

On the forces acting between interacting objects

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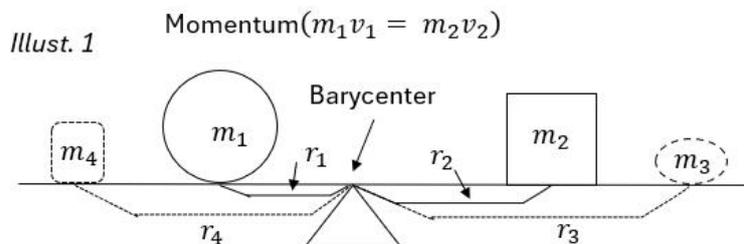
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Abstract

If there are two objects of unknown mass. To determine the ratio of their weights, in nature, all we need is a weightless rod and a fulcrum. If we place two objects on the rod and fulcrum at the balance point, then the balance point becomes the barycenter. Measuring the distance between them from the barycenter yields the relationship $m_1 r_1 = m_2 r_2$. Now, differentiating each distance with respect to time yields $p = m_1 v_1 = m_2 v_2$, which defines Newton's first law of motion, momentum. Therefore, interacting between two objects has two opposing momenta. Newton's second law is the law of acceleration. Differentiating one momentum of the two momenta with respect to time gives $F = ma$. Since interaction is mutually attractive forces between two objects, it is the sum of the two forces between two objects. In other words, $2F = 2ma$. Newton's third law, the law of action and reaction, is two forces with opposite directions repel each other, but the result is the same as interaction. In Newton's law of universal gravitation, gravity also applies equally to the interaction between two objects. The gravitational force is $2F = 2G \frac{Mm}{r^2}$. In the former case, kinetic energy is $E_k = mv^2$, and gravitational energy is $E_g = \frac{2GMm}{r}$. If the two energies are equal, the relation between the two becomes $v^2 = \frac{2GM}{r}$. This is the same equation given in the Einstein field equation. When an object moves at the speed of light, all of the object's mass convert to energy, which is Einstein's energy-mass equivalence principle, $E = mc^2$. Furthermore, if we extend the Newton's law of universal gravitation, the law of universal gravitation between three bodies becomes $2F = 2G \left(\frac{m_1 m_2}{r_{12}^2} + \frac{m_1 m_3}{r_{13}^2} + \frac{m_2 m_3}{r_{23}^2} \right)$.

A. Weight and Momentum

A scale is a tool that can quantify the weight of anything. In nature, a rod and a fulcrum may replace scales. While these two cannot measure the weight of each object, they can determine its barycenter. To compare the weights of two or more objects, place them on the rod and find the point of bary-



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center where they are balanced. The point where each object laid is also its own barycenter.

By measuring the distance between each object and the barycenter, we can determine which of the two objects is lighter. Comparing the weights of two objects m_1 and m_2 , if the distance between object m_1 and the fulcrum is r_1 and the distance between object m_2 and the fulcrum is r_2 , the following equation holds:

$$m_1 r_1 = m_2 r_2 \text{ or } \frac{r_1}{r_2} = \frac{m_2}{m_1}. \quad (1)$$

If we know the weight of one, we can also know the weight of the other.

Here, the point where the fulcrum is located is the point of application, which is the barycenter of the two objects m_1 and m_2 . While the barycenter, or the point of application, is an imaginary point, it tells that the product of each weight and the distance from the barycenter to the object is constant.

For multi-body problems, including the three-body problem, the following holds true: If we place object m_3 on the right side of the rod, the barycenter will shift to the right; if we place object m_4 on the left side, the barycenter will shift to the left.

Simplifying these, we see that the sum of the weights of each object on the left multiplied by their distances to the barycenter is equal to the sum of the weights of each object on the right side by their distances to the barycenter,

$$m_1 r_1 + m_4 r_4 = m_2 r_2 + m_3 r_3. \quad (2)$$

The three-body problem generally refers to the case where one object is placed on the left side of a rod and two on the right side. However, it also applies to binary systems, where two objects are placed on the left side and one object on the right. The multi-body problem extends the Eq. (2) to apply to complex systems.

Since distance divided by time equals velocity, differentiating the Eq. (1) with respect to time yields Newton's first law, the law of inertia,

$$m_1 \frac{dr_1}{dt} = m_2 \frac{dr_2}{dt}, \text{ or } m_1 v_1 = m_2 v_2. \quad (3)$$

Now, the Eq. (1) is transformed from weight under gravity to mass, and the two momenta interact between the two objects, which, as defined by Newton, are unaffected by external forces such as gravity.

In other words, we can see that the momentum, which is the product of the mass of each object and the velocity to the barycenter, is equal. If only object m_1 is placed on the rod, the fulcrum will move toward the barycenter of object m_1 . Since the distance to the barycenter is zero, the

momentum also becomes zero. This indicates that the object is at rest.

The law of inertia states that a moving object's state of motion remains constant unless acted upon by an external force. Therefore, from the Eq. (3), we have:

$$p = m_1v_1 = m_2v_2 . \quad (4)$$

Newton's first law, the law of inertia, defines momentum as $p = mv$ by taking one of two equals of the Eq. (4) above. If $p_1 = m_1v_1$ and $p_2 = m_2v_2$, then $p_1 = p_2$, so interacting objects have always two opposing momenta. As long as interaction exists, two momenta always exist. However, since the two momenta are in opposite directions, they may be calculated as follows:

$$p_1 + p_2 = 0. \quad (5)$$

In other words, it was assumed that inertial interactions maintain balance, resulting in zero momentum. This occurs when two objects with the same momentum collide, and when momentum reaches zero, the energy generated is understood to disappear. However, after the collision, the energy is simply converted to another form of energy, not destroyed.

Newton's second law, the law of acceleration, defines the change in force required to accelerate an object at rest with zero momentum or moving at a constant velocity v_1 to a velocity v_2 . This law is derived by differentiating one of the two momenta of the Eq. (4) with respect to time.

An object with zero momentum or moving at a constant velocity v will not change its state of motion unless acted upon by an external force. Therefore, a change in inertia requires the application of an external force. The application of an external force represents a change in inertia, and a change in inertia occurs when an object at rest or moving at a constant velocity v accelerates or decelerates, thereby changing its velocity. A change in velocity represents a change in the interaction of forces. Force is derived by differentiating the momentum with respect to time, which is called acceleration,

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = ma. \quad (6)$$

Force is the result of an interaction, as an object with inertia accelerates or decelerates under the influence of an external force. This interaction requires more than just a single force; it involves two forces acting simultaneously, interacting with each other. Therefore, it is expressed as follows:

$$2F = 2 \frac{dp}{dt} = 2m \frac{dv}{dt} = 2ma. \quad (7)$$

The same holds true for Newton's third law, the law of action and reaction. For an object to move on its own, it must exert its own force. This is called action and reaction.

The change in kinetic energy E_k when accelerating from v_1 to v_2 is as follows:

$$E_k = \int_{r_1}^{r_2} 2Fdr = \int_{r_1}^{r_2} 2 \left(\frac{dp}{dt} \right) dr = 2m \int_{v_1}^{v_2} vdv = mv_2^2 - mv_1^2. \quad (8)$$

The kinetic energy $E_k = \frac{1}{2}mv^2$ by Émilie du Châtelet, is the result obtained by calculating only one of the two forces interacting between two objects. In Eq. (8), if the velocity of the moving object $v = c$, $E = mc^2$ can be obtained, which means that all matters of the moving object is converted into energy according to Einstein's energy-mass equivalence principle. Therefore, the kinetic energy is given by $mc^2 \left(1 - \frac{v^2}{c^2} \right)$, which is the same as the Lagrangian. This explains that the kinetic energy and the energy-mass equivalence principle have the same roots, rather than coming from other sources.

B. Gravitational Interactions

Interaction refers to a force that pulls against each other. This mutual pulling is like two people playing tug-of-war while holding onto a rope, pulling with both hands instead of just one. In other words, the giving and receiving forces occur simultaneously. Gravity is also an interaction.

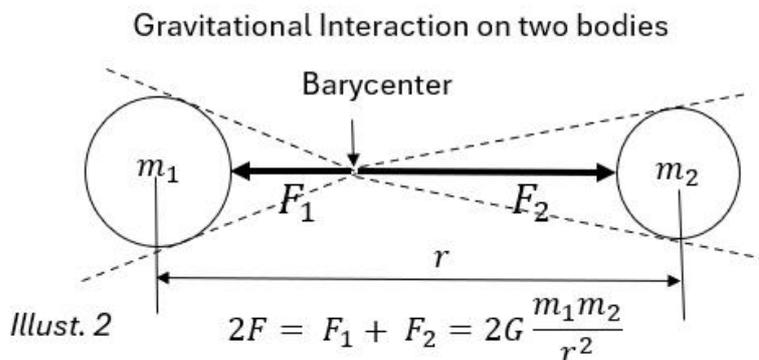
For an object at rest or moving at a constant speed to accelerate or slow down, it must be pushed or pulled by another object. Objects with inertia cannot accelerate on their own without interaction.

Therefore, acceleration is the simultaneous application of pushing and pulling forces. Interaction means the simultaneous application of two forces, so calculating only one force is incorrect. In the case of universal gravitation, two opposing forces interact with each other with equal magnitude around the barycenter or center of gravity. Repulsive and pulling forces are simply different expressions of the same force, acting in different directions.

Newton's law of universal gravitation is expressed as follows:

$$F = G \frac{m_1 m_2}{r^2}. \quad (9)$$

There are two interacting forces relative to the barycenter, but only one of them is defined.



As we can see in the above illustration, F_1 and F_2 have the same magnitude. Therefore,

$$\vec{F}_{12} = \vec{F}_1 + \vec{F}_2, \quad (10)$$

$$\vec{F}_{21} = \vec{F}_2 + \vec{F}_1.$$

\vec{F}_{12} is the force from m_1 to m_2 , and \vec{F}_{21} is the force from m_2 to m_1 , which are equal in magnitude and opposite in direction.

Therefore, since the law of universal gravitation has two interacting forces, it can be expressed as follows by multiplying both sides by two,

$$2F = \vec{F}_{12} + \vec{F}_{21} = 2G \frac{m_1 m_2}{r^2}. \quad (11)$$

Equations (9) and (11) are fundamentally identical, but the amount of energy varies depending on whether there are one or two forces interacting.

From this, the energy E_g between objects under gravitational force can be obtained as follows:

$$E_g = \int_r^\infty 2F dr = \int_r^\infty \frac{2GMm}{r^2} dr = \frac{2GMm}{r}. \quad (12)$$

If the kinetic energy E_k obtained from the equation (8) and the gravitational energy E_g from the equation (12) are equal, then the following relationship is given,

$$mv^2 = \frac{2GMm}{r}. \quad (13)$$

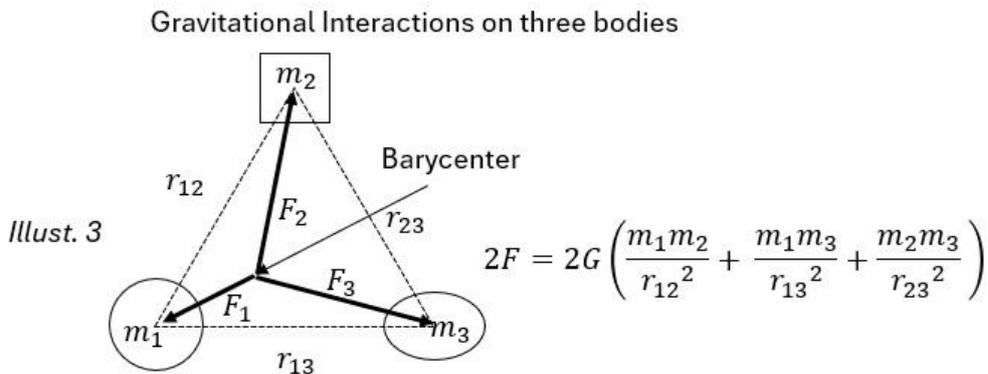
If we eliminate m from both sides of the above, we have,

$$v^2 = \frac{2GM}{r}. \quad (14)$$

This is the same result as the equation given by the Einstein Field Equation.

C. Three-body problem

The three-body problem, by adding an additional object, m_3 , to the two-body problem, attempts to elucidate the force relationships among the three-body interactions.



Similar to the interaction of three bodies on a scale, it begins with the assumption that the sum of the products of the distances from the common barycenter of the three bodies is constant. In a three-body problem, the three bodies each maintain a force balance, starting from their respective barycenter.

From the above, we can identify that,

$$\vec{F}_1 = \vec{F}_2 = \vec{F}_3. \quad (15)$$

Therefore,

$$\begin{aligned} \vec{F}_{12} &= \vec{F}_1 + \vec{F}_2, \\ \vec{F}_{21} &= \vec{F}_2 + \vec{F}_1. \end{aligned} \quad (16)$$

And gravitational interaction between two objects becomes,

$$2F = \vec{F}_{12} + \vec{F}_{21} = 2 \frac{Gm_1m_2}{r_{12}^2}. \quad (17)$$

Cyclical procedures provide with,

$$2F = 2G \left(\frac{m_1m_2}{r_{12}^2} + \frac{m_1m_3}{r_{13}^2} + \frac{m_2m_3}{r_{23}^2} \right). \quad (18)$$

We can see that when m_3 is removed from the three-body problem, it is reduced to a two-body problem.

References

- [1] [https://en.wikipedia.org/wiki/Barycenter_\(astronomy\)](https://en.wikipedia.org/wiki/Barycenter_(astronomy))
- [2] https://en.wikipedia.org/wiki/Newton's_law_of_universal_gravitation
- [3] https://en.wikipedia.org/wiki/Newton's_laws_of_motion
- [4] https://en.wikipedia.org/wiki/Kinetic_energy
- [5] [https://en.wikipedia.org/wiki/Lagrangian_\(field_theory\)](https://en.wikipedia.org/wiki/Lagrangian_(field_theory))
- [6] https://en.wikipedia.org/wiki/Coriolis_force
- [7] https://en.wikipedia.org/wiki/Two-body_problem
- [8] https://en.wikipedia.org/wiki/Three-body_problem
- [9] https://en.wikipedia.org/wiki/N-body_problem