remains a crucial actor in a larger theory managing any type of acceleration-field. With this idea in mind, the TEQ allows a formal generalization of this discussion in that sense that the logical sentence which has been involved in the discussion when the acceleration-field is Newtonian:

$$|\frac{c}{2}. \mathbf{g} = \frac{k}{c^2.r^2}.(\mathbf{n} \wedge \mathbf{M}) > \rightarrow | \Omega > = \frac{c}{2}.|\mathbf{rot_r\, g} > = \frac{k}{c^2.r^3}.[Q].|\mathbf{M}>$$

… can eventually be extrapolated in writing (hypothesis 1: the whirl tensor deforms any classical cross product):

$$\Omega = \frac{c}{2}. \mathbf{rot_r\, g} = \frac{k}{c^2.r^2}.[\mathbf{n}, \mathbf{M}]_{[A]}$$

With (fundamental relation connecting a deformed cross product and a classical one):

$$|\frac{c}{2}. \mathbf{g} = \frac{k}{c^2.r^2}. (\mathbf{n} \wedge \mathbf{M}) > \xrightarrow{\cdots [A]} \frac{k}{c^2.r^2}. |[\mathbf{n}, \mathbf{M}]_{[A]} > = \frac{k}{c^2.r^2}. \{[A]^t.[J]\}. |\mathbf{n} \wedge \mathbf{M}>$$

And (hypothesis 2: the Pythagorean matrix $T_2(o)(\partial_r, \Gamma)$ contains information concerning the Thirring-Lense effect and related to the concept of rotation):

$$|\frac{c}{2}. \mathbf{g} = \frac{k}{c^2.r^2}. (\mathbf{n} \wedge \mathbf{M}) > \rightarrow | \Omega > = \frac{c}{2}. |\mathbf{rot_r\, g}> \sim T_2(o)(\partial_r, \Gamma).|\mathbf{M}>$$

The consequence of these hypotheses is:

$$\frac{k}{c^2.r^2}. \{[A]^t.[J]\}. |\mathbf{n} \wedge \mathbf{M}> \sim T_2(o)(\partial_r, \Gamma).|\mathbf{M}>$$

Hence, when the effective deforming matrix is not degenerated:

$$|\mathbf{n} \wedge \mathbf{M}> = K. \{[A]^t. [J]\}^{-1}.T_2(o)(\partial_r, \Gamma).|\mathbf{M}>$$

The classical cross product $\mathbf{n} \wedge \mathbf{M}$ is pseudo-trivially decomposed and there is an obligatory constraint associated with this situation:

$$|_{[J]}\Phi(\mathbf{n}) - K. \{[A]^t.[J]\}^{-1}.T_2(o)(\partial_r, \Gamma)| = 0$$

Within the TQE, any kern [N] is the sum of a symmetric matrix and of an axial rotation matrix whatever the polynomial $\Lambda(\mathbf{n})$ associated with a decomposition of a given deformation of **g** is (i.e.: degenerated or not [c; p.31]).

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One may symbolically write:

$$[N] = \tfrac{1}{2}. [Hess_n \Lambda(\mathbf{n})] - |A|._{[J]}\Phi(_\wedge\mathbf{s}), |A| = \pm 1$$

Here, one can affirm that $K.T_2(o)(\partial_r, \Gamma)$ is the pseudo-trivial main part of the decomposition of $\frac{k}{c^2.r^2}.[\mathbf{n}, \mathbf{M}]_{[A]}$; hence, one must write:

$$\{[A]^t.[J]\}.[N] = K.T_2(o)(\partial_r, \Gamma)$$

Therefore, in a pre-Euclidean context (i.e.: when $[A] = [J]$, $|A| = -1$):

$$\tfrac{1}{2}. [Hess_n \Lambda(\mathbf{n})] + _{[J]}\Phi(_\wedge\mathbf{s}) = [N] = K. T_2(o)(\partial_r, \Gamma)$$

$$|_{[J]}\Phi(\mathbf{n}) - K. T_2(o)(\partial_r, \Gamma)| = |-\tfrac{1}{2}. [Hess_n \Lambda(\mathbf{n})] + _{[J]}\Phi(\mathbf{n} -_\wedge\mathbf{s})| = 0$$

Before going further, one should remark that this way of thinking imposes two relations:

$$[Hess_n \Lambda(\mathbf{n})] = K. \{T_2(o)(\partial_r, \Gamma) + T^t_2(o)(\partial_r, \Gamma)\}$$

$$_\wedge\mathbf{s} = \frac{K}{2}. \mathbf{rot}_r \Gamma$$

A confrontation with the Newtonian case yields:

$$\tfrac{1}{2}. [Hess_n \Lambda(\mathbf{n})] = \frac{G.M}{r^3}. [Q], \; _\wedge\mathbf{s} = \mathbf{0}, \; |-\frac{G.M}{r^3}. [Q] + _{[J]}\Phi(\mathbf{n})| = 0$$

This result is coherent with what has already been obtained previously in a similar context.

Definition: Kern-compatible Perian matrix

A kern-compatible Perian matrix is a Perian matrix of which the symmetric part can be identified with at least one Hessian matrix involved in the kern of some decomposition.

Example for the pedagogy

In the exploration at hand, a Perian matrix is involved in the decomposition of $\mathbf{g} \cong \mathbf{n} \wedge \mathbf{M}$; it is kern-compatible each time one can meaningfully write:

$$\alpha. Id_3 + \beta. T_2(\otimes)(\mathbf{n}, \mathbf{n}) = \tfrac{1}{2}. [Hess_n \Lambda(\mathbf{n})]$$

At this stage, one may recall the existence of two classes of kerns. The class I is related to non-degenerated polynomials $\Lambda(\mathbf{n})$. The class II is related to degenerated polynomials $\Lambda(\mathbf{n})$. The kerns in class II are in general Pythagorean tables which are built with the classical (not deformed) tensor product: $\otimes$. They involve a pair of vectors, e.g.: $[N] = T_2(\otimes)(\mathbf{n}, \mathbf{m})$. In the case at hand, kerns in the class II can be envisaged for pairs $(\mathbf{n}, \mathbf{m} = \mathbf{n})$ when $(\alpha, \beta) = (0, 1)$.

The Jacobian matrix $T_2(o)(\partial_r, \Gamma)$ as Perian matrix

In a pre-Euclidean context ([A] = [J]), one can accept to work with a generic Jacobian matrix $T_2(o)(\partial_r, \Gamma)$ which is a Perian matrix (materialisation of hypothesis 2; starting point):

$$T_2(o)(\partial_r, \Gamma) = \alpha(\Gamma) . \, Id_3 + \beta(\Gamma) . \, T_2(\otimes)(\mathbf{n}, \mathbf{n}) + \chi(\Gamma) ._{[J]}\Phi(\mathbf{rot_r}\, \Gamma)$$

… in adding the conditions:

$$\lim_{\Gamma \to \Gamma_{(Newton)}} \alpha(\Gamma) = -\left(\frac{G.M}{r^3}\right), \quad \lim_{\Gamma \to \Gamma_{(Newton)}} \beta(\Gamma) = 3.\left(\frac{G.M}{r^3}\right)$$

… because one already knows that $\mathbf{rot_r}\, \Gamma_N = 0$. This hypothesis is associated with an exigence which is symbolized through the relation:

$$|(t_P.\delta\rho) . \, Id_3 - \gamma_0 . T_2(o)(\partial_r, \Gamma)| = P(\delta\rho) = |(t_P.\delta\rho - \gamma_0.\alpha) . \, Id_3 - \gamma_0.\beta . \, T_2(\otimes)(\mathbf{n}, \mathbf{n}) - \gamma_0.\chi ._{[J]}\Phi(\mathbf{rot_r}\, \Gamma)|$$

Or equivalently, see Annex 01 below and [b; annex 5.1], through the calculations:

$$(t_P.\delta\rho)^3 - \gamma_0 . \, div\, \Gamma . (t_P.\delta\rho)^2 + (\gamma_0)^2 . \, £ . (t_P.\delta\rho) - (\gamma_0)^3 . \, |T_2(o)(\partial_r, \Gamma)|$$

$$=$$

$$(\alpha')^3 + (\alpha')^2 . \, \beta' . \, ||\mathbf{n}||^2 + (\chi')^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . (\alpha') + \beta' . (\chi')^2 . (\mathbf{n} . \mathbf{rot_r}\, \Gamma)^2$$

$$=^2$$

$$(t_P.\delta\rho - \gamma_0.\alpha)^3 - \gamma_0.\beta . (t_P.\delta\rho - \gamma_0.\alpha)^2 + (\gamma_0)^2.\chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . (t_P.\delta\rho - \gamma_0.\alpha) - (\gamma_0)^3.\beta . \, \chi^2 . (\mathbf{n} . \mathbf{rot_r}\, \Gamma)^2$$

$$=$$

$$(t_P.\delta\rho - \gamma_0 . \, \alpha)^3 - \gamma_0 . \, \beta . (t_P.\delta\rho - \gamma_0 . \, \alpha)^2 + (\gamma_0)^2 . \, \chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . (t_P.\delta\rho - \gamma_0 . \, \alpha)$$

$$- (\gamma_0)^3 . \, \beta . \, \chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . \cos^2(\mathbf{n}, \mathbf{rot_r}\, \Gamma)$$

$$=$$

$$(t_P.\delta\rho)^3 - 3 . \, \gamma_0 . \, \alpha . (t_P.\delta\rho)^2 + 3 . (\gamma_0)^2 . \, \alpha^2 . (t_P.\delta\rho) - (\gamma_0)^3 . \, \alpha^3$$

$$- \gamma_0 . \, \beta . (t_P.\delta\rho)^2 + 2 . (\gamma_0)^2.\alpha . \, \beta . (t_P.\delta\rho) - (\gamma_0)^3 . \, \beta . \, \alpha^2$$

$$+ (\gamma_0)^2 . \, \chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . (t_P.\delta\rho) - (\gamma_0)^3 . \, \chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . \, \alpha$$

$$- (\gamma_0)^3 . \, \beta . \, \chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2 . \cos^2(\mathbf{n}, \mathbf{rot_r}\, \Gamma)$$

$$=$$

$$(t_P.\delta\rho)^3$$

$$- \gamma_0 . (3.\alpha + \beta) . (t_P.\delta\rho)^2$$

$$+ (\gamma_0)^2 . (3 . \, \alpha^2 + 2 . \, \alpha . \, \beta + \chi^2 . \, ||\mathbf{rot_r}\, \Gamma||^2) . (t_P.\delta\rho)$$

---

[2] Recall that $||\mathbf{n}||^2 = 1$.

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$$- (\gamma_0)^3. \{\alpha^3 + \beta. \alpha^2 + \chi^2. ||\mathbf{rot_r}\,\Gamma||^2. \alpha + \beta. \chi^2. ||\,\mathbf{rot_r}\,\Gamma||^2. \cos^2(\mathbf{n}, \mathbf{rot_r}\,\Gamma)\}$$

The coherence imposes three identifications:

$$\text{div}\,\Gamma = 3. \alpha + \beta$$

$$\pounds = 3. \alpha^2 + 2. \alpha. \beta + \chi^2. ||\mathbf{rot_r}\,\Gamma||^2$$

$$|T_2(o)(\partial_r, \Gamma)| = \alpha^3 + \beta. \alpha^2 + \chi^2. ||\mathbf{rot_r}\,\Gamma||^2. \alpha + \beta. \chi^2. ||\,\mathbf{rot_r}\,\Gamma||^2. \cos^2(\mathbf{n}, \mathbf{rot_r}\,\Gamma)$$

Testing the hypothesis with the Newtonian fields

To test the solidity of the extrapolation, let consider a Newtonian field. The hypothesis which has been proposed previously furnishes three identifications[3]:

$$0 = 3. \alpha + \beta$$

$$- 3. \left(\frac{G.M}{r^3}\right)^2 = 3. \alpha^2 + 2. \alpha. \beta$$

$$2. \left(\frac{G.M}{r^3}\right)^3 = \alpha^3 + \beta. \alpha^2$$

The first one is encouraging and coherent with what is already known concerning the Newtonian limits. In injecting the first relation into the second and into the third one, one easily recovers:

$$\alpha(\Gamma_N) = - \frac{G.M}{r^3}$$

... and, after that:

$$\beta(\Gamma_N) = 3. \left(\frac{G.M}{r^3}\right)$$

One gets the expected results.

The allowed acceleration-fields with $\mathbf{rot_r}\,\Gamma = 0$

The three relations ensuring the coherence of the hypothesis automatically induce a domain of definition for the acceleration-fields Γ. Let illustrate this affirmation with an example concerning all acceleration-fields with a vanishing whirl tensor:

$$\mathbf{rot_r}\,\Gamma = 0$$

This subset contains all Newtonian fields and some others. They are characterized through three relations:

$$\text{div}_r\,\Gamma = 3. \alpha + \beta \Rightarrow \beta = (\text{div}_r\,\Gamma - 3. \alpha)$$

$$\Downarrow$$

---

[3] $\forall$ a, b = 1, 2, 3: $\frac{\partial \Gamma^a}{\partial x^b} = \frac{G.M}{r^3}. (\delta^{ab} - \frac{3}{r^2}. x^a. x^b) \Rightarrow \frac{\partial \Gamma^a}{\partial x^a} = \frac{G.M}{r^3}. \{1 - 3.\left(\frac{x^a}{r}\right)^2\} \Rightarrow \text{div}\,\Gamma_N = 0$

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$$£ = 3.\,\alpha^2 + 2.\,\alpha.\,\beta = 3.\,\alpha^2 + 2.\,\alpha.\,(\text{div}_r\,\Gamma - 3.\,\alpha) = \alpha.\,(2.\,\text{div}_r\,\Gamma - 3.\,\alpha)$$

$$|T_2(o)(\partial_r, \Gamma)| = \alpha^3 + \beta.\,\alpha^2 = \alpha^2.\,(\alpha + \text{div}_r\,\Gamma - 3.\,\alpha) = \alpha^2.\,(\text{div}_r\,\Gamma - 2.\,\alpha)$$

As long as the coefficient $\alpha(\Gamma)$ doesn't vanish:

$$\alpha(\Gamma) \neq 0:$$

$$\frac{2.£}{\alpha(\Gamma)} - \frac{3.|T_2(o)(\partial_r,\Gamma)|}{\alpha^2(\Gamma)} = \text{div}_r\,\Gamma$$

$$\frac{£}{\alpha(\Gamma)} - \frac{2.|T_2(o)(\partial_r,\Gamma)|}{\alpha^2(\Gamma)} = \alpha(\Gamma)$$

One can isolate $\alpha(\Gamma)$ with the help of the first relation:

$$\text{div}_r\,\Gamma.\,\alpha^2(\Gamma) - 2.\,£.\,\alpha(\Gamma) + 3.\,|T_2(o)(\partial_r, \Gamma)| = 0$$

Concretely:

- When $\text{div}_r\,\Gamma = 0$ and $£ \neq 0$:

$$\alpha(\Gamma) = \frac{3}{2}.\,\frac{|T_2(o)(\partial_r,\Gamma)|}{£}$$

The generic formalism for the Jacobian matrices in this category is:

$$T_2(o)(\partial_r, \Gamma) = \frac{3}{2}.\,\frac{|T_2(o)(\partial_r,\Gamma)|}{£}.\,[Q]$$

The value of $\alpha(\Gamma)$ must be reinjected into the second relation which can be rewritten as:

$$\alpha^3(\Gamma) - £.\,\alpha(\Gamma) + 2.\,|T_2(o)(\partial_r, \Gamma)| = 0$$

Hence, here, the domain of definition is given through the relation:

$$(\frac{3}{2}.\,\frac{|T_2(o)(\partial_r,\Gamma)|}{£})^3 + \tfrac{1}{2}.\,|T_2(o)(\partial_r, \Gamma)| = 0$$

It contains two sub-domains:

➢ The first one contains only one element, the null-matrix:

$$|T_2(o)(\partial_r, \Gamma)| = 0 \Rightarrow T_2(o)(\partial_r, \Gamma) = [0]$$

➢ The second one contains Jacobian matrices which are proportional to the quadrupolar tensor:

$$27.\,|T_2(o)(\partial_r, \Gamma)|^2 + 4.\,£^3 = 0 \Rightarrow T_2(o)(\partial_r, \Gamma) = \pm i.\,\sqrt{\frac{£}{3}}.\,[Q]$$

Within this theory, the presence of i (such that i² + 1 = 0) in a factor will not systematically discard it if the result is real. The Newtonian case illustrates perfectly this decision.

- When $\text{div}_r \, \Gamma \neq 0$:

$$\alpha_\pm(\Gamma) = \frac{\pounds \pm \sqrt{\pounds^2 - 3.\text{div}_r\Gamma.|T_2(o)(\partial_r,\Gamma)|}}{\text{div}_r\Gamma}$$

The calculations are a little bit more complicated, but/and they always furnish three domains of definition for which the hypothesis is valid. Let write for convenience:

$$\Delta = \pounds^2 - 3.\,\text{div}\,\Gamma.\,|T_2(o)(\partial_r, \Gamma)|$$

➢ When $\Delta = 0$:

$$\pounds^2 = 3.\,\text{div}_r\,\Gamma.\,|T_2(o)(\partial_r, \Gamma)|$$

$$\alpha(\Gamma) = \frac{\pounds}{\text{div}_r\Gamma} = \sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}}$$

$$T_2(o)(\partial_r, \Gamma)$$

$$=$$

$$\sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}}.\,\text{Id}_3 + (\text{div}_r\,\Gamma - 3.\sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}}).\,T_2(\otimes)(\mathbf{n}, \mathbf{n})$$

$$=$$

$$\sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}}.\,[Q] + \text{div}_r\,\Gamma.\,T_2(\otimes)(\mathbf{n}, \mathbf{n})$$

The reinjection of $\alpha(\Gamma)$ into the second relation of coherence is precising the three domains of validity for this case:

$$(\sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}})^3 \pm 3.\,|T_2(o)(\partial_r, \Gamma)| + 2.\,|T_2(o)(\partial_r, \Gamma)| = 0$$

- ○ The Jacobian matrix is degenerated:

$$|T_2(o)(\partial_r, \Gamma)| = 0 \Rightarrow T_2(o)(\partial_r, \Gamma) = \text{div}_r\,\Gamma.\,T_2(\otimes)(\mathbf{n}, \mathbf{n})$$

- ○ ...

$$\frac{1}{\text{div}_r\Gamma}.\sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}} + 5 = 0 \Rightarrow T_2(o)(\partial_r, \Gamma) = \text{div}_r\,\Gamma.\{- 5.\,[Q] + T_2(\otimes)(\mathbf{n}, \mathbf{n})\}$$

- ○ ...

$$\frac{1}{\text{div}_r\Gamma}.\sqrt{\frac{3.|T_2(o)(\partial_r,\Gamma)|}{\text{div}_r\Gamma}} - 1 = 0 \Rightarrow T_2(o)(\partial_r, \Gamma) = \text{div}_r\,\Gamma.\{[Q] + T_2(\otimes)(\mathbf{n}, \mathbf{n})\}$$

➢ When $\Delta > 0$. The details of this case will be examined later.
➢ When $\Delta < 0$. The details of this case will be examined later.

The premisses of an interpretation

All situations characterized by a non-vanishing divergence are potentially interesting because in the "interior Newtonian case" the divergence is related to an eventually non-vanishing density of matter per unit of volume; hence, … to the presence of a particle.

Since only a few number of situations exists (i.e.: two when the divergence is null, three when the divergence is not null whilst $\Delta =0$, etc.), one gets the strong temptation to associate each domain with a given type of particles.

The plausibility of this interpretation is reenforced since one knows that some Perian matrices can be treated as anti-commutative, [e], (resp. commutative, [f]) operators.

The Newtonian-like fields and their strange properties

Before going further on the road attempting to realize a junction between the variations of acceleration-fields and particles, I want to make a remark concerning the polynomial form $P(\delta\rho)$.

Definition: Newtonian-like field

A Newtonian-like field is characterized through the relation:

$$t_P . \, \delta\rho = \gamma_0 . \, \mathrm{div}\,\Gamma$$

Due to the information which has been obtained in § Deepening the analysis (in fact: in deepening the consequences of hypothesis 1), the variation $\delta\rho$ can always be decomposed in a sum containing the three roots of $P(\delta\rho)$:

$$\delta\rho_1(r) + \delta\rho_2(r) + \delta\rho_3(r) = \frac{\gamma_0}{t_P} . \, \mathrm{div}_r\,\Gamma$$

The polynomial form P($\delta\rho$) when the acceleration-field is Newtonian-like

The Newtonian-like fields reduce any polynomial form $P(\delta\rho)$ to:

$$P_{\text{N-like}}(\delta\rho) = (\gamma_0)^2 . \{£. \, \delta\rho - \gamma_0 . \, |T_2(o)(\partial_r, \Gamma)|\} = (\gamma_0)^3 . \{£. \, \mathrm{div}_r\,\Gamma - |T_2(o)(\partial_r, \Gamma)|\}$$

The roots of these reduced polynomials coincide then with a factorization of the determinant of the Jacobian matrix:

$$|T_2(o)(\partial_r, \Gamma)| = £. \, \mathrm{div}_r\,\Gamma$$

Let now examine the consequence of hypothesis 2:

$$£. \, \mathrm{div}_r\,\Gamma$$

$$=$$

$$(3. \, \alpha^2 + 2. \, \alpha. \, \beta + \chi^2. \, ||\mathbf{rot}_r\,\Gamma||^2).(3. \, \alpha + \beta)$$

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$$=$$

$$9.\, \alpha^3 + 9.\, \alpha^2.\, \beta + (2.\, \beta^2 + 3.\, \chi^2.\, ||\mathbf{rot_r\, \Gamma}||^2).\, \alpha + \chi^2.\, ||\mathbf{rot_r\, \Gamma}||^2.\, \beta$$

$$=$$

$$\alpha^3 + \beta.\, \alpha^2 + \chi^2.\, ||\mathbf{rot_r\, \Gamma}||^2.\, \alpha + \beta.\, \chi^2.\, ||\mathbf{rot_r\, \Gamma}||^2.\, \cos^2(\mathbf{n, rot_r\, \Gamma})$$

$$=$$

$$|T_2(o)(\partial_r, \Gamma)|$$

Both polynomials can only be equated when (i) the coefficient $\alpha(\Gamma)$ vanishes and (ii) the vector $\mathbf{rot_r\, \Gamma}$ is parallel and in the same direction than the unitary vector $\mathbf{n}$:

$$\{\delta\rho = \gamma_0.\, div\, \Gamma_{\text{N-like}}\} \Rightarrow \{\alpha(\Gamma_{\text{N-like}}) = 0,\, \cos(\mathbf{n, s}) = 1 \Leftrightarrow \exists\, \sigma \in R^+ : \mathbf{rot_r\, \Gamma = \sigma.\, n}\}$$

In this theory, if the polynomial form $P_{\text{N-like}}(\delta\rho)$ has solutions, the generic formalism of the Jacobian matrices related to these solutions is:

$$T_2(o)(\partial_r, \Gamma_{\text{N-like}}) = \beta(\Gamma_{\text{N-like}}).\, T_2(\otimes)(\mathbf{n, n}) + \sigma.\, \chi(\Gamma_{\text{N-like}}).\, {}_{[J]}\Phi(\mathbf{n}),\, \sigma \in R^+$$

$$|T_2(o)(\partial_r, \Gamma_{\text{N-like}})| = \pounds_{\text{N-like}}.\, div_r\, \Gamma_{\text{N-like}} = \beta(\Gamma_{\text{N-like}}).\, \chi^2(\Gamma_{\text{N-like}}).\, ||\mathbf{rot_r\, \Gamma_{\text{N-like}}}||^2$$

Furthermore, there are two different expressions representing the divergence of the allowed acceleration fields if $\pounds$ does not vanish ($\pounds \neq 0$):

$$div_r\, \Gamma_{\text{N-like}} = \frac{t_P.\,\delta\rho}{\gamma_0} = \frac{|T_2(o)(\partial_r, \Gamma)|}{\pounds}$$

From which it is easy to get another expression for the variations of ρ:

$$t_P.\,\delta\rho = \gamma_0.\,\frac{|T_2(o)(\partial_r, \Gamma)|}{\pounds}$$

Conversely, when $\mathbf{rot_r\, \Gamma = \sigma.\, n}$, the determinant of the Perian matrices one is working with can be factorized [b; § 5.1, pp. 53-54]:

$$|T_2(o)(\partial_r, \Gamma)| = |\alpha.\, Id_3 + \beta.\, T_2(\otimes)(\mathbf{n, n}) + \sigma.\, \chi.\, {}_{[J]}\Phi(\mathbf{n})| = (\alpha + \beta).\, (\alpha^2 + \sigma^2.\, \chi^2)$$

If $\alpha = 0$, one recovers:

$$|T_2(o)(\partial_r, \Gamma)| = \beta.\, \sigma^2.\, \chi^2 = \pounds.\, div_r\, \Gamma$$

Then:

$$P(\delta\rho) = (t_P.\, \delta\rho)^2.\, (t_P.\, \delta\rho - \gamma_0.\, div\, \Gamma)$$

The polynomial form $P(\delta\rho)$ has solutions if either $\delta\rho = 0$ or the acceleration-field is Newton-like: $t_P.\, \delta\rho = \gamma_0.\, div\, \Gamma$.

The allowed acceleration-fields for any $rot_r\,\Gamma$

One can now go a step further and state that:

$$£.\,div_r\,\Gamma = 9.\,\alpha^3 + 9.\,\alpha^2.\,\beta + (2.\,\beta^2 + 3.\,\chi^2.\,||rot_r\,\Gamma||^2).\,\alpha + \beta.\,\chi^2.\,||rot_r\,\Gamma||^2$$

Whilst:

$$9.\,|T_2(o)(\partial_r,\,\Gamma)| = 9.\,\alpha^3 + 9.\,\beta.\,\alpha^2 + 9.\,\chi^2.\,||rot_r\,\Gamma||^2.\,\alpha + 9.\,\beta.\,\chi^2.\,||rot_r\,\Gamma||^2.\,cos^2(n,\,rot_r\,\Gamma)$$

"Is there a possibility to equalize these expressions?" The answer is positive under two conditions:

$$\beta^2 = 3.\,\chi^2.\,||rot_r\,\Gamma||^2 \Leftrightarrow \beta = \pm\,\sqrt{3}.\,\chi.\,||rot_r\,\Gamma||$$

$$1 = 9.\,cos^2(n,\,rot_r\,\Gamma) \Leftrightarrow cos(n,\,rot_r\,\Gamma) = \pm\frac{1}{3}$$

$$(n,\,\widehat{rot_r\Gamma}) \cong +\,70{,}53° \text{ and } (n,\,\widehat{rot_r\Gamma}) \cong 109{,}47°$$

The Perian matrices:

$$T_2(o)(\partial_r,\,\Gamma) = \alpha.\,Id_3 \pm \sqrt{3}.\,\chi.\,||rot_r\,\Gamma||.\,T_2(\otimes)(n,\,n) + \chi.\,{}_{[J]}\Phi(rot_r\,\Gamma)$$

Such that :

$$9.\,|T_2(o)(\partial_r,\,\Gamma)| = £.\,div_r\,\Gamma,\,cos(n,\,rot_r\,\Gamma) = \pm\frac{1}{3}$$

... validate the relation:

$$|(t_P.\delta\rho).\,Id_3 - \gamma_0.T_2(o)(\partial_r,\,\Gamma)| = P(\delta\rho) = |(t_P.\delta\rho - \gamma_0.\alpha).\,Id_3 - \gamma_0.\beta.\,T_2(\otimes)(n,\,n) - \gamma_0.\chi.\,{}_{[J]}\Phi(rot_r\,\Gamma)|$$

This sentence means that they allow the calculations of the roots of $P(\delta\rho)$.

The difficulties

An interpretation for the allowed acceleration-fields with any $rot_r\,\Gamma$?

It may be a pure hazard but, the two angles between n and $rot_r\,\Gamma^4$ also exist in specific families of molecules. The value $70{,}53°$ can for example be associated to SiO_2-rings (big peak for O-O-O and small peak for Si-Si-Si) [06; figure 4.10 a) and b), p. 44] whilst the value $109{,}47°$ can be related to CH_4.

As extensively explained in [06], there are diverse theoretical approaches trying to understand what is experimentally observed.

One can cite: the classical dynamics applied to molecules, the Car-Parinello Method, the density functional theory, the pseudo-potential principle, the velocity Verlet-Algorithm,

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[4] ... allowing to consider that the Jacobian matrix $T_2(o)(\partial_r,\,\Gamma)$ containing all first-order partial derivations of an acceleration-field Γ of which the rotational (whirl tensor) is not necessary null can be identified with a Perian matrix giving the opportunity to calculate variations $\delta\rho$ of a density of matter per unit of volume have values which...

the BKS potential, the Ewald summation and the Valence Shell Electron Pair Repulsion (VSEPR). The density functional theory involves the gravitational potential and, because of this fact, seems to be the closest way of thinking to the one which is developed in this document.

Nevertheless, the coincidence is perhaps only a misleading indication because the angles which have been obtained in this document concern the rotational of the acceleration-field at hand (it must not obligatorily be Newtonian) and an arbitrarily positioned unitary position-vector **n**, ... not the angle between (e.g.) three atoms of Silicium.

Representing the Si-Si-Si
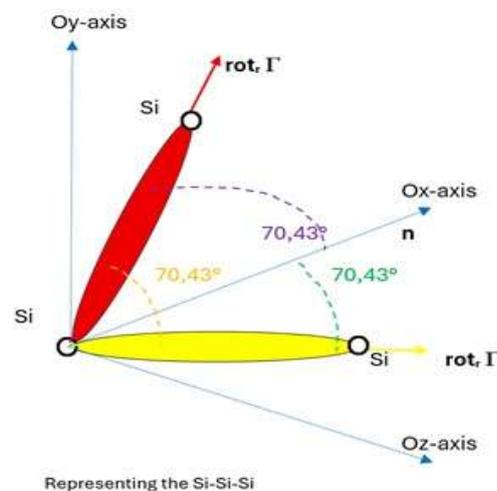
Anyway, one can state an unexpected coincidence between (i) mathematical conditions allowing to involve the Perian matrices in a description furnishing variations of the density of matter per unit of volume due to the variations of an acceleration-field and (ii) angles which have been experimentally measured in molecules but for which the actual explanation seems to be an electromagnetic repulsion.

A minima, this fact gives rise to a discussion about the interplay between electromagnetism and gravitation inside molecules.

There exists Perian matrices which are the main parts of decomposed (and eventually deformed) cross products and, simultaneously, the representations of Euler-Rodrigues parametrisations. Therefore, one can easily imagine that a nucleus and its captive electrons form a polymorphic pulsating whole, the constituents of which are obliged to stay in privileged regions due to physical laws.

The interferences between gravitation and electromagnetism in vacuum

All what happens in vacuum is the delayed result of interactions that occurred very far away. But they are obliged to interfere locally, here and now.

Fields of different nature interfere in these empty regions because of their wave nature: electromagnetic and gravitational waves exist.

It can therefore be assumed that the electromagnetic fields act (i) on themselves, resulting in interferences patterns; and (ii) on the geometry, resulting in variations of the metric.

The point (ii) in previous sentence is a bit misleading or at least opening a debate because a given geometry is in principle the signature of the positioning of material objects at a given instant. Classically, distant material sources (black holes, stars, planets, etc.) move and influence the motion of each other's. Therefore: "Is it still meaningful to speak of geometry in a region containing neither material bodies nor particles but only fields?"

The junction with the subatomic particles

The procedure which has been explained in [b; § 4.7] is interesting but it can only be applied to a part of the Perian matrices, more precisely: to the non-injective images of pairs (**a**, **n**). The non-injectivity constrains the formalism of these matrices:

$$T_2(o)(\partial_r, \Gamma) = \alpha(\Gamma).\ Id_3 + \beta(\Gamma).\ T_2(\otimes)(\mathbf{n}, \mathbf{n}) + \chi(\Gamma).\ _{[J]}\Phi(\mathbf{n}) = [M(\mathbf{a}, \mathbf{n})]$$

... and imposes an interdependence between the components of both arguments in the pair (**a**, **n**) at hand [b; Lemma 4.5, p.50]:

$$(n^1)^2 = (n^2)^2 = (n^3)^2 = constant = -\frac{d\alpha}{d\beta} = \left(\frac{d\chi}{d\beta}\right)^2$$

A deepening of this information is needed:

- Firstly, the equality $T_2(o)(\partial_r, \Gamma) = [M(\mathbf{a}, \mathbf{n})]$ can only be written when $\mathbf{rot_r}\ \Gamma = \chi(\Gamma).\ \mathbf{n}$ with $\chi(\Gamma) > 0$. In that case, the relations of coherence are:

$$div\ \Gamma = 3.\ \alpha + \beta,\ \ £ = 3.\ \alpha^2 + 2.\ \alpha.\ \beta + \chi^2,\ |T_2(o)(\partial_r, \Gamma)| = (\alpha + \beta).\ (\alpha^2 + \chi^2)$$

The Newtonian-like fields belong to the set of acceleration-fields which can be associated with these Perian matrices if $\alpha(\Gamma) = 0$. Hence, for these fields, the relations of coherence are simplified and written:

$$div_r\ \Gamma_{N-like} = \beta,\ £ = \chi^2,\ |T_2(o)(\partial_r, \Gamma_{N-like})| = \beta.\ \chi^2 = div_r\ \Gamma_{N-like}.\ £, \chi > 0$$

The Newtonian-like acceleration-fields are associated with the matrices:

$$T_2(o)(\partial_r, \Gamma_{N-like}) = div_r\Gamma._{N-like}\ T_2(\otimes)(\mathbf{n}, \mathbf{n}) + \sqrt{\frac{|T_2(o)(\partial_r,\Gamma)|}{div_r\Gamma}}.\ _{[J]}\Phi(\mathbf{n}) = [M(\mathbf{a}, \mathbf{n})]$$

$$\mathbf{a}_\pm : (0, div_r\Gamma_{N-like}, \sqrt{\frac{|T_2(o)(\partial_r,\Gamma)|}{div_r\Gamma}}),\ div_r\ \Gamma_{N-like} \neq 0$$

- The first and the third components of **a** can vary by respect for the second one and the square of the variations of the third component is minus one times the product of the variations of the first one times the variations of the second one:

$$(d\chi)^2 = -d\alpha.\ d\beta$$

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If one supposes that the matrices are differences related to the polynomial form $P(\delta\rho)$:

$$d\alpha = t_P . \delta\rho = \gamma_0 . \, div_r \, \mathbf{\Gamma}_{\text{N-like}}, \quad d\beta = div_r \, \mathbf{\Gamma}_{\text{N-like}}, \quad d\chi = \pm \sqrt{\pounds}$$

Then, for Newtonian-like fields, the constraints on **a** can be rewritten as:

$$-\frac{d\alpha}{d\beta} = \left(\frac{d\chi}{d\beta}\right)^2 = \frac{t_P . \delta\rho}{div\mathbf{\Gamma}} = \frac{\pounds}{(div\mathbf{\Gamma})^2} = \text{constant} = \gamma_0$$

The constant γ_0 characterizing the empty regions commands the evolution of the components of **a**. Since the admissible Newtonian-like fields respect the relations:

$$|T_2(o)(\partial_r, \mathbf{\Gamma}_{\text{N-like}})| = \pounds . \, div_r \, \mathbf{\Gamma}_{\text{N-like}}$$

$$div_r \, \mathbf{\Gamma}_{\text{N-like}} = \frac{t_P . \delta\rho}{\gamma_0} = \frac{|T_2(o)(\partial_r, \mathbf{\Gamma})|}{\pounds}$$

It follows that:

$$|T_2(o)(\partial_r, \mathbf{\Gamma}_{\text{N-like}})| = \gamma_0 . \, (div_r \, \mathbf{\Gamma}_{\text{N-like}})^3 = \frac{(t_P . \delta\rho)^3}{\gamma_0^2}$$

- The squares of each component of the second argument, **n**, are equal and equal to a constant. This exigence contains eight subcases:

+++ / ++- / +-+ / -++ / --+ / -+- / +-- /---

But they correspond to only four generic matrices $\gamma_0 . T_\kappa$ for $\kappa = 0, 1, 2, 3$:

$$T_0 = {+}{+}{+} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = {-}{-}{-} = T_7$$

$$T_1 = {-}{+}{+} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} = {+}{-}{-}$$

$$T_2 = {+}{-}{+} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = {-}{+}{-}$$

$$T_3 = {+}{+}{-} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} = {-}{-}{+}$$

- The junction with useful matrices in physics of particles is obtained for Perian matrices that can be identified with a matrix studied in [05]:

$$\lambda . \, Id_3 + \phi . \gamma_0 . T_2(\otimes)(\mathbf{n}_0, \mathbf{n}_0) + \xi . \sqrt{\gamma_0} ._{[J]}\Phi(\mathbf{n}_0)$$
$$\mathbf{n}_0: (1, 1, 1)$$

$$\Phi = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}, \lambda = \frac{a}{\sqrt{3}} - \frac{b.cos\omega}{\sqrt{6}}, \phi.\,\gamma_0 = \frac{b.cos\omega}{\sqrt{6}}, \xi.\,\sqrt{\gamma_0} = -\frac{i.b.sin\omega}{\sqrt{6}}$$

Hence, the interesting matrices have the formalism:

$$\lambda.\,Id_3 + \phi.\,\gamma_0.\,T_2(\otimes)(\mathbf{n_0}, \mathbf{n_0}) + \xi.\,\sqrt{\gamma_0}._{[J]}\Phi(\mathbf{n_0}) = [M(\mathbf{a'}, \mathbf{n_0})]$$
$$\mathbf{a'} : (\lambda, \phi.\,\gamma_0, \xi.\,\sqrt{\gamma_0})$$

One may remark that :

$$T_2 = Id_3 + {}_{[J]}\Phi(\mathbf{n_0})$$

$$T_0 + T_1 + T_2 + T_3 = 4.\,Id_3$$

- The identification:

$$[M(\mathbf{a'}, \mathbf{n_0})] = [M(\mathbf{a}, \mathbf{n})]$$

... imposes the conditions:

$$\mathbf{n_0} = \mathbf{n}$$

$$\mathbf{a'} = \mathbf{a}$$

There is no immediate physical argument justifying the first condition[5]. The second condition is equivalent to:

$$0 = \frac{a}{\sqrt{3}} - \frac{b.cos\,\omega}{\sqrt{6}}$$
$$\gamma_0.\,\phi = \frac{b.cos\,\omega}{\sqrt{6}} = \gamma_0.\,div_r\,\Gamma$$
$$\xi.\,\sqrt{\gamma_0} = -\frac{i.b.sin\,\omega}{\sqrt{6}} = \pm\sqrt{\frac{|T_2(o)(\partial_r,\Gamma)|}{div_r\Gamma}}) = \pm\sqrt{\gamma_0}.\,\sqrt{\pounds}$$

If this identification would be meaningful within the context of this exploration, the procedure which has been proposed in [05] and recalled in [b] would give a set of densities:

$$t_P.\,\delta\rho_1 = \frac{a}{\sqrt{3}} + \frac{2.b.cos\,\omega}{\sqrt{6}} = \frac{3.b.cos\,\omega}{\sqrt{6}} = 3.\,\gamma_0.\,div_r\,\Gamma$$

$$t_P.\,\delta\rho_2 = \frac{a}{\sqrt{3}} - \frac{b.cos\,\omega}{\sqrt{6}} + \frac{b.sin\,\omega}{\sqrt{2}} = i.\,\sqrt{\gamma_0}.\,\sqrt{3.\pounds}$$

$$t_P.\,\delta\rho_3 = \frac{a}{\sqrt{3}} - \frac{b.cos\,\omega}{\sqrt{6}} - \frac{b.sin\,\omega}{\sqrt{2}} = -i.\,\sqrt{\gamma_0}.\,\sqrt{3.\pounds}$$

At this stage, it seems to be important to compare these results with the eigenvalues of $T_2(o)(\partial_r, \Gamma)$ for the Newtonian-like fields. A coincidence would be the

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[5] A document is in preparation.

starting point for the demonstration of an equivalence between the TEQ, and the ideas proposed in [05]. For now, one can only state that:

$$t_P. (\delta\rho_1 + \delta\rho_2 + \delta\rho_3) = 3. \gamma_0. \text{div}_r \Gamma$$

$$(t_P)^3. \delta\rho_1. \delta\rho_2. \delta\rho_3 = 9. (\gamma_0)^2. \text{div}_r \Gamma. £ = 9. (\gamma_0)^2. |T_2(o)(\partial_r, \Gamma_{N\text{-like}})| = (t_P. \delta\rho)^3$$

These relation suggest introducing the average variation: $\delta\rho = 1/3. (\delta\rho_1 + \delta\rho_2 + \delta\rho_3)$ to get a formulation corresponding to the remark accompanying the definition of Newtonian-like fields.

Conclusion

The question motivating this exploration was: "Are the neutrinos indirectly related to variations of acceleration-fields?" The omnipresence of the quadrupolar tensor, its formalism (it is a truncated Perian matrix), the fact that a rotational can be interpreted as a tool deforming the classical cross products (hypothesis 1) led to consider that any Jacobian matrix $T_2(o)(\partial_r, \Gamma)$ with a physical meaning must be a Perian matrix (hypothesis 2) of which each eigenvalue represents an admissible variation of the local density of matter per unit of volume, equivalently a given type of sub-atomic particle (postulate).

The postulate is justified by the fact that a variation related to given type of sub-atomic particles in a first frame must be related to the same type in another frame. Its validity is reenforced by the fact that the hypothesis 2 is drastically reducing the number of admissible Jacobian matrices.

The hypothesis 1 explains why the admissible Jacobian matrices are important in a pre-Euclidean context (this context can be associated with quasi-empty regions) and why a C6 symmetry appears when one concretely calculates the variations $\delta\rho$.

Concretely, for a given configuration of a Newton-field and of its variations, the quadrupolar tensor carries three possible values for the variations of the spatial density of matter per unit of volume. This mathematical fact can be explained within the TEQ in interpreting it as the main part of decompositions related to the deformation of the classical cross product $\mathbf{n} \wedge \mathbf{M}$ through a whirl tensor (rotational).

Since the quadrupolar tensor is a truncated Perian matrix (of which the third coefficient is null), one bets that the Jacobian matrix containing the spatial variations of any acceleration-field can always be identified with a Perian matrix (hypothesis 2). This supposition generates a small set of constraints on the coefficients of this matrix, and these coefficients can be related to the divergence $\text{div}_r \Gamma$, to the quantity £ (see Annex 2 below) and to the determinant $|T_2(o)(\partial_r, \Gamma)|$ of the Jacobian matrix. This way of thinking allows the definition of acceleration-fields which are compatible with the hypothesis 2.

The attention is attracted by a subset of Newtonian-like fields. They are represented by truncated Perian matrices (of which the first coefficient is null) and characterized by an

alinement between the rotational of the field and the unitary-position vector. Furthermore, these matrices can be related to the procedure which has been explained in [05] and recalled in [b]. More precisely, they can be related to quark and lepton mass matrices associated with a cyclic permutation invariant form.

This exploration lies on the hope achieving a junction between electromagnetism and gravitation in studying the possibilities to connect two logics: the one of the standard model of particles and the one studying the variations of acceleration-fields (anyone). Although it has hopefully brough some interesting lightning on a promising intuition, it would be false to affirm that the junction has effectively been completely realized here. It would be better to say that it only did try to bring and to formalize arguments reenforcing the conviction that acceleration-fields may be at the origin of some sub-atomic phenomenon that one may eventually identify with already known particles. More work is needed.

Personal works

[a] Vide de Maxwell et cordes cosmiques ; vixra:2405.0053, 13 pages.

[b] Matrices périennes ordinaires : lien avec le problème des trois générations ; vixra:2510.0013, 62 pages.

[c] The (E) Question in a three-dimensional space - Analysing a subset of linear systems with the intrinsic method; vixra:2503.0200, 32 pages.

[d] The so-called extrinsic method - A method to decompose deformed tensor (resp. Lie) products; vixra:2406.0127, 12 pages.

[e] Matrices périennes - Recherche d'un lien avec les opérateurs anticommutatifs; vixra:2509.0014, 33 pages.

[f] Algebraic Dynamics - First Stones; vixra:2509.0108, 49 pages.

Annexes

Annex 01: The polynomial form P($\delta\rho$)

Let calculate the eigenvalues of the quadrupolar tensor.

$$P(\delta\rho)$$

$$=$$

$$- \begin{vmatrix} \gamma.\dfrac{\partial\Gamma^1}{\partial x^1} - t_P.\delta\rho & \gamma.\dfrac{\partial\Gamma^1}{\partial x^2} & \gamma.\dfrac{\partial\Gamma^1}{\partial x^3} \\ \gamma.\dfrac{\partial\Gamma^2}{\partial x^1} & \gamma.\dfrac{\partial\Gamma^2}{\partial x^2} - t_P.\delta\rho & \gamma.\dfrac{\partial\Gamma^2}{\partial x^3} \\ \gamma.\dfrac{\partial\Gamma^3}{\partial x^1} & \gamma.\dfrac{\partial\Gamma^3}{\partial x^2} & \gamma.\dfrac{\partial\Gamma^3}{\partial x^3} - t_P.\delta\rho \end{vmatrix}$$

$$=$$

$$-(\gamma.\tfrac{\partial\Gamma^1}{\partial x^1} - t.\,\delta\rho).\{(\gamma.\tfrac{\partial\Gamma^2}{\partial x^2} - t.\,\delta\rho).(\gamma.\tfrac{\partial\Gamma^3}{\partial x^3} - t.\,\delta\rho) - \gamma^2.\tfrac{\partial\Gamma^3}{\partial x^2}.\tfrac{\partial\Gamma^2}{\partial x^3}\}$$

$$+ \gamma . \frac{\partial \Gamma^1}{\partial x^2} . \{\gamma . \frac{\partial \Gamma^2}{\partial x^1} . (\gamma . \frac{\partial \Gamma^3}{\partial x^3} - t. \, \delta\rho) - \gamma^2 . \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^3}\}$$

$$- \gamma . \frac{\partial \Gamma^1}{\partial x^3} . \{\gamma^2 . \frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^2} - \gamma . \frac{\partial \Gamma^3}{\partial x^1} . (\gamma . \frac{\partial \Gamma^2}{\partial x^2} - t. \, \delta\rho)\}$$

$$=$$

$$(\gamma . \frac{\partial \Gamma^1}{\partial x^1} - t. \, \delta\rho) . \{- \gamma^2 . (\frac{\partial \Gamma^2}{\partial x^2} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^3}) + \gamma . t. \, \delta\rho . (\frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3}) - t^2 . (\delta\rho)^2\}$$

$$+ \gamma . \frac{\partial \Gamma^1}{\partial x^2} . \{\gamma^2 . (\frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^3}) - \gamma . t. \, \delta\rho . \frac{\partial \Gamma^2}{\partial x^1}\}$$

$$- \gamma . \frac{\partial \Gamma^1}{\partial x^3} . \{\gamma^2 . (\frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^2} - \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^2}) + \gamma . t. \, \delta\rho . \frac{\partial \Gamma^3}{\partial x^1}\}$$

$$=$$

$$- \gamma^3 . \frac{\partial \Gamma^1}{\partial x^1} . (\frac{\partial \Gamma^2}{\partial x^2} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^3}) + \gamma^2 . t. \, \delta\rho . \frac{\partial \Gamma^1}{\partial x^1} . (\frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3}) - \gamma . t^2 . (\delta\rho)^2 . \frac{\partial \Gamma^1}{\partial x^1}$$

$$+ \gamma^2 . t. \, \delta\rho . (\frac{\partial \Gamma^2}{\partial x^2} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^3}) - \gamma . t^2 . (\delta\rho)^2 . (\frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3}) + t^3 . (\delta\rho)^3$$

$$+ \gamma^3 . \frac{\partial \Gamma^1}{\partial x^2} . (\frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^3}) - \gamma^2 . t. \, \delta\rho . \frac{\partial \Gamma^1}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^1}$$

$$- \gamma^3 . \frac{\partial \Gamma^1}{\partial x^3} . (\frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^2} - \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^2}) - \gamma^2 . t. \, \delta\rho . \frac{\partial \Gamma^1}{\partial x^3} . \frac{\partial \Gamma^3}{\partial x^1}$$

$$=$$

$$t^3 . (\delta\rho)^3 - \gamma . t^2 . (\delta\rho)^2 . (\frac{\partial \Gamma^1}{\partial x^1} + \frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3})$$

$$+ \gamma^2 . t. \, \delta\rho . \{\frac{\partial \Gamma^1}{\partial x^1} . (\frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3}) + (\frac{\partial \Gamma^2}{\partial x^2} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^3}) - (\frac{\partial \Gamma^1}{\partial x^3} . \frac{\partial \Gamma^3}{\partial x^1} + \frac{\partial \Gamma^1}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^1})\}$$

$$- \gamma^3 . \{\frac{\partial \Gamma^1}{\partial x^1} . (\frac{\partial \Gamma^2}{\partial x^2} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^3}) - \frac{\partial \Gamma^1}{\partial x^2} . (\frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^3}) + \frac{\partial \Gamma^1}{\partial x^3} . (\frac{\partial \Gamma^2}{\partial x^1} . \frac{\partial \Gamma^3}{\partial x^2} - \frac{\partial \Gamma^3}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^2})\}$$

$$=$$

$$t^3 . (\delta\rho)^3 - \gamma . \text{div}_r \, \Gamma . t^2 . (\delta\rho)^2 + \gamma^2 . \pounds . t. \, \delta\rho - |T_2(o)(\partial_r, \Gamma)|$$

With:

$$\text{div}_r \, \Gamma = \frac{\partial \Gamma^1}{\partial x^1} + \frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3}$$

$$\pounds = (\frac{\partial \Gamma^1}{\partial x^1} . \frac{\partial \Gamma^2}{\partial x^2} - \frac{\partial \Gamma^1}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^1}) + (\frac{\partial \Gamma^2}{\partial x^2} . \frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2} . \frac{\partial \Gamma^2}{\partial x^3}) + (\frac{\partial \Gamma^3}{\partial x^3} . \frac{\partial \Gamma^1}{\partial x^1} - \frac{\partial \Gamma^1}{\partial x^3} . \frac{\partial \Gamma^3}{\partial x^1})$$

This result allows the calculation of variations $\delta\rho$ in any region with coefficient of "gravitational permeability" γ.

In this case (recall):

$$\Gamma = -G.M.\frac{\mathbf{n}}{r^2} \Leftrightarrow \forall\, a = 1, 2, 3:\ \Gamma^a = -\,G.M.\frac{x^a}{r^3}$$

$$r = \{(x^1)^2 + (x^2)^2 + (x^3)^2\}^{1/2} > 0$$

This context allows the calculations of:

- The first order partial derivates of the field:

$$\forall\, a, b = 1, 2, 3:\ \frac{\partial \Gamma^a}{\partial x^b} = \frac{G.M}{r^3}.(\delta^{ab} - \frac{3}{r^2}.\, x^a.\, x^b)$$

- The divergence of the field, outside of the source:

$$\text{div}_r\Gamma$$

$$=$$

$$\frac{\partial \Gamma^1}{\partial x^1} + \frac{\partial \Gamma^2}{\partial x^2} + \frac{\partial \Gamma^3}{\partial x^3}$$

$$=$$

$$\frac{G.M}{r^3}.\{\{1 - \frac{3}{r^2}.\,(x^1)^2\} + \{1 - \frac{3}{r^2}.\,(x^2)^2\} + \{1 - \frac{3}{r^2}.\,(x^3)^2\}\}$$

$$=$$

$$\frac{G.M}{r^3}.\{3 - \frac{3}{r^2}.\,\{(x^1)^2 + (x^2)^2 + (x^3)^2\}\}$$

$$=$$

$$0$$

- The quantity £:

$$£$$

$$=$$

$$(\frac{\partial \Gamma^1}{\partial x^1}.\frac{\partial \Gamma^2}{\partial x^2} - \frac{\partial \Gamma^1}{\partial x^2}.\frac{\partial \Gamma^2}{\partial x^1}) + (\frac{\partial \Gamma^2}{\partial x^2}.\frac{\partial \Gamma^3}{\partial x^3} - \frac{\partial \Gamma^3}{\partial x^2}.\frac{\partial \Gamma^2}{\partial x^3}) + (\frac{\partial \Gamma^3}{\partial x^3}.\frac{\partial \Gamma^1}{\partial x^1} - \frac{\partial \Gamma^1}{\partial x^3}.\frac{\partial \Gamma^3}{\partial x^1})$$

$$=$$

$$(\frac{G.M}{r^3})^2.\{\{1 - \frac{3}{r^2}.\,(x^1)^2\}.\{1 - \frac{3}{r^2}.\,(x^2)^2\} - \frac{9}{r^4}.\,(x^1)^2.\,(x^2)^2\}$$

$$+ (\frac{G.M}{r^3})^2.\{\{1 - \frac{3}{r^2}.\,(x^2)^2\}.\{1 - \frac{3}{r^2}.\,(x^3)^2\} - \frac{9}{r^4}.\,(x^2)^2.\,(x^3)^2\}$$

$$+ (\frac{G.M}{r^3})^2.\{\{1 - \frac{3}{r^2}.\,(x^3)^2\}.\{1 - \frac{3}{r^2}.\,(x^1)^2\} - \frac{9}{r^4}.\,(x^3)^2.\,(x^1)^2\}$$

$$=$$

$$\left(\tfrac{G.M}{r^3}\right)^2 . \left\{1 - \tfrac{3}{r^2} . \{(x^1)^2 + (x^2)^2\}\right\} + \left(\tfrac{G.M}{r^3}\right)^2 . \left\{1 - \tfrac{3}{r^2} . \{(x^2)^2 + (x^3)^2\}\right\} + \left(\tfrac{G.M}{r^3}\right)^2 . \left\{1 - \tfrac{3}{r^2} . \{(x^3)^2 + (x^1)^2\}\right\}$$

$$=$$

$$\left(\tfrac{G.M}{r^3}\right)^2 . \left\{3 - \tfrac{6}{r^2} . \{(x^1)^2 + (x^2)^2 + (x^3)^2\}\right\}$$

$$=$$

$$-3 . \left(\tfrac{G.M}{r^3}\right)^2$$

- The determinant of the matrix $|T_2(o)(\partial_r, \Gamma)|$; calculation left to the readers.

These results allow the calculation of the polynomial form $P(\delta\rho)$ in the Newtonian case, outside of the gravitational sources.

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Bibliography

[01] The neutrino mass puzzle; CERN courier, 4 May 2024.

[02] The Classical Theory of Fields, third revised English Edition, Course of Theoretical Physics Volume 2, Copyright © 1971 Pergamon Press Ltd. Printed in Great Britain.

[03] Lehrbuch der theoretischen Physik, Band II klassische Feldtheorie; 12.überarbeite Auflage ©Akademie Ver lag GmbH, Berlin 1992, ISBN 3-05-501550-9, 480 p.

[04] Allgemeine Relativitätstheorie; 4. Auflage, © 2003, 1998, 1995 Spektrum Akademischer Verlag GmbH Berlin Heidelberg, ISBN 3-8274-1356-7, 343 p.

[05] Quark and Lepton Mass Matrices with a cyclic permutation Invariant form; arXiv:hep-phy/0005137v1 15 May 2000.

[06] Mischler, Klaus: Molekulardynamik-Simulation zur Struktur von SiO_2-Oberflächen mit adsorbiertem Wasser. Dissertation zur Erlangung des Grades „Doktor der Naturwissenschaften" am Fachbereich Physik der Johannes-Gutenberg-Universität in Mainz; openscience.ub.uni-mainz.de, 2002, 119 p.