MATHEMATICAL FOUNDATIONS OF THE TEMPORAL STRUCTURE OF THE RUNNING MOVEMENT SYSTEM AND ITS UNITY

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Abstract

Based on a geometric model of the periods of support and flight of running steps and a subogram of double-step movements, the optimal temporal structure of running locomotives is shown in the article. The hypothesis is substantiated that the unity of the "Double step running" object-system is due to the "golden" proportional relations of the object-system to its subsystems and between the subsystems themselves. The revealed patterns of the human locomotion system are not only of theoretical value for biomechanics, but also of practical value for robotics.

1 Introduction

The modern understanding of motor actions is based on a system-structural approach that allows us to consider the body as a moving system. The system-structural approach to the study of movements is implemented in N.A. Bernstein's theory of the structurality of movements. The scientist claims that "Movement is not a chain of details, but a structure that differentiates into details" [1, 2, 3]. His own research allows us to scientifically reasonably assume the presence of certain patterns of the human locomotion system [4, 5, 6, 7, 8]. Identifying the patterns of the human locomotion system is a significant biomechanics problem. In this regard, the present study has been undertaken.

2 The main part

The object of this study is running locomotion. The subject of the study was the temporal structure of the running movement system.

As a visual representation of the ratio of the duration of the studied movements, a geometric model of the optimal running method was created (figure 1) [4, 6, 7]. This model contains segments commensurate with the duration of the depreciation phase of the reference period (segment AB, red), the repulsion phase of the reference period (segment BO, blue), and the unsupported period (segment OB_1, green). The guides AO, OA_1 form the final shape of the geometric model in the form of right triangles.

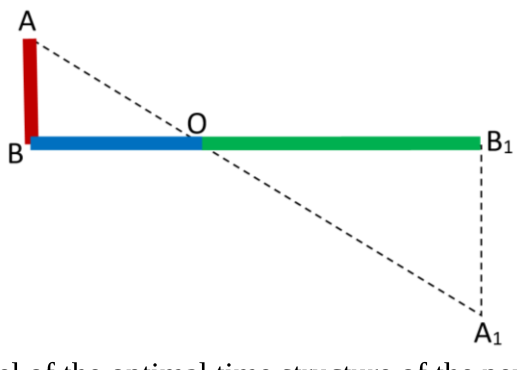

Figure 1 is a geometric model of the optimal time structure of the periods of support and flight of running steps, where: AB +BO is a commensurate display of the duration of the reference period (AB/BO = 0.618...); AB is the duration of depreciation; BO is the duration of repulsion; OB_1 is the duration of the unsupported period in running; $AO/OA_1 = 0.618…$

The optimal running technique (using the example of the geometric model shown in Figure 1) arises in conditions when, in relation to the "golden" proportion, the duration of the repulsion phase is correlated with the duration of the depreciation phase in the reference period and, accordingly, the duration of the unsupported period is correlated with the duration of the repulsion phase of the reference period according to formula 1 [6].

$$BO/AB=OB_1/BO \tag{1}$$

Using digital data on the duration of depreciation of the reference period of a running step and the geometric model of the periods of support and flight of running steps in running (Figure 1), as well as the Pythagorean theorem, it is possible to calculate the duration of repulsion of the reference period of a running step according to formula 2.

$$BO=\sqrt{(AO^2–AB^2)} \tag{2}$$

Knowing the duration of the repulsion of the reference period of the running step and using the geometric model of the periods of support and flight of running steps in running (Figure 1), it is possible to calculate the duration of the unsupported period of the running step in accordance with formula 3.

$$OB_1=BO\times1{,}618\ldots \tag{3}$$

Knowing the value of the running activity coefficient (KA), it is possible to calculate the ratio of the duration of individual periods and phases of a running step in the following way [6]. The value of this coefficient (for example, 1.3) is divided by the value of the "golden ratio" – 1.618 ... according to the proposed formula (4). We obtain the relative value of the phase of repulsion of the reference period ($Ф_{от}$ = 0.8034441853748632...). If we subtract the obtained value of the "repulsion" phase from the conventional unit, we get the relative value of the "depreciation" phase ($Ф_{ам}$ =0.1965558146251368...) (formula 5). Thus, using relatively simple mathematical operations, we obtain a series of relative values (coefficients) of two periods of running stride – repulsion (depreciation and repulsion) and flight in the form of the following series of relative values: 0.196...∝ 0.803... ∝ 1.3.

$$Ф_{от}=КА/1{,}618\ldots \tag{4}$$

$$Ф_{ам}=1– Ф_{от} \tag{5}$$

Empirical verification of the hypothesis about the presence of optimal temporal structures in running locomotives with saving physical effort (or when running "for endurance") was carried out in studies [4, 5, 6].

By the structure of the system, we mean a set of stable connections between the elements of the system that ensure the integrity of the system and its identity. At the same time, the presence of connections between the elements of the system leads to the appearance of new properties in the integrated system that are not inherent in the elements individually.

The structural elements in the system are not located by themselves, but are connected to each other. By such a connection we mean any kind of relationship between the parts of the system. The maximum number of heterogeneous (with different ratios) connections in a system is determined by the number of possible combinations between the elements. If, for example, the system consists of five elements, then the maximum number of heterogeneous connections for it is 10 (excluding repetitions). In this case, only one relationship (relation) is allowed between the two elements. The formation of more than one (in this case, two) structural links is possible if the system is built on the basis of the so–called "golden" ratio or the principle of the "golden" ratio

(Figure 2). In this case, for example, for two subsystems, not one, but two homogeneous (with the same ratio) structural relationships arise. A similar number of homogeneous connections among the many subsystems in a system can characterize its unity: the more homogeneous connections the parts of the system have, the more unified the whole system appears.

Figure 2 – Division according to the "Golden" ratio, where B/A=C/B

An increase in the number of subsystems in such a system leads to a relatively large increase in its homogeneous structural relationships in accordance with formula 6:

$$A=n\times2–2 \tag{6}$$

where: A is the number of structural links; n is the number of elements in the structure.

The unity of the "Double step running" object-system is probably due to the "golden" proportional relations of the object-system to its subsystems and between the subsystems themselves [5]. For example, the ratio of the "golden" proportion between the fixation of the forearm and its movement, between the movement of the shin and its fixation (Figures 3, 4). Thus, the unity of the temporal structure of individual movements of running locomotives may be due to the largest (of possible variants) number of "golden" ratios between subsystems, and hence the largest number structural connections (mathematical relations) between them. Any other coefficient shows a smaller number of structural connections in the system under consideration (equalities 7-14).

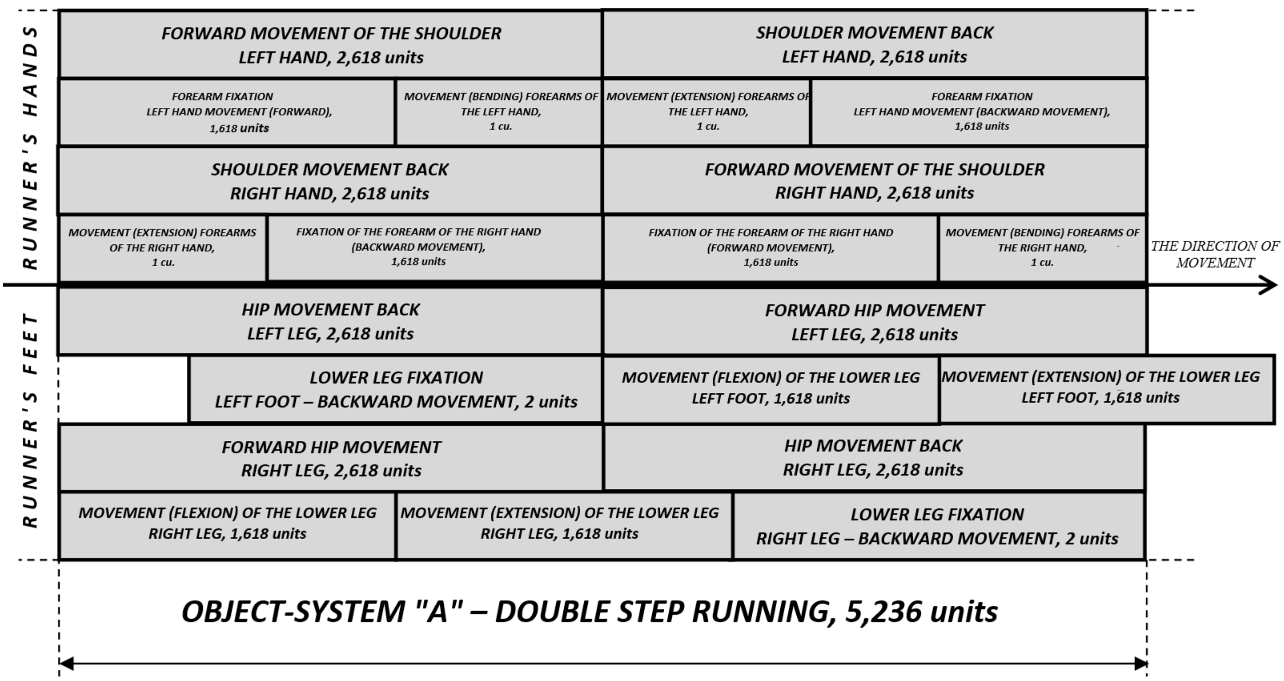

Figure 3 is a sub–graph of the temporal structure of double-step movements in running [5], where "Φ" ((√5+1)/2=1,618) and "ϕ" ((√5+1)/2)-1=0,618) – constants of the "golden" proportion

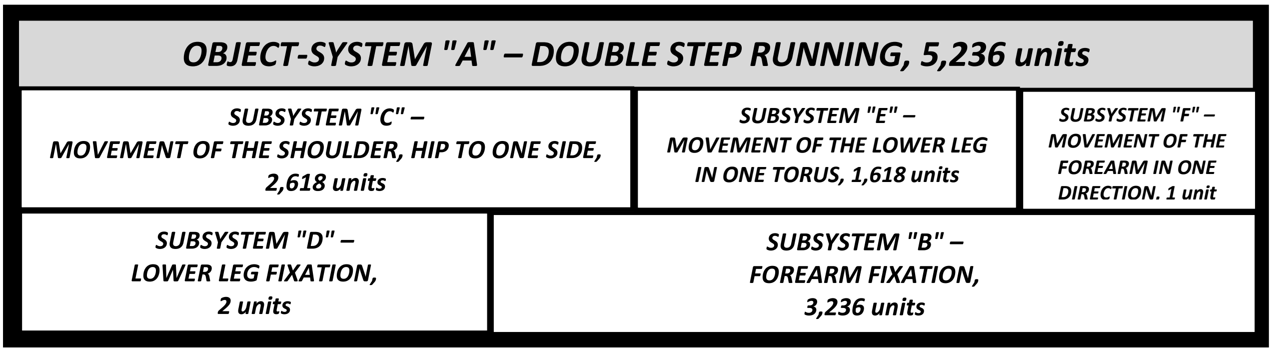

Figure 4 is a block diagram of the duration of athletes' movements in the "double step" cycle of running as the relationship of the object-system "A" to the objects-subsystems "B", "C", "D", "E", "F" [5], where the components of an object-system with a size of 5,236 conventional units (cu) are subsystems with sizes 1; 1,618; 2; 2,618; 3,236 cu.

$$\Phi=1{,}618=A/B=B/D=C/E=E/F=(E+E)/D=B/(F+F)=(C+E)/C=(B+F)/C \quad (7)$$

$$A=C+E+F=B+D=B+F+F=D+E+E=C+C \quad (8)$$

$$B=E+E \quad (9)$$

$$C=E+F \quad (10)$$

$$D=F+F \quad (11)$$

$$A=C\times D=B\times E=F\times A=C\times E+F=D\times E+(F+F) \quad (12)$$

$$B=D\times E=E\times(F+F) \quad (13)$$

$$C=E\times E \quad (14)$$

Equality 12 is noteworthy: the numerical value of the system object "A" is equal to the product of two different subsystems included in the system (C×D=B×E=F×A).

The patterns of the human locomotion system are not only a theoretical issue of biomechanics, but also a practical component for robotics. Namely, an important part of building or creating a humanoid robot is solving the problem of programming the movements of the "running" machine. The basic mathematical algorithms of this locomotion in a robot should be based on the laws of biomechanics similar to human locomotion. In general, using the laws of biomechanics of human locomotion will significantly improve the quality of movements in humanoid robots.

3 Conclusion

1. Based on a geometric model of the periods of support and flight of running steps and a subogram of double-step movements, the optimal temporal structure of running locomotives is shown.

2. The hypothesis is substantiated that the unity of the object-system "Double step running" is due to the "golden" proportional relations of the object-system to its subsystems and between the subsystems themselves.

3. The revealed patterns of the human locomotion system are not only of theoretical value for biomechanics, but also of practical value for robotics.

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