# Clustering Words by Graph based AHC Variants

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***Abstract*—In this research, we propose and apply the graph based AHC variants to the word clustering. The initial AHC version which clusters graphs was previous proposed as an approach to the word clustering. In this research, we mention the three AHC variants: one where the data clustering proceeds in the bottom-up direction with the similarity threshold, one where it allows any merge of more than two pairs, and one where clusters are merged based on the radius. In this research, we modify the three AHC variants into the graph-based versions, as well as the initial AHC version. As the goal of this research, we improve the clustering performance, by modifying them so.**

## I. Introduction

The word clustering is defined as the process of partitioning a group of words into subgroups by their similarities. The process of clustering words depending on their spellings is called lexical word clustering or syntactical word clustering. The word clustering which is covered in this research is called semantic word clustering where words are clustered depending on their meanings; the lexical word clustering is excluded from this research. It is assumed that the word clustering to which what is proposed in this research is applied is the hard clustering where each word is arranged into only one cluster, but it will be expanded into the soft word clustering or the hierarchical word clustering in future research. An individual word which is entity is given as a single string with its own meaning.

This research is motivated by the successful application of the graph based AHC algorithm to the word clustering in the previous research. Words were encoded into graphs, instead of numerical vectors, and the similarity between two graphs was defined based on their edges. The AHC algorithm was modified into the graph-based version which processes graphs directly and applied to the word clustering. Its better performance than the traditional version was empirically validated in clustering words in various domains. We derive some AHC variants and modify them into their graph-based version as the approaches to the word clustering.

The idea of this research is to modify the three AHC variants into the graph-based versions as the approaches to the word clustering. In the previous work, the initial AHC algorithm was modified into the version which takes a graph as its input, and its word clustering performance was improved. In this research, we propose the three AHC variants: the version which clusters data items in the bottom-up direction with the parameter, similarity threshold, the version where it is allowed to merge more than one pair in each iteration, and the version the radius from a cluster is used for merging it with other clusters. In this research, we modify the three AHC variants into the graph-based versions by the similarity metric between graphs and apply them to the semantic word clustering. This research is intended to improve the performances of the AHC variants.

Let us mention some benefits from this research. The huge dimensionality and the sparse distribution as the problems in encoding words into numerical vectors are solved by encoding them into graphs. Its variants are proposed for improving the clustering speed instead of the AHC algorithm. The AHC variants are modified into the graph-based versions for improving the clustering performance as well as the clustering speed. The better approaches to the word clustering are provided with respect to both the clustering performance and the clustering speed, as the goal of this research.

Let us mention the organization of this paper. In Section II, we explore the previous works which are relevant to this study. In Section III, we describe in detail what is proposed in this research. In Section IV, we mention the significances of this research and the remaining tasks. This paper is composed of the four sections.

## II. Previous Works

This section is concerned with the previous works which are relevant to this research. It is intended to expand the style of modifying the AHC algorithm to its three variants. The directions of exploring previous works are derivation of AHC variants, modification of the standard AHC algorithm into its graph-based version, and the cases of encoding a text into a graph. The distinguishable points of this research are derivation of three AHC variants from the standard version, modification of them into graph-based versions, and their applications to the word clustering. This section is intended to explore previous works for providing the background for this research.

Let us explore the previous cases of deriving AHC variants. In 2012, Murtagh and Contreras mentioned the possibility of deriving various versions of AHC algorithm and a particular variant which uses nearest neighbors [3]. In 2013, Sasirekha and Baby presented various similarity metrics between vectors and strategies of defining a similarity between clusters [4]. In 2015, Nazari et al. proposed the AHC algorithm which merges two clusters based on their common nearest neighbors [5]. In the AHC algorithms which are mentioned in above previous literatures, only one pair of clusters is merged in each iteration.

Let us explore the previous works which are concerned with the application of the modified version of the standard AHC algorithm to word clustering. In 2016, Jo initially proposed that the graph-based version should be applied to the word clustering [6]. In 2018, Jo validated empirically its performances in the word clustering [8]. In 2023, he prepares the journal article which describes the graph-based AHC algorithm and its application to the word clustering for its publication [11]. What is provided by the previous works becomes the basis for this research.

Let us explore the previous works on representation of a text into a graph. In 2004, the process of representing a text into a graph was mentioned by Hensman [1]. In 2007, encoding a text into a graph for doing data mining tasks was covered by Jin and Srihari [2]. In 2020, the matching criteria of graphs which represent texts was proposed by Osman and Barukub [10]. The scheme of encoding a text into a graph and the similarity matric between graphs may be different in this research.

Let us mention the differences of this research from the previous works which are explored above. In this research, we derive the AHC variants as fast clustering algorithms by allowing merging multiple cluster pairs for each iteration. The three AHC variants are modified into graph-based versions and adopted for implementing the word clustering system. In this research, the graph based AHC variants are developed as clustering algorithms as well as classification algorithms. We will describe the modified AHC variants in Section III.

## III. Proposed Work

This section is concerned with what is proposed in this research. In Section III-A, we describe the process of encoding a word into a graph and the proposed similarity metric. In Section III-B, we describe the standard version of AHC algorithm where the proposed similarity metric is adopted. In Section III-C, we derive the three varaints from the standard version. In Section III-D, we present the system architecture of the word clustering system.

### A. Similarity Metric

This section is concerned with the graph as a word representation and the similarity between graphs. In representing a word into a graph, a vertex indicates a text, and an edge indicates a similarity between texts. The similarity between edges is computed, before computing the similarity between graphs, viewing a graph as a set of edges. The similarity between graphs is computed by averaging over the similarities of all possible pairs of edges. In this section, we describe the process of encoding a word into a graph and the process of computing the similarity between graphs.

The process of encoding words into graphs is illustrated in Figure 1. In the vertex definition, from a text collection, texts which are relevant to the word are retrieved as the vertices. In the edge definition, the similarities among the retrieved texts are computed as edges. In the edge selection, the edges with their higher similarities are selected for representing a graph. Refer to [7] for studying the encoding process in more detail.

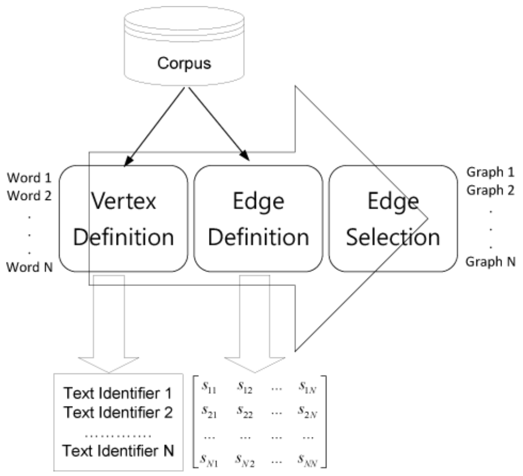


Figure 1. Process of Encoding Words into Graphs

The three cases of computing the similarity between two edges are illustrated in Figure 2. Each weight which is associated with its own edge is assumed to be a normalized value between zero and one, and if both nodes are identical to each other, the average over two weights is the similarity between two edges. If either of the two nodes is identical to each other, the product of two weights is the similarity between two edges. If neither of two nodes is identical, zero is the similarity between them. The similarity between two edges is always a normalized value between zero and one.

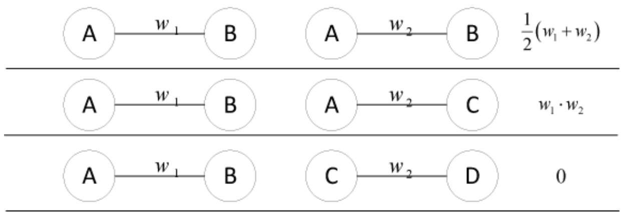

A $w_1$ B A $w_2$ B $\frac{1}{2}(w_1+w_2)$
A $w_1$ B A $w_2$ C $w_1 \cdot w_2$
A $w_1$ B C $w_2$ D 0

Figure 2. Three Cases of Comparing Two Edges with Each Other

Let us mention the process of computing the similarity between graphs as a semantic similarity between words. The two graphs which represent words are notated by edge sets, $G_1 = \{e_{11}, e_{12}, \ldots, e_{1|G_1|}\}$ and $G_2 = \{e_{21}, e_{22}, \ldots, e_{2|G_2|}\}$. Two edges, $e_{1i} \in G_1$ and $e_{2j} \in G_2$, are associated with their own weights, $w_{1i}$ and $w_{2j}$, and the similarity between two edges, $e_{1i}$ and $e_{2j}$ by equation (1), (2), or (3), depending on the cases which are mentioned above,

$$sim(e_{1i}, e_{2j}) = \frac{1}{2}(w_{1i} + w_{2j}) \tag{1}$$

$$sim(e_{1i}, e_{2j}) = w_{1i} \times w_{2j} \tag{2}$$

$$sim(e_{1i}, e_{2j}) = 0 \tag{3}$$

The similarity between two graphs is computed by equation (4),

$$sim(G_1, G_2) = \frac{1}{|G_1| \times |G_2|} \sum_{i=1}^{|G_1|} \sum_{j=1}^{|G_2|} sim(e_{1i}, e_{2j}). \tag{4}$$

The value which is generated from equation (4), is always given as a normalized value between zero and one as shown in equation (5),

$$0 \leq sim(G_1, G_2) \leq 1. \tag{5}$$

Let us make some remarks on the process of encoding a word into a graph and computing the similarity between graphs. A word is encoded into a graph by the vertex definition, the edge definition, and the edge weighting. The three cases are considered for computing the similarity between edges. The similarity between graphs is computed by equation (4). In the future research, we consider representing a word into multiple graphs.

## B. Initial Version of Graph based AHC Algorithm

This section is concerned with the initial version of graph based AHC algorithm. The version of AHC algorithm was initially proposed as the approach to the text clustering by Jo in 2019 [9]. The similarity metric between graphs which was described in the previous section is used for computing the similarity between clusters. The AHC algorithm proceeds clustering data items with the bottom-up direction. This section is intended to describe the initial version of AHC algorithm from which its variants are derived.

The initial status for applying the AHC algorithm to the word clustering is illustrated in Figure 3. The words as clustering targets are encoded into graphs by the process, which was described in Section III-A, and they are notated by a set, $Tr = \{G_1, G_2, \ldots, G_N\}$. The clusters are defined as many as data items, as $C_1, C_2, \ldots, C_N$, and each cluster includes a data item as $C_i = \{G_i\}$. The initial status which is represented in Figure 00 is notated by $C_1 = \{G_1\}, C_2 = \{G_2\}, \ldots, C_N = \{G_N\}$. The cluster with only one item is called singleton, and singletons are constructed as many as data items in the initial status.

Graph 1
Graph 2
...
Graph N

Figure 3. Initial Clusters in using AHC Algorithm: Singletons

The process of computing the similarity between two clusters is illustrated in Figure 4. The two clusters are notated by two sets of items, $C_1 = \{G_{11}, G_{12}, \ldots, G_{1|C_1|}\}$ and $C_2 = \{G_{21}, G_{22}, \ldots, G_{2|C_2|}\}$. The similarity between clusters, $C_1$ and $C_2$, is computed by equation (6),

$$sim(C_1, C_2) = \frac{1}{|C_1| \cdot |C_2|} \sum_{i=1}^{|C_1|} \sum_{j=1}^{|C_2|} sim(G_{1i}, G_{2j}). \tag{6}$$

If the similarity metric which was described in Section III-A is adopted, the similarity between two clusters is always a normalized value between zero and one, as expressed in equation (7),

$$0 \leq sim(C_1, C_2) \leq 1. \tag{7}$$

The complexity of computing the similarity between two clusters is expressed by $O(|C_1||C_2|)$.

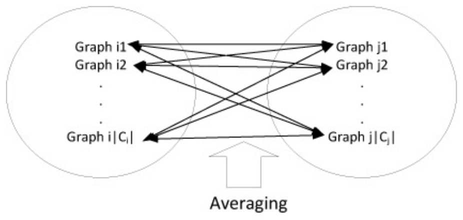
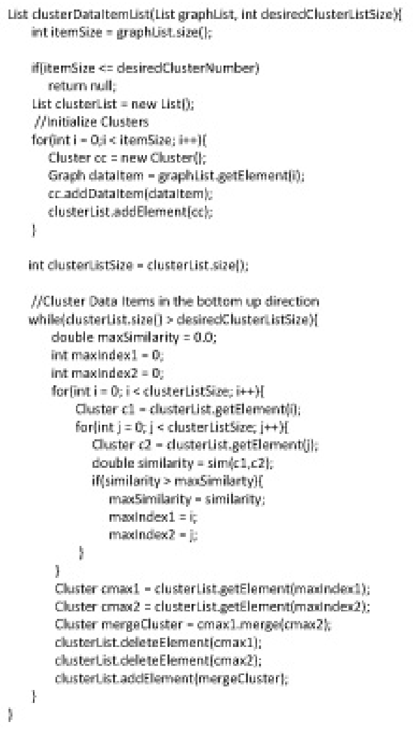


Figure 4. Similarity between Clusters

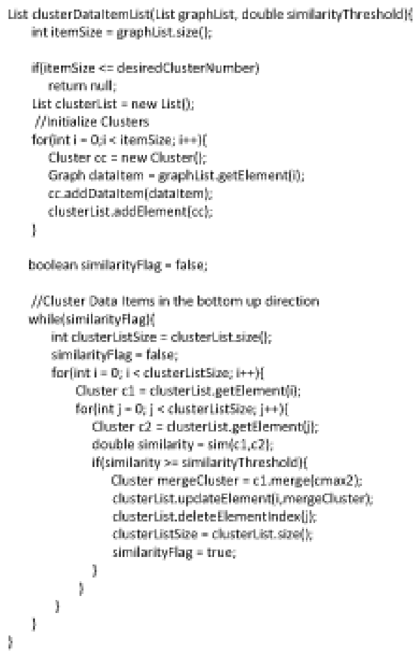
The process of clustering data items is illustrated as pseudo codes in Figure 5. The AHC algorithm begins with the status where singletons are given as many as data items. All possible pairs of clusters are generated from the current cluster list, their similarities are computed by equation (6), and the cluster pair with the highest similarity is merged into a cluster. In the AHC algorithm, the data items are clustered by iterating computing similarities of all possible cluster pairs and merge a cluster pair into a cluster. In each iteration, one cluster is decreased, and this iteration continues until reaching the desirable number of clusters.

Let us make some remarks on the initial version of the AHC algorithm from which we derive the variants. The initial status in applying the AHC algorithm to data clustering is that there are singletons as many as data items. The similarity between clusters is computed by averaging the similarities of all possible pairs from them. Data items are clustered by iterating computing the similarities of all possible cluster pairs and merge the pair with its maximal similarity into one cluster. In its variant, we will consider allow more than one pair of clusters to be merged.



```
List clusterDataItemList(List graphList, int desiredClusterListSize){
    int itemSize = graphList.size();

    if(itemSize <= desiredClusterNumber)
        return null;
    List clusterList = new List();
     //Initialize Clusters
    for(int i = 0;i < itemSize; i++){
        Cluster cc = new Cluster();
        Graph dataItem = graphList.getElement(i);
        cc.addDataItem(dataItem);
        clusterList.addElement(cc);
    }

    int clusterListSize = clusterList.size();

    //Cluster Data Items in the bottom up direction
    while(clusterList.size() > desiredClusterListSize){
        double maxSimilarity = 0.0;
        int maxIndex1 = 0;
        int maxIndex2 = 0;
        for(int i = 0; i < clusterListSize; i++){
            Cluster c1 = clusterList.getElement(i);
            for(int j = 0; j < clusterListSize; j++){
                Cluster c2 = clusterList.getElement(j);
                double similarity = sim(c1,c2);
                if(similarity > maxSimilarty){
                    maxSimilarity = similarity;
                    maxIndex1 = i;
                    maxIndex2 = j;
                }
        }
        Cluster cmax1 = clusterList.getElement(maxIndex1);
        Cluster cmax2 = clusterList.getElement(maxIndex2);
        Cluster mergeCluster = cmax1.merge(cmax2);
        clusterList.deleteElement(cmax1);
        clusterList.deleteElement(cmax2);
        clusterList.addElement(mergeCluster);
    }
}
```

Figure 5. Initial Version of AHC Algorithm



```
List clusterDataItemList(List graphList, double similarityThreshold){
    int itemSize = graphList.size();

    if(itemSize <= desiredClusterNumber)
        return null;
    List clusterList = new List();
     //Initialize Clusters
    for(int i = 0;i < itemSize; i++){
        Cluster cc = new Cluster();
        Graph dataItem = graphList.getElement(i);
        cc.addDataItem(dataItem);
        clusterList.addElement(cc);
    }

    boolean similarityFlag = false;

    //Cluster Data Items in the bottom up direction
    while(similarityFlag){
        int clusterListSize = clusterList.size();
        similarityFlag = false;
        for(int i = 0; i < clusterListSize; i++){
            Cluster c1 = clusterList.getElement(i);
            for(int j = 0; j < clusterListSize; j++){
                Cluster c2 = clusterList.getElement(j);
                double similarity = sim(c1,c2);
                if(similarity >= similarityThreshold){
                    Cluster mergeCluster = c1.merge(cmax2);
                    clusterList.updateElement(i,mergeCluster);
                    clusterList.deleteElementIndex(j);
                    clusterListSize = clusterList.size();
                    similarityFlag = true;
                }
            }
        }
    }
}
```

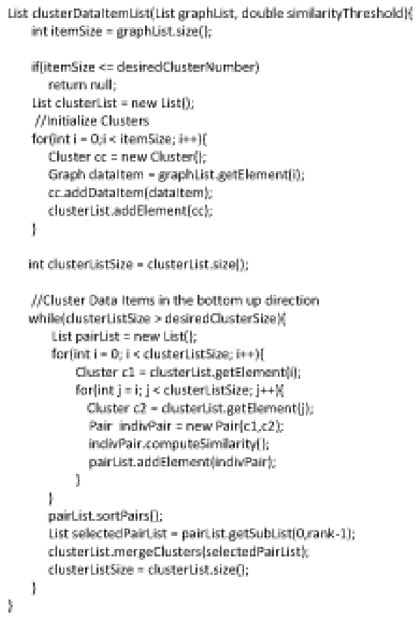
Figure 6. Graph based AHC Variant 1: Similarity Threshold Based AHC Algorithm

### C. AHC Variants

This section is concerned with the variants which are derived from the AHC algorithm which was covered in Section III-B. The difference from the traditional version of AHC algorithm is to adopt the similarity metric between graphs for computing the similarity between clusters. In this section, we derive the three variants by merging cluster pairs by adopt the threshold-based selection, allowing merge of multiple pairs in each iteration, and merging clusters within the radius. In this research, we adopt the three variants of AHC algorithm for implementing the word clustering system. This section is intended to describe the three variants of AHC algorithm.

In Figure 6, we illustrate the process of clustering data items by the first variant of AHC algorithm. The clusters are initialized with singletons like the initial version which was described in Section III-B. All possible cluster pairs are generated, and the cluster pairs whose similarities are more than or equality to the similarity threshold are merged. If there is no pair which satisfies the condition, clustering data items is terminated. In this variant, the number of clusters is decreased variably in this variant.

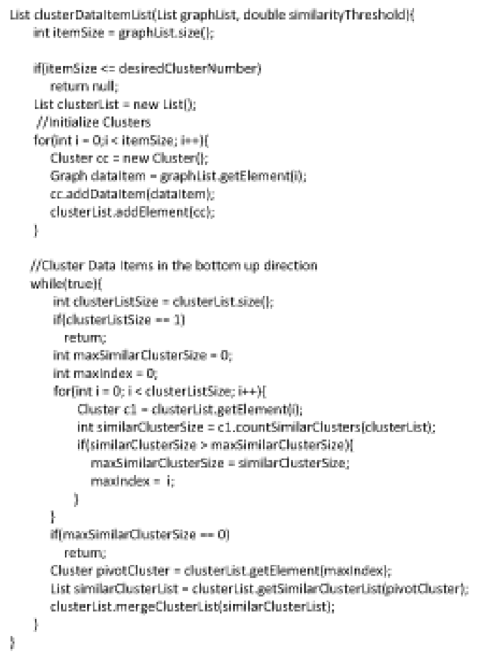
We illustrate the process of clustering data items by the second variant of AHC algorithm in Figure 7 as a pseudo code. The number of cluster pairs which are merged into each iteration is given as an additional parameter in this variant. All possible cluster pairs are generated, and the similarity is computed for each pair. The cluster pairs are sorted by the descending order of the similarity, and the pairs within the rank are merged. The difference of this variant from the initial version is to merge multiple cluster pairs, instead of one pair.



```
List clusterDataItemList(List graphList, double similarityThreshold){
    int itemSize = graphList.size();

    if(itemSize <= desiredClusterNumber)
        return null;
    List clusterList = new List();
     //Initialize Clusters
    for(int i = 0;i < itemSize; i++){
        Cluster cc = new Cluster();
        Graph dataItem = graphList.getElement(i);
        cc.addDataItem(dataItem);
        clusterList.addElement(cc);
    }

    int clusterListSize = clusterList.size();

    //Cluster Data Items in the bottom up direction
    while(clusterListSize > desiredClusterSize){
        List pairList = new List();
        for(int i = 0; i < clusterListSize; i++){
            Cluster c1 = clusterList.getElement(i);
            for(int j = i; j < clusterListSize; j++){
              Cluster c2 = clusterList.getElement(j);
              Pair  indivPair = new Pair(c1,c2);
              indivPair.computeSimilarity();
              pairList.addElement(indivPair);
            }
        }
        pairList.sortPairs();
        List selectedPairList = pairList.getSubList(0,rank-1);
        clusterList.mergeClusters(selectedPairList);
        clusterListSize = clusterList.size();
    }
}
```

Figure 7. Graph based AHC Variant 2: Merge of Multiple Pairs

In Figure 8, we illustrate the process of clustering data items by the last variant of AHC algorithm. The similarity threshold is given as an external parameter, and the number of other clusters whose similarity is greater than or equality to the similarity threshold is counted. In each iteration, the cluster with its maximal number of its similar clusters is appointed as the pivot, and the pivot cluster and its similar ones are merged into one cluster. This variant iterates selecting the pivot cluster in the current cluster list and merge the pivot and its similar ones, as the process of clustering data items. If there is no cluster with its similar cluster, the iteration is terminated.



```
List clusterDataItemList(List graphList, double similarityThreshold){
     int itemSize = graphList.size();

     if(itemSize <= desiredClusterNumber)
        return null;
     List clusterList = new List();
      //Initialize Clusters
     for(int i = 0;i < itemSize; i++){
        Cluster cc = new Cluster();
        Graph dataItem = graphList.getElement(i);
        cc.addDataItem(dataItem);
        clusterList.addElement(cc);
     }

    //Cluster Data Items in the bottom up direction
    while(true){
        int clusterListSize = clusterList.size();
        if(clusterListSize == 1)
          return;
        int maxSimilarClusterSize = 0;
        int maxIndex = 0;
        for(int i = 0; i < clusterListSize; i++){
             Cluster c1 = clusterList.getElement(i);
             int similarClusterSize = c1.countSimilarClusters(clusterList);
             if(similarClusterSize > maxSimilarClusterSize){
                maxSimilarClusterSize = similarClusterSize;
                maxIndex = i;
             }
        }
       if(maxSimilarClusterSize == 0)
          return;
       Cluster pivotCluster = clusterList.getElement(maxIndex);
       List similarClusterList = clusterList.getSimilarClusterList(pivotCluster);
       clusterList.mergeClusterList(similarClusterList);
    }
}
```

Figure 8. Graph based AHC Variant 3: Radius based AHC Algorithm

Let us make some remarks on the three variants of AHC algorithm which are adopted as the approaches to the word clustering. In each iteration of the first variant, cluster pairs with their similarities which are greater than or equal to the threshold are merged. In each iteration of the second variant, a fixed number of cluster pairs with their highest similarities are merged. In each iteration of the third variant, a pivot cluster and its similar clusters are merged. We may derive more variants from the AHC algorithm by setting different policies on merging clusters.

## D. System Architecture

This section is concerned with the architecture and the execution flow of the word clustering system. In Section III-C, we described the three AHC variants which are adopted for implementing the word clustering system. The initial version of AHC algorithm which is mentioned in Section III-B is replaced by its variants in the word clustering system which was proposed. The process of executing the system is to encode words into graphs and cluster them. This section is intended to describe the word clustering system which is implemented in this study.

The process of encoding the words which are given as clustering targets into graphs. The offline clustering where all words are given at a time is assumed in implementing the word clustering system. The words are encoded into graphs by the process which is described in Section III-A. In the system, the graphs which represent the words are clustered by the AHC algorithms which are described in Section III-C. The online clustering will be considered where words are given as a continual stream in the next research.

The system architecture of the word clustering system is illustrated in Figure 9. The role of encoding module is to encode words into graphs. The similarity computation module which is a nested module computes the similarity between numerical vectors by the process, which is described in Section III-A, they are clustered by one among the AHC variants which are described in Section III-C in the clustering module. Clusters of graphs are generated in the clustering module, they are decoded into words, and word clusters are generated as the final output from the system. The upgrading point of the system architecture which is proposed in [9] is to replace the initial version of AHC algorithm by its variants.

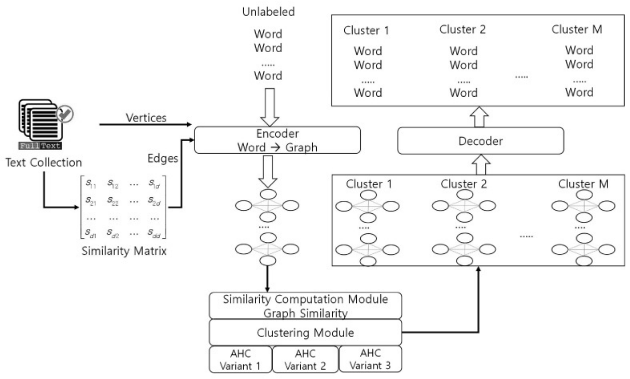


Figure 9. System Architecture

The execution flow of the word clustering system is illustrated in Figure 10. The words are encoded into graphs by the word encoding process. The similarity metric between graphs is used for computing the similarity between clusters. Data items are clustered by one which is selected among the three AHC variants which were described in Section III-C. The external parameter, similarity threshold, should be controlled in the first and the third variant for reaching the desired number of clusters.

Let us make some remarks on the proposed version of word clustering system which is presented in Figure 9 as its architecture. The offline clustering is assumed in implementing the system; this system should be expanded into doing

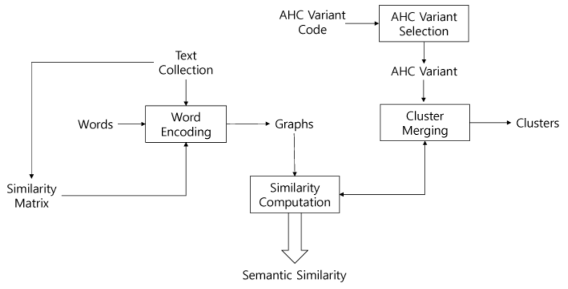


Figure 10. System Flow

the online clustering in the next research. The upgrading point of this system is to replace the initial version of AHC algorithm by one of the three variants which are described in Section III-C. Words are encoded into graphs, the similarity between clusters is computed by equation (4), and clustering data items proceeds by one of the three AHC variants. We expect the improvement of the clustering performance and speed by replacing the initial AHC algorithm by its variants.

## IV. Conclusion

Let us mention the significances of this research as the conclusion. We derive the three variants from the standard version by allowing multipe clusters to be merged at a time. We encode words into graphs and apply the similarity metric among them to the AHC variants as well as the standard version. We design the word clustering system by adopt the AHC variants with the proposed similarity metric. In the next research, we will implement the word clustering system as a real system.

Let us mention the remaining tasks as the further discussions on this research. It is necessary to improve the speed of computing the similarity between graphs in the implementation level. Other clustering algorithms such as the divisive clustering algorithm and the online linear clustering algorithm should be modified into the graph-based versions. The proposed versions of AHC variants should be applied to the text clustering as well as the word clustering. The word clustering system should be implemented by adopting the proposed approaches as a real program.

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