An omitted term in the Lorentz force formula

Jan Sandtner [1]

Abstract:

If the well-known Lorentz force formula is applied for nonstationary cases, then an additional term containing changing electric field must be adjoined. This term has dimension of force too. When in space, where the field is located, pure vacuum is present, then a rightful question arises, where does this force apply. Some considerations about electromagnetic momentum and compensating mechanical momentum are mentioned.

Lorentz force formula onto electric current has the well-known form

$\mathbf{F} = Q\,[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$

where Q is electric charge, **E** is electric field, **v** velocity and **B** magnetic induction.

This form is used for stationary and nonstationary cases. However, in nonstationary cases the formula may be augmented by an additional term

$(\delta\mathbf{D}/\delta t \times \mathbf{B})\, dV$

where **D** is Maxwell's displacement and dV volume element. The $\delta\mathbf{D}/\delta t$ represents a displacement current density, therefore the whole term has dimension of force, but no free charges are present.

The complete formula for Lorentz force will be

$$\mathbf{F} = Q\,[\mathbf{E} + (\mathbf{v} \times \mathbf{B})] + (\delta\mathbf{D}/\delta t \times \mathbf{B})\, dV \qquad (1)$$

Consider the last term. There are two eventualities: The volume element dV contains either a polarizable dielectricum, i.e. a matter with bounded charges (solid, liquid, gas, or plasma) or pure vacuum. In the first case, the force acts onto bounded charges. But what about the second case? There is no conductive current flowing; there is a changing electric field only. There arises a rightful question, where do these forces apply. The simplest answer is: It is nonphysical (otherwise we would need an ether's resurrection).

The effects of vacuum polarization become significant when the external fields approach the Schwinger limit [1], which is:

$E_c = 1.32 \times 10^{18}$ V/m

$B_c = 4.41 \times 10^{9}$ T

These effects break the linearity of Maxwell's equations and therefore break the superposition principle [2].

Now consider interesting special situations:

In cases with constant **B** and periodic **D** (e.g. sine wave), the resulting force will oscillate. However, if also **B** is periodic with the same angular frequency ω and phase as the δ**D**/δt, then the force will be pulsating and points in *one* direction.

Consider the following:

$D = D_0 \sin(\omega t)$, and after finding its derivative: $\delta D/\delta t = \omega D_0 \cos(\omega t)$

$B = B_0 \cos(\omega t)$

Therefore, the product (δD/δt) x B will be proportional to $\cos^2(\omega t)$, which is always positive.

Look at the problem from another point of view:

Dividing the formula (1) by volume, a more compact expression for force density will be obtained

$\mathbf{f} = \rho_e [\mathbf{E} + (\mathbf{v} \times \mathbf{B})] + (\delta\mathbf{D}/\delta t \times \mathbf{B})$

or in another form

$\mathbf{f} = \rho_e \mathbf{E} + (\mathbf{j}_e \times \mathbf{B})] + (\delta\mathbf{D}/\delta t \times \mathbf{B})$

where ρ_e is charge density and $\mathbf{j}_e$ conductive current density. Note again that δ**D**/δt has dimension of current density.

Electromagnetic field possesses momentum, whose density is given by

$\mathbf{g}_{EM} = \mathbf{D} \times \mathbf{B}$

Considering the system remains at rest, another compensating momentum must exist.

This momentum is purely mechanical [3], relativistic in nature, and its density is given by

$\mathbf{g}_m = \rho_m \times \mathbf{u}$

where ρ_m is mass density and **u** vector velocity field.

References:

[1] Wikipaedia: “Vacuum polarization”

[2] Gerd Leuchs, et al.: “Physical Mechanisms Underpinning the Vacuum Permittivity”, Physics 2023, 5, 179–192, https://doi.org/10.3390/physics5010014

[3] David Babson, et al.: “Hidden momentum, field momentum, and electromagnetic impulse” Am. J. Phys., Vol. 77, No. 9, Sept. 2009, pp. 826-833

1) jan.sandtner@eblcom.ch