Thermal noise harnessing

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Abstract:

Every resistor is a source of a thermal noise with frequencies reaching up to THz. The noise available power is given by bandwidth, but independent of the frequency in which the noise is generated. To harness the noise power, some nonlinearity in the system should be present. With modern semiconductors, such as Schottky diodes or CMOS rectifiers, contemporary employed in RFID devices, the GHz part of the noise frequency range may be used. By means of RLC resonant circuits or PCB resonators connected in a cascade, a sufficient noise voltage can be attained. After rectifying, the noise energy may be stored in a condenser as a charge. Noise energy in a resonant circuit can be considered as that of a one sole molecule in an ideal gas. Even if the noise available power in a resonant circuit is negligibly small, the energy can be greatly increased by letting the power integrate over a long-time interval. In the appendix, a numerical example is presented.

Keywords: Thermal noise harnessing, RLC resonant circuits, small signal Schottky diode, RF-harvesting, CMOS multistage rectifiers, PCB resonators, RFID devices, Second law.

Article:

Every resistor is a source of a white noise with frequencies reaching up to THz. Average root-mean-square (rms) voltage is given by a well-known formula

$$\langle u_R \rangle = \sqrt{4kTR\,\Delta f}$$

where

$\langle u_R \rangle$... rms voltage at resistor in V

k … Boltzmann constant 1.38×10^{-23} J/K,

T … absolute temperature in K,

R … resistance in ohms

Δf … bandwidth in Hz.

Available power from the resistor is given

$$P_{av} = kT\,\Delta f$$

or

$$P_{av} = \frac{\langle u_R \rangle^2}{4R}$$

The problem of thermal noise is treated in many textbooks, and no attempt has been made to give an exhaustive treatment here (see [1], [2], [3]).

If two or more resistors are connected in series, the available power remains the same, but the rms voltages will be increased. Therefore, by changing the circuit's parameters, the available power cannot be increased, but the resulting rms voltage can. Notice that any external current flowing through the resistor doesn't influence the resistor's thermal noise.

In the following we will consider, instead of white noise, a narrow-band noise only. The noise available power is given by the bandwidth, but independent of the frequency in which the noise is generated.

The bandwidth is given by the following formula

$$\Delta f = \frac{\omega_0}{4Q}$$

where

ω_0 ... resonant angular frequency in rad/s

Q .... RLC circuit figure of merit

By means of parallel RLC resonant circuits (R-resistance, L-inductance, C-capacitance), the white noise can be converted into a sine wave with well-defined frequency, changing amplitude and phase. Such a transformed signal may be more appropriate for rectifying.

The rms voltage generated at the condenser will be

$$\langle u_C \rangle = \sqrt{\frac{kT}{C}}$$

which surprisingly doesn't depend on resistance.

The narrow band noise instantaneous voltages are generated according to a Gaussian distribution. However, the noise envelope obeys Rayleigh distribution, i.e., there is some probability that the noise magnitude will be greater (or smaller) than its rms value during a finite time (see Fig. 2 at [12]). Note that a similar situation occurs for a white noise too, but the time intervals are almost infinitely small.

If two or more uncorrelated noise voltages are superimposed, the resulting noise voltage is equal to the square root of the sum of the noise voltages squared. The result cannot be smaller than any of the voltages.

$$\langle u \rangle_{tot} = \sqrt{\sum_{i=1}^{n} \langle u_i \rangle^2}$$

By means of a few identical parallel RLC resonant circuits or PCB resonators (Printed Circuit Board) [10], [11], connected in a cascade, a sufficient noise voltage can be attained. PCB resonators are usually designed for very high Q values; in our case, in contrast, we need a resonator with a rather low Q that can be easily realizable by intentionally increasing resonator losses. Then sufficient bandwidth will be attained.

To harness the noise power, some nonlinearity in the system should be present. With modern semiconductors, such as Schottky diodes [9] or CMOS rectifiers (Complementary Metal Oxid Semiconductors) [4], [5], [6], [7], [8] contemporary employed in RFID devices (Radio Frequency Identification Devices), the GHz part of the noise frequency range may be used. After rectifying, the noise energy may be stored in a condenser as a charge. Even if the noise available power in a resonant circuit is negligibly small, the energy can be greatly increased by letting the power integrate over a long-time interval.

Conclusions:

Therefore, in principle, by increasing the number of RLC circuits, an arbitrary rms output voltage can be achieved. However, the available power remains the same as for *one* RLC circuit. Noise energy in a resonant circuit can be considered as that of a one sole molecule in an ideal gas.

This contemplation is outlined in the bottom-up direction, i.e. from the well-known theory of thermal noise and resonant circuits. No attempt has been made with a view to elucidating the resistors' thermal balancing. In the appendix, a numerical example is presented.

Universities are highly encouraged to perform this low-technology experiment at GHz frequencies with multiple PCB resonators and Schottky diode or CMOS rectifiers.

Any comments would be highly welcome.

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Appendix:

Microwave oven frequency: $f_0 = 2.45$ GHz $= 2.45 \times 10^9$ Hz

Inductor series resistance: $R_s = 50\ \Omega$

RLC circuit's figure of merit: Q = 15

Bandwidth: $\Delta f = \frac{\omega_0}{4Q} = \frac{2\pi\times 2.45\times 10^9}{4\times 15} = 2.56 \times 10^8$ Hz = 256 MHz

Inductance: $L = \frac{QR_s}{\omega_0} = \frac{15\times 50}{2\pi\times 2.45\times 10^9} = 4.87 \times 10^{-8}$ H = 48.7 nH

Capacitance : $C = \frac{1}{Q\,R_s\,\omega_0} = \frac{1}{15\times 50\times 2\pi\times 2.45\times 10^9} = 8.7 \times 10^{-14}$ F = .087 pF

Boltzmann constant k: 1.38×10^{-23} J/K

Thermal energy at room temperature T = 300 K: $k\,T = 4.14 \times 10^{-21}$ J

rms voltage at series resistor:

$\langle u_{R_s}\rangle = \sqrt{4kTR_s\Delta f} = \sqrt{4 \times 4.14 \times 10^{-21} \times 50 \times 2.56 \times 10^8} = 1.46 \times 10^{-5}$ V = 14.6 μV

Available power: $P_{av} = \frac{\langle u_{R_s}\rangle^2}{4R_s} = \frac{(1.46\times 10^{-5})^2}{4\times 50} = 1.07 \times 10^{-12}$ W = 1.07 pW

Using another formula:

$$P_{av} = kT\Delta f = 4.14 \times 10^{-21} \times 2.56 \times 10^8 = 1.07 \times 10^{-12}\ \text{W} = 1.07\ \text{pW}$$

rms voltage at the condenser: $\langle u_C\rangle = \sqrt{\frac{kT}{C}} = \sqrt{\frac{4.14\times 10^{-21}}{8.7\times 10^{-14}}} = 2.18 \times 10^{-4}$ V = 218 μV

or another expression: $\langle u_C\rangle = Q\langle u_R\rangle = 15 \times 1.46 \times 10^{-5} = 2.18 \times 10^{-4}$ V = 218 μV

Inductor (or series resistor) current: $\langle i_L\rangle = \sqrt{\frac{kT}{L}} = \sqrt{\frac{4.14\times 10^{-21}}{4.87\times 10^{-8}}} = 2.92 \times 10^{-7}$ A = 292 nA

Power loss at the series resistor:

$P_{Rs} = \langle i_L\rangle^2 R_S = (2.92 \times 10^{-7})^2 \times 50 = 4.26 \times 10^{-12}$ W = 4.26 pW

A parallel resistor to the condenser (considering a non-resistive inductor) can replace the series one:

Parallel resistance: $R_p = Q^2R_s = 15^2 \times 50 = 1.125 \times 10^4\ \Omega = 11.25$ kΩ

Power loss at the parallel resistor:

$P_{Rp} = \frac{\langle u_C\rangle^2}{R_p} = \frac{(2.18\times 10^{-4})^2}{1.125\times 10^4} = 4.26 \times 10^{-12}$ W = 4.26 pW

which is the same as for the series resistor.

Considerations above concern an unloaded RLC circuit only. For a loaded (and simultaneously matched) case, some changes need to be made.

Note that if the product QR_s remains constant, then L, C and $\langle u_C\rangle$ will be the same. To achieve a matched output, the original losses at the series resistor must be equally distributed between the series resistor and the parallel one, which also represents a load. To achieve it, the series resistor must be halved, while the Q factor will be doubled.

$R_s' = R_s / 2 = 25\ \Omega$, $Q' = 2Q = 30$

Parallel resistance $R_p' = Q'^2 R_s' = 30^2 \times 25\ \Omega = 2.25 \times 10^4\ \Omega = 22.5\ k\Omega$

Output current for a matched load will be:

$$i_{out} = \frac{\langle u_C \rangle}{R_p'} = \frac{2.18 \times 10^{-4}}{2.25 \times 10^4} = 9.69 \times 10^{-9}\ A = 9.69\ nA$$

If more identical RLC circuits are connected as a cascade in series:

rms total output voltage: $(u_C) = (n)^{1/2} (u_C)$ and the load resistor R_{pn} parallel to the cascade will be

$$R_{pn} = n\, R_p'$$

where n is a number of RLC circuits.

Consider n = 25, then $\langle u_C \rangle_{TOT} = \sqrt{25}\, \langle u_C \rangle = 5 \times 2.18 \times 10^{-4}\ V = 1.09 \times 10^{-3}\ V = 1.09\ mV$

at the matched load resistor $R_{pn} = 25 \times R_p' = 25 \times 2.25 \times 10^4\ \Omega = 5.63 \times 10^5\ \Omega = 563\ k\Omega$

By adding the load resistor, the original Q = 15 and bandwidth will be restored.

The output voltage is ample to be rectified with modern Schottky diodes (or CMOS) rectifiers. Note that the input resistance of the rectifier represents the load resistor.