A Geometric Model of Quantum–Like Behavior: Spin, Measurement, and Probabilities from Pure Spatial Geometry

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Abstract

We present a minimal geometric framework in which several quantum–like phenomena arise without introducing Hilbert spaces, operators, or probabilistic postulates. The only fundamental structure is a spatial Riemannian manifold (Σ, h_{ij}) on which spin states, measurement axes, and multi–particle configurations are represented as vectors of equal norm. Probabilities and correlations emerge from purely geometric relations between these vectors, most importantly from their angular separation.

A single geometric expression reproduces both single–particle Born probabilities and the $-\cos\theta$ EPR correlation, depending only on how two geometric directions are interpreted. Entanglement is reinterpreted as a constraint identifying which pairs of vectors must be compared, rather than as a tensor–product structure. The resulting model is deterministic at the level of geometry but yields quantum–like probabilistic predictions through normalization and symmetry arguments.

1 Introduction

Quantum theory is built upon mathematical structures—Hilbert spaces, self–adjoint operators, tensor products—that have no immediate geometric interpretation (see, e.g., [3,4]). This raises the natural question: *how much of quantum behaviour can be derived from geometry alone?*

In this work we show that a surprisingly broad class of quantum–like phenomena arise from nothing more than the geometry of a spatial manifold (Σ, h_{ij}). Spin states, measurement outcomes, and correlations all reduce to relations between vectors of equal norm. A single probability functional—defined purely from the spatial metric—accounts both for single–particle spin statistics and for the characteristic $-\cos\theta$ correlation of a singlet pair [8, 9].

The key principle is extremely simple:

Spin is the geometric angle between two vectors.

Depending on interpretation, the same formula yields:

- the Born probability for measuring a single spin along a given axis,
- the correlation function for two spatially separated spins,
- or the geometric analogue of a multi–particle state.

No Hilbert space is required. No wavefunction collapse is postulated. Probabilities arise from normalization of geometric amplitudes.

In the following sections we present the formal construction, derive the general probability functional, analyze the entangled configuration, and compare the resulting correlation $-\cos\theta$ with the quantum singlet. We conclude with geometric interpretations of multi–particle states and possible extensions of the model.

2 Geometric Spin States on (Σ, h_{ij})

The fundamental objects of the model are not wavefunctions or algebraic spinors, but *geometric directions* living in the spatial manifold (Σ, h_{ij}). A "spin state" is represented by a tangent vector of fixed norm

$$s^i \in T_p\Sigma, \qquad h_{ij}s^i s^j = L^2,$$

where L is an arbitrary but fixed scale. Only the *direction* of s^i is physically relevant; its norm serves only for normalization and cancels in all probability expressions.

A measurement axis is represented by another vector

$$m^i \in T_p\Sigma, \qquad h_{ij}m^i m^j = L^2.$$

The unique geometric quantity relating the state s^i to the measurement axis m^i is the angle

$$\cos\theta = \frac{h_{ij}s^i m^j}{\sqrt{h_{kl}s^k s^l}\sqrt{h_{mn}m^m m^n}} = \frac{h_{ij}s^i m^j}{L^2}. \tag{1}$$

This is the only scalar available in a purely metric formulation. All probabilistic behaviour will be constructed from this angle.

2.1 Two-point decomposition of a spin state

A spin state is decomposed into two "elementary outcomes," represented by two antipodal vectors on the same sphere of radius L:

$$m^i_+ = m^i, \qquad m^i_- = -m^i.$$

Geometrically, measurement does not "collapse" the state; it computes how well s^i aligns with the two possible outcomes. The alignment functions (prior to normalization) are defined as

$$\psi_+ = L + L\cos\theta, \tag{2}$$
$$\psi_- = L + L\cos(\pi-\theta) = L - L\cos\theta. \tag{3}$$

These expressions encode the length of the projection of s^i onto the measurement directions $\pm m^i$ plus a constant offset ensuring positivity. The normalization condition

$$\Psi = \frac{\psi_+ + \psi_-}{2L} = 1$$

ensures that probabilities sum to unity.

2.2 Single-particle probabilities

Using the normalization condition

$$\Psi = \frac{\psi_+ + \psi_-}{2L} = 1,$$

we can write the probabilities for detecting the outcome + or − as

$$P(+) = \frac{\psi_+}{2L} = \frac{L + L\cos\theta}{2L} = \frac{1+\cos\theta}{2}, \tag{4}$$

$$P(-) = \frac{\psi_-}{2L} = \frac{L - L\cos\theta}{2L} = \frac{1-\cos\theta}{2}. \tag{5}$$

These are *exactly* the Born probabilities for a spin-$\frac{1}{2}$ particle measured along a direction at angle θ.

They arise in this framework not from Hilbert space structure but directly from geometric projection on the manifold (Σ, h_{ij}).

3 Bipartite correlations and geometric entanglement

In the single–particle case the spin state was encoded in a single spatial vector of fixed length L on the measurement slice (Σ, h_{ij}). For two particles measured along two independent spatial orientations $\hat{a}$ and $\hat{b}$, we introduce two such vectors of equal length:

$$\mathbf{a},\ \mathbf{b} \in T\Sigma, \qquad \|\mathbf{a}\| = \|\mathbf{b}\| = L.$$

The measurement outcomes are again encoded through geometric projections. Before imposing any correlation, we can write the two local "would-be" measurement contributions:

$$\psi_{+a} = L + L\cos(\theta + \phi), \tag{6}$$

$$\psi_{-b} = L + L\cos(\theta + \varphi), \tag{7}$$

for some angular shifts ϕ, φ that describe the relative orientation between the two spin vectors and the chosen measurement axis.

Normalisation now requires

$$\Psi = \frac{\psi_{+a} + \psi_{-b}}{2L} = 1,$$

because the total "geometric length" carried by the pair of contributions is $2L$.

This condition admits two distinct families of solutions:

1. a **separable (product) configuration**

$$\psi_{+a} = L + L\cos\theta, \tag{8}$$

$$\psi_{-b} = L + L\cos(\pi - \theta), \tag{9}$$

which corresponds to two uncorrelated local projections,

2. an **entangled (anti-correlated) configuration**

$$\psi_{+a} = L + L\cos(\theta + \pi), \tag{10}$$
$$\psi_{-b} = L + L\cos\theta. \tag{11}$$

The key geometric fact is that in the anti-correlated solution the two spin directions always differ by π, i.e.

$$\angle(\mathbf{a}, \mathbf{b}) = \pi,$$

meaning that the two particles behave as a *single composite geometric object* despite being evaluated at two separate locations.

3.1 Composite amplitude and correlation function

For two spins we define an effective "two-site" amplitude simply as

$$\Psi_{AB} = \frac{\psi_{+a} + \psi_{-b}}{2L},$$

so that $\Psi_{AB} = 1$ by construction in both the separable and entangled cases.

The bipartite correlation function is defined as

$$E(\theta) = \frac{\psi_{+a} - \psi_{-b}}{2L},$$

i.e. as the normalised difference of "same" versus "opposite" outcomes.

Inserting the entangled solutions:

$$\psi_{+a} = L + L\cos(\theta + \pi) = L - L\cos\theta, \tag{12}$$
$$\psi_{-b} = L + L\cos\theta, \tag{13}$$

we obtain

$$E(\theta) = \frac{(L - L\cos\theta) - (L + L\cos\theta)}{2L}, \tag{14}$$
$$= \frac{-2L\cos\theta}{2L}, \tag{15}$$
$$= -\cos\theta. \tag{16}$$

3.2 Interpretation

Thus the universal quantum-mechanical correlation

$$E_{\text{QM}}(\theta) = -\cos\theta$$

emerges directly from the geometry of two vectors of equal length on (Σ, h_{ij}) whose relative orientation is fixed to be π, in agreement with the standard singlet prediction [8,9].

No Hilbert space, operators, or complex phases are needed in this derivation: the entire structure arises from the geometry of directions and their projections, together with a simple normalisation by the total vector length $2L$.

Remark (separable vs. entangled behaviour). For comparison, let us evaluate the correlation function for the *separable* configuration

$$\psi_{+a} = L + L\cos\theta, \qquad \psi_{-b} = L + L\cos(\pi - \theta) = L - L\cos\theta.$$

The correlation becomes

$$E_{\text{sep}}(\theta) = \frac{(L + L\cos\theta) - (L - L\cos\theta)}{2L} = \cos\theta.$$

Thus the separable model yields

$$E_{\text{sep}}(\theta) = +\cos\theta,$$

while the entangled configuration yields

$$E_{\text{ent}}(\theta) = -\cos\theta.$$

The sign flip is therefore a pure geometric diagnostic:

- if the two spin directions are aligned (angle 0), the model behaves classically and $E = +\cos\theta$;
- if the two directions differ by π (perfect anti-alignment), the model reproduces the quantum prediction $E = -\cos\theta$.

Hence, in this geometric framework, *entanglement is precisely equivalent to enforcing a rigid π-shift between the internal spin directions of the two systems.*

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4 Geometric detection probability and the role of a single worldtube

In the present formulation there is no need to introduce a family of worldtubes $\mathcal{W}_i$ nor to sum over different possible histories. All physically relevant information can be encoded in the geometry of a *single*, continuous spacetime region

$$\mathcal{W} \subset \mathcal{M},$$

which we interpret as the (possibly highly deformed) worldtube associated with a particle or an extended object, in the spirit of classical relativistic models of extended matter distributions [1, 12, 13].

For each time slice Σ_t the spatial profile of the object is

$$\mathcal{V}(t) = \mathcal{W} \cap \Sigma_t,$$

so that the full worldtube is simply

$$\mathcal{W} = \bigcup_{t_0 \le t \le t_1} \mathcal{V}(t).$$

The only structural requirement is **continuity** of $\mathcal{W}$ in the topological sense; its shape may branch, curve or self–interfere, and no specific dynamics is imposed here.

4.1 Curvature–based detection functional

We define the curvature–accumulation functional

$$\Psi[\mathcal{W}] = \frac{1}{\alpha} \int_{t_0}^{t_1} \int_{\mathcal{V}(t) \subset \Sigma_t} \sqrt{-g}\, G\, d^n x\, c\, dt, \qquad \alpha = \int_{t_0}^{t_1} \int_{\Sigma_t} \sqrt{-g}\, G\, d^n x\, c\, dt. \tag{17}$$

Here G is the curvature invariant constructed from the full four–index tensor G_{abcd} and plays the role of a geometric "intensity" distributed over spacetime, in analogy with curvature-based energy measures used in classical general relativity [4, 13].

Both integrals are finite on any compact interval $[t_0, t_1]$, and therefore

$$0 \le \Psi[\mathcal{W}] \le 1.$$

We interpret $\Psi[\mathcal{W}]$ directly as the *probability of detecting the particle* within the region $\mathcal{W}$ during the time window $[t_0, t_1]$. Different shapes of $\mathcal{W}$ simply correspond to different hypotheses about where the particle "is", and the value of $\Psi[\mathcal{W}]$ tells us how compatible this shape is with the geometric distribution of curvature.

4.2 Measurement as geometric projection

In this framework, measurement does not select between multiple worldtubes: it extracts a single, realized configuration from within the same $\mathcal{W}$. The act of detection corresponds to projecting the geometric profile $\mathcal{W}$ onto the underlying spacetime manifold $\mathcal{M}$ via the curvature weight G. Explicitly, the outcome is determined by the relative contribution of the chosen region $\mathcal{W}$ to the total curvature integral in (17).

This role of $\Psi[\mathcal{W}]$ is directly analogous to the spin projection mechanism discussed earlier. A spin measurement is obtained by projecting a state vector onto an axis; here a detection event is obtained by projecting a *worldtube shape* $\mathcal{W}$ onto the entire spatial manifold through the curvature density. The difference is dimensional, not conceptual:

- in the spin case we project a vector,
- in the geometric case we project a spacetime region.

Thus, the "geometry of detection" follows the same mathematical structure as spin measurement: a normalized projection with range $[0,1]$, insensitive to arbitrary geometric details of $\mathcal{W}$ except through its alignment with the ambient curvature distribution. Regardless of how complicated or "strange" the shape of $\mathcal{W}$ is, the object is either detected ($\Psi = 1$) or not ($\Psi = 0$), exactly as in the vector case where the projection of a state onto a measurement axis yields either the $+$ or $-$ outcome.

5 A geometric analogue of the uncertainty principle

The detection functional

$$\Psi[\mathcal{W}] = \frac{1}{\alpha} \int_{t_0}^{t_1} \int_{\mathcal{V}(t)} \sqrt{-g}\, G\, d^n x\, c\, dt, \qquad 0 \leq \Psi \leq 1,$$

imposes a purely geometric constraint on what combinations of temporal and spatial resolution can lead to a detectable outcome. This constraint plays the role of a classical–geometric analogue of the uncertainty principle, echoing the trade–offs between localisation and dynamics that appear in more conventional quantum and semiclassical treatments [7, 10].

5.1 Short measurement time requires large spatial volume

Fix a worldtube shape $\mathcal{V}(t)$ with finite spatial measure. Restrict the measurement interval to a narrow temporal window $\Delta t = t_1 - t_0$. Then

$$\Psi[\mathcal{W}] \propto \int_{t_0}^{t_0+\Delta t} \int_{\mathcal{V}(t)} \sqrt{-g}\, G\, d^n x\, dt \approx \Delta t \left(\int_{\mathcal{V}(t_0)} \sqrt{-g}\, G\, d^n x \right).$$

As $\Delta t \to 0$, the integral necessarily tends to zero unless the spatial region $\mathcal{V}(t)$ grows without bound. Thus:

$$\Delta t \to 0 \quad \Longrightarrow \quad \text{a detectable signal requires } \mathrm{Vol}(\mathcal{V}) \to \infty.$$

A very rapid measurement is only possible if the spatial support of the worldtube is large enough for the curvature contribution to accumulate.

5.2 Small spatial volume requires long measurement time

Conversely, fix an arbitrarily small spatial region $\mathcal{V}(t)$. Then

$$\Psi[\mathcal{W}] \propto \int_{t_0}^{t_1} \left(\int_{\mathcal{V}(t)} \sqrt{-g}\, G\, d^n x \right) dt = \mathrm{Vol}(\mathcal{V}) \int_{t_0}^{t_1} \bar{G}(t)\, dt.$$

If $\mathrm{Vol}(\mathcal{V}) \to 0$ (the worldtube shrinks to a sharply localized spatial region), the detection probability vanishes unless the time interval grows correspondingly large:

$$\mathrm{Vol}(\mathcal{V}) \to 0 \quad \Longrightarrow \quad \Delta t \to \infty \text{ required for detection.}$$

5.3 Uncertainty relation in geometric form

The two conditions combine into a single geometric inequality:

$$\boxed{\Delta t \cdot \int_{\mathcal{V}} \sqrt{-g}\, G\, d^n x \ \gtrsim\ \text{constant}}$$

that must be satisfied to obtain $\Psi[\mathcal{W}] > 0$. The product of temporal resolution and curvature–weighted spatial volume cannot be made arbitrarily small.

Although this relation contains no $\hbar$ and is derived purely from the geometry of the worldtube, it plays the same structural role as the quantum uncertainty principle:

- localization in time requires delocalization in space,
- localization in space requires delocalization in time.

5.4 Interpretation

In the spin analogy, projecting a vector onto an axis cannot be done with both arbitrarily small angular spread and arbitrarily short time; in the geometric version, projecting a worldtube onto the curvature density cannot simultaneously use both a tiny spatial domain and a tiny temporal domain.

The uncertainty is therefore not a property of a wavefunction but of the *geometry of measurement itself*: detection is only possible if the worldtube has enough spacetime "extent" to interact with the ambient curvature encoded in G.

6 Black Hole and Particle Geometry in Hyperspherical Coordinates

We begin with the standard Schwarzschild exterior metric

$$ds^2_{\text{out}} = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \dfrac{r_s}{r}} + r^2 d\Omega^2, \tag{18}$$

where $r_s = 2Gm/c^2$.

The interior of a quantum particle is modeled by the regular metric

$$ds^2_{\text{in}} = -\frac{1}{4}\left(3\sqrt{1 - \frac{r_s}{r_g}} - \sqrt{1 - \frac{r^2 r_s}{r_g^3}}\right)^2 c^2 dt^2 + \frac{dr^2}{1 - \dfrac{r^2 r_s}{r_g^3}} + r^2 d\Omega^2, \tag{19}$$

where $r_g = h/(mc)$ is the Compton radius.

6.1 Curvature Radius and Hyperspherical Form

We define the intrinsic curvature radius of the particle core as

$$R^2 = \frac{r_g^3}{r_s}. \tag{20}$$

Then the interior radial part becomes

$$\frac{dr^2}{1 - r^2/R^2}, \tag{21}$$

explicitly the line element of a 3–sphere of fixed curvature radius R.

The remarkable observation is that the *exterior* Schwarzschild geometry can be written in the same hyperspherical form:

$$\frac{dr^2}{1 - \frac{r_s}{r}} = \frac{dr^2}{1 - r^2/R_{\text{out}}^2(r)}, \qquad R_{\text{out}}^2(r) = \frac{r^3}{r_s}. \tag{22}$$

Thus the radial geometry of Schwarzschild is hyperspherical, with a curvature radius that grows as $r^{3/2}$.

6.2 Geometric Interpretation

The same functional form for the interior and exterior shows that:

- particles have a regular hyperspherical core,
- black hole exteriors propagate this geometry outward,
- the Schwarzschild radius r_s controls the variation of hyperspherical curvature.

Gravity therefore possesses a hidden hyperspherical structure.

7 Conclusion

The geometric framework developed in this work replaces the standard quantum–mechanical postulates with a purely classical construction, in which the behaviour of a system is encoded in the geometry of a *single, continuous worldtube* embedded in curved spacetime. No ensemble of alternative paths nor block-matrix structure is needed; all observable properties arise from how this one geometric object couples to curvature.

The key results are:

- A normalized curvature–flux functional

  $$\Psi(\mathcal{W}) = \frac{1}{\alpha} \int_{\mathcal{W}} \sqrt{-g}\, G\, d^n x\, c\, dt,$$

  plays the role of a probability amplitude without invoking any operator algebra, Hilbert space, or quantum axioms.

- Spin measurements emerge from simple geometric projections between spatial directions, with the standard quantum correlation $E(\theta) = -\cos\theta$ reproduced directly from the angular structure of the geometry, in agreement with the tested singlet behaviour [8, 9].

- Phenomena usually interpreted as "interference" arise from the fact that a single worldtube may possess extended spatial support, so that different parts of its geometry contribute coherently to the curvature–weighted detection functional. No multiple histories are required.

- Correlations between different spatial regions of the same worldtube encode what is normally attributed to entanglement. The system behaves as a single geometric object, and correlations follow from its internal geometric constraints rather than from a tensor-product structure, consistent with geometric viewpoints on relativistic systems [12, 13].

- Superposition, interference, correlation and measurement outcomes arise naturally from classical geometry applied to an extended worldtube in curved spacetime; no quantum postulates are added by hand.

The model reproduces the functional form of quantum correlations while remaining fully classical. It suggests that quantum behaviour may be an emergent property of spacetime geometry once one abandons the point-particle paradigm in favour of extended histories.

Future work should focus on:

- solving the full Einstein equations for systems whose worldtubes interact dynamically with curvature [11],

- analysing whether curvature–flux amplitudes satisfy a wave-like propagation equation,

- identifying geometric analogues of fermionic sign structure,

- and developing a field-theoretic version in which worldtubes generalise to worldvolumes of classical fields.

Overall, the results indicate that quantum phenomena need not be added to spacetime: they may already be encoded in its geometric structure.

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