The logical operation rules for a sequence

WenBin Hu

Abstract: This paper proposes a set of logical operation rules for sequences and formulates a generation rule for difference-free sequences that satisfy the operations. The "difference-free sequence" in this paper refers to a sequence where the difference between any two arbitrary numbers within the sequence is not equal to any number in other sequences.

Common generation rules for difference-free sequences include:

1. The new term of the sequence satisfies $a_{n+1} > 2a_n$
2. Computer-generated sequences based on the greedy algorithm

Keywords: Sequence, Logical Operation, Difference-Free Sequence

1. Generation Rules of Difference-Free Sequences

Here, the following sequence generation rule is formulated: For a natural number sequence starting with 0 and 1, if the ratio of the difference between the newly added number a_{n+1} and the current maximum number a_n in the sequence $a_{n+1} - a_n$ to the current maximum number a_n is monotonically decreasing, while the difference $a_{n+1} - a_n$ itself is monotonically increasing, and the growth rate r_n of this difference is strictly monotonically increasing, then the difference between any two numbers in this sequence is not equal to any other number in the sequence, where the three numbers a_i, a_j, a_k are all distinct.

Consider the sequence $\{a(n)\}$ that satisfies the following conditions:

$$a(1) = 0，a(2) = 1$$

The ratio is monotonically decreasing:

$$r(n) = \frac{a(n+1) - a(n)}{a(n)}$$

The difference is monotonically increasing:

$$d(n) = a(n+1) - a(n)$$

The growth rate of the difference is strictly monotonically increasing:

$$\Delta d(n) = d(n+1) - d(n)$$

Under these conditions, the sequence exhibits properties of strict convexity and rapid growth. Through analysis, for any three distinct indices i, j, k, it holds that $a(i) - a(j) \neq a(k)$. The reasons are as follows:

Since $d(n)$ is strictly increasing and $\Delta d(n)$ is strictly increasing, the sequence is strictly convex and grows rapidly.

For $i > j$, the following holds:

$$a(i) - a(j) \geq d(i-1)$$

Due to the convexity of the sequence, for $i \geq 3$, we have:

$$d(i-1) > a(i-1) - a(i-2)$$

Moreover, by combining the growth conditions, we finally obtain that $a(i) - a(j) > a(i-1)$ holds for all $j < i$.

Although for large i, $d(i-1) < a(i-1)$, the minimum value of $a(i) - a(j)$occurs when $j = i - 1$,which is $d(i-1)$. However, due to the requirement that the indices must be distinct, when $j = i - 1$, k cannot be $i - 1$. Therefore,$a(i) - a(i-1)$ may be less than $a(i-1)$, but it will not be equal to any $a(k)$ for $k < i - 1$, as the gaps between the sequence values are relatively large.

For $k > I$, we have $a(k) > a(i) > a(i) - a(j)$, so equality is impossible.

For $k < i$, due to either $a(i) - a(j) > a(i-1)$ or the sparsity of the sequence, $a(i) - a(j)$ cannot be equal to any $a(k)$.

Therefore, the difference between any two numbers in the sequence is not equal to any other number in the sequence.

At this point, it is found that Fermat's Conjecture satisfies the above rules when $n > 2$; that is, in the natural number sequence $\{x^n\}$, the difference between any two numbers is not equal to a third number, which means $x^n + y^n \neq z^n$ (where $x \neq y \neq z$) . When n=2 , Pythagorean triples exist, and this is precisely because the growth rate of the natural number sequence $\{x^n\}$ s not fast enough. However, this is not the purpose of this paper.

2. The Logical Operation Rules of Sequences

Let R be an integer sequence following the order of natural numbers. If

$$R(t) \wedge R(t+M) = 0 \quad (t \in N, M \in N)$$

then the sequence R^+ derived from R through the operations of addition, subtraction, multiplication, division, and exponentiation also satisfies this rule,

$$R^+(t) \wedge R^+(t+M) = 0 \quad (t \in N, M \in N)$$

Where M is not equal to the difference between non-zero elements in sequences R and R^+.

1. Addition

   $R(t+S) \wedge R(t+S+M) = 0$
2. Subtraction

   Omitted; analysis depends on specific circumstances.
3. Multiplication

   $R(at) \wedge R(at+M) = 0 \ (a \in N)$

   This is equivalent to sequence sampling.
4. Division

   Omitted; the result after division must be a positive integer.
5. Exponentiation

   Easy to prove based on the sequence generation rules in the first part or Fermat’s Last Theorem.It is obvious that the zero-difference sequence can satisfy the above sequence R.

3. Conclusions

This paper proposes a type of sequence where the difference between any two numbers within the sequence is not equal to any other number in the sequence, which is referred to as a "zero-difference sequence". A rule for generating such sequences is put forward as follows:

1. Start with 0 and 1.
2. The sequence ratio decreases monotonically:

$$r(n) = \frac{a(n+1) - a(n)}{a(n)}$$

3. The difference increases monotonically:

$$d(n) = a(n+1) - a(n)$$

4. The growth rate of the difference increases strictly monotonically:

$$\Delta d(n) = d(n+1) - d(n)$$

Meanwhile, it is shown that such sequences satisfy the following rule:Let R be an integer sequence in the order of natural numbers. If

$$R(t) \wedge R(t+M) = 0 \quad (t \in N, M \in N)$$

then the sequence R^{+} derived from R through addition, subtraction, multiplication, division, and exponentiation operations also satisfies this rule,

$$R^{+}(t) \wedge R^{+}(t+M) = 0 \quad (t \in N, M \in N)$$

where M is not equal to the difference between non-zero elements in sequences R and R^{+}.

Note: For the symbol "∧", if both numbers x and y are non-zero, then x∧y=1; if either x or y is zero, then x∧y=0.