

A Relationship Between position and number

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Abstract

By observing the relative positions of odd composite numbers in the set of odd natural numbers up to a given n , the positions of the prime numbers can be logically derived by subtraction. Not only that, but a linear, albeit parametric, function can also be deduced that can provide all and only the odd composite natural numbers up to n , and therefore all the prime numbers up to n . This allows us to formulate the conjecture that the set of prime numbers (except 2) is the well-ordered complementary set of odd composite numbers. This ordering can also be seen using the Cartesian line $y = 2x + 1$. Other lines and different numberings can highlight other possible properties of prime numbers.

Have you ever wondered about the order of prime numbers and their connection to positive integers' positions? These questions intrigued me, compelling me to explore the intricate relationship between the position and number of prime integers. Starting with the observation that, aside from 2, prime numbers are all odd, I began ordering them numerically.

I organized them into two columns, associating odd numbers with the first column and even numbers with the second. This arrangement revealed a fascinating pattern:

p	Odd	Even
0	1	2
1	3	4
2	5	6
3	7	8
4	9	10
5	11	12
6	13	14
7	15	16
8	17	18
9	19	20
10	21	22
...		

An idea started taking shape—a real connection between the positions corresponding to odd numbers and the numbers themselves. The following relationship emerged:

n	p	n+p+n
3	1	4
4	7	10
13	16	29
5	2	7
12	17	29
22	27	49
7	3	10
17	24	41
31	38	69
9	4	13
22	31	53
40	49	89
11	5	16
16	27	43
38	49	87
60	60	120
13	6	19
32	45	77
58	71	129
15	7	22
37	52	89
67	82	149
17	8	25
42	59	101
76	93	169
19	9	28
47	66	113
85	104	189
104	104	208

Continuing this pattern, each new position identified through the sum of the odd number and its position, and again with the number, produced a relationship connecting the odd number and the position:

$$p' = p + kn$$

These positions exclusively correspond to positive composite numbers, implying that the unoccupied positions in the table are solely those occupied by prime numbers.

This relationship can be expressed as the parametric equation: $y = kx + (k-1)/2$ where y represents a positive integer like x , and k is an odd positive integer. If there are integer values of k and x that satisfy the equation, y is a composite number; otherwise, it is a prime number.

With this equation, a program can be devised to test the primality of a number. Examples include positions tested for primality, accompanied by their relative execution times on a basic laptop.

The subsequent tests revealed that the last tested position, 10272, with a value of $2 \times 10272 + 1$, is not prime:

Program to test the primality of positions, published by Gasbion 01/13 Returns position and relative prime number or position, first divisor and its composite number

Position to be tested from (minimum 4, to exit 0): 10262

To: 10272

10262 5 20525

10263 13 20527

10264 3 20529

10265 7 20531

10266 20533

10267 3 20535

10268 11 20537

10269 19 20539

10270 3 20541

10271 20543

10272 5 20545

It took 0.1839585304260254 seconds

Below is the Python program developed based on the parametric equation:

```
# Python program to test the primality of positions # Python program to test the primality of
positions import math import time
# Python program to test the primality of positions
import math
import time
while True:
    def main():
        print('Program to test the primality of positions, published by Gasbion 01/13')
        print('Returns position and relative prime number or position, first divisor and its
composite number')

        a = int(input('Position to be tested from (minimum 4, to exit 0): '))
        if a == 0:
            exit()

        b = int(input('To: '))
        p = a
        k = 3

        t_ini = time.time()

        while a <= b:
            while k <= math.sqrt(2 * p + 1):
                if (p - int(k / 2)) % k == 0:
                    print(p, k, 2 * p + 1)
                    break
                k += 2
            else:
                print(p, 2 * p + 1)

            p += 1
            k = 3
```

```

if p > b:
    break

```

```

t_fin = time.time()
print("It took", str(t_fin - t_ini), "seconds")

```

```

if __name__ == '__main__':
    main()

```

This original result, born from observation and intuition, enhances our understanding of prime numbers: the **prime numbers constitute the well-ordered set complementary to that of the odd composite natural numbers.**

Another intriguing revelation emerged from the Cartesian representation of the line $y=2x+1$. By considering this line applied to positive integers, we can obtain all odd integers y as x varies between positive integers. By adding the values of x and y , we obtain the new value $x+y$, which, when assigned to the variable x through the same equation, gives us a new value y' for example:

```

x+y  y'
1+3  9
2+5  15
3+7  21
4+9  27

```

This process generates composite numbers, showcasing the marvelous capability of the Cartesian graphic representation of the straight line to provide all prime numbers (except 2) through the linear system composed by:

```

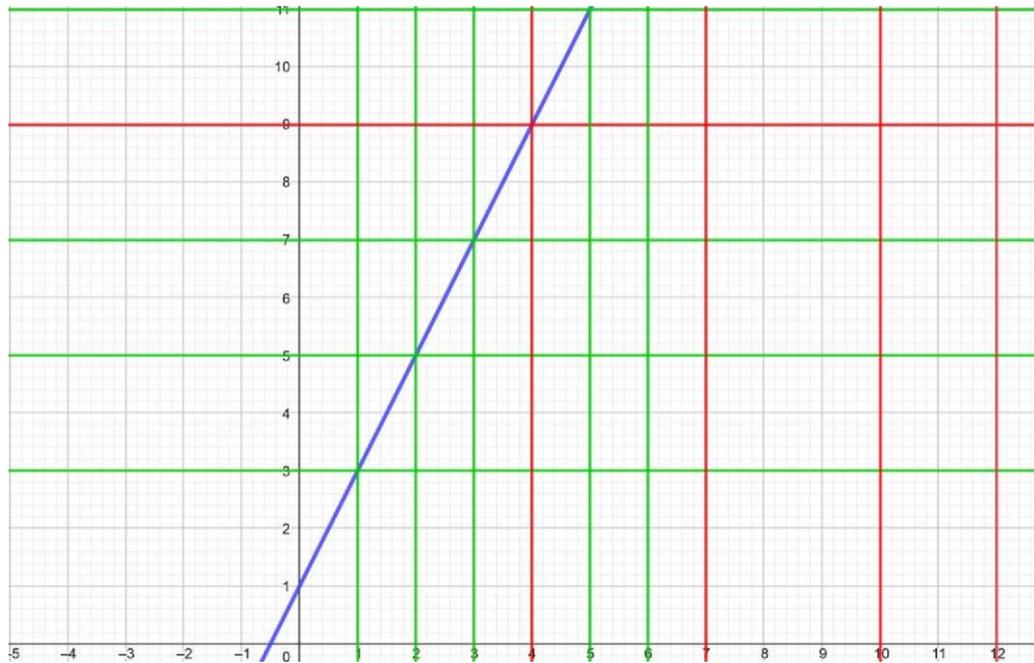
y = 2x+1
x' = x+y+ky
y' = 2x'+1

```

Changing the value of the positive integer k in the inequality $x' \neq x+y+ky$ provides all positive integer values of y' , representing all prime numbers. A similar concept applies to the Cartesian representations of the lines: $y = 3x$ $y = 3x + 1$ $y = 3x + 2$

For example, the second line describes positions occupied by composite numbers, while the first and third lines describe positions occupied by both composite and prime numbers. By multiplying the position values obtained from the first and third lines by 2 and adding one, we get whole numbers corresponding to both prime and composite numbers.

This concept opens up avenues of exploration, allowing us to study positions through Cartesian representations applied to positive integers.



Another study, correlated with the concept of position and applied to a different function and calculation, is constituted by Cartesian representations (applied to positive integers) that involve numbering with modulo 12.

Numbering by twelve instead of ten, we have the following table:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
 22 23 24
 25 26 27 28 29 30 31 32 33 34 35 36
 37 38 39 40 41 42 43 44 45 46 47 48
 49 50 51 52 53 54 55 56 57 58 59 60
 Etc...

Prime numbers were found only in four columns:

0 1 5 7 11 13 17 19 23
 2 25 29 31 35
 3 37 41 43 47
 4 49 53 55 59
 5 61 65 67 71
 6 73 77 79 83

7 85 89 91 95 Etc...

Specifically, in the columns corresponding to one, five, seven, and eleven.
 Taking the column of one and extracting the square root of the numbers...

1 1
 13 3,605551
 25 5
 37 6,082763
 49 7
 61 7,81025

73 8,544,004
 85 9,219,544
 97 9,848,858
 109 10,440,31
 121 11

Numbers with an integer root correspond to multiples of 12: the square of these numbers, represented by the curve $y = x^2$, is equal to the k-th position of the number multiplied by twelve plus one, as in the formula $n^2 = 12k + 1$.

Here, k corresponds to the position in column one of the number, starting from the zero position, which corresponds to n equal to one. Continuing, it corresponds to the position for the number thirteen, and so on.

Conversely, we obtain the position p, equal to $(n-1) / 12$.

In this set of square roots (which correspond to whole numbers), all prime numbers are included, although not exclusively, except for two and three: From the above reports, we obtain the positions p that correspond to composite numbers and those that correspond to prime numbers, both represented by the line $y = 12x + 1$:

P	1	13	3.605551	1.898829
 2 | 25 | 5 | 2.236068 |
 3 | 37 | 6.082763 | 2.466326 |
 4 | 49 | 7 | 2.645751 |
 5 | 61 | 7.81025 | 2.794682 |
 6 | 73 | 8.544004 | 2.923013 |
 7 | 85 | 9.219544 | 3.03637 |
 8 | 97 | 9.848858 | 3.138289 |
 9 | 109 | 10.44031 | 3.231146 |
 10 | 121 | 11 | 3.316625 |
 ... | ... | ... | ... | ...

The following table shows the values of positions p that correspond only to composite numbers ($n = p * 12 + 1$). Positions not present in the table, thus constructed and extended up to a certain p, represent positions corresponding to prime numbers ($n = p * 12 + 1$):

2 4 10 14 24 30 44 70 80.....
 7 11 21 27 41 49 67 99 111.....
 12 18 32 40 58 68 90 128 142.....
 17 25 43 53 75 87 113 157 173.....
 22 32 54 66 92 106 136 186 204.....
 27 39 65 79 109 125 159 215 235.....
 32 46 76 92 126 144 182 244 266.....
 Etc...

And below are positions p that correspond to prime numbers (such as integer radicals of n), represented in the curve $y = \sqrt{12x + 1}$. Note that not for all values of x does the function return positions occupied by prime numbers!

4 49 7
 10 121 11
 14 169 13
 24 289 17
 30 361 19

44 529 23
 70 841 29
 80 961 31 114 1369 37

It should be noted that the latter end either by 0 or by 4 except for position two which corresponds to the prime number 5.



The representation of integers in the form of twelve is also very interesting with regard to twin primes: numbering for twelve all the prime numbers (but 2 and 3) can be arranged around the rectangle of side 2 and 4 respective distances starting from 5 in the lower right passing to 7 in the upper right, to 11 in the upper left to 13 in the lower left and continuing to rotate then counter-clockwise (it would be the same also in a clockwise direction starting with the 5 in the lower left). It is interesting to note that all quadruples of numbers, in this way, are generated 2 by 2 from the possible pairs of numbers that generate twin primes (and not). If we distribute them on four columns, we see that they have (roughly) the same number of prime numbers: infinite. But these 4 columns are generated as we have seen two by two from the possible twin primes, is it enough to say that they could themselves be infinite?

