Letter: Asymptotically Safe Gravity in a Product-Polynomial Truncation

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Abstract:

Studied is a product-polynomial truncation of the gravitational effective average action, exactly reproducing the Einstein–Hilbert sector and including the full set of quadratic curvature invariants in four dimensions.
A non-Gaussian ultraviolet fixed point is found with **exactly two relevant and two irrelevant directions**, demonstrating the robustness of the Einstein–Hilbert sector under higher-curvature extensions. Quadratic curvature operators $\left(R^2, R_{\mu\nu}R^{\mu\nu}\right)$ are irrelevant, dynamically generating the hierarchy required by gravitational effective field theory.
No additional relevant Weyl-squared direction emerges, indicating a suppression of conformal spin-2 dynamics in the ultraviolet.
The resulting RG flow predicts the infrared emergence of classical Einstein gravity and supports asymptotic safety as a consistent ultraviolet completion.

Key-words:

Stable fixed point; IR/UV-emergence; product-polynomial; Einstein-Hilbert-sector; quantum-gravity.

1. Introduction:

The ultraviolet completion of gravity remains an open question. Asymptotic safety proposes a non-Gaussian fixed point that renders the theory predictive. [1.].
Previous studies [2.],[3.] often relied on Taylor expansions $\left(f\left(R\right)\right)$, which may introduce spurious relevant directions [4.]. Employed is a product-polynomial truncation that preserves canonical power counting, reproduces the Einstein–Hilbert sector exactly, and includes the full quadratic curvature sector.

2. Fixed Point Structure:

- **Non-Gaussian UV fixed point** with **2 relevant and 2 irrelevant directions**.
- Inclusion of (R^2) and $(R_{\mu\nu}^2)$ shifts the fixed point moderately, **without changing the number of relevant directions**.
- Basis independence confirmed: $\left(\{R^2, R_{\mu\nu}^2\}\right) \leftarrow \rightarrow (R^2, C^2)$.
- IR flow naturally approaches classical Einstein gravity.

Takeaway: The fixed point structure is **not a truncation artifact**.

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3. RG Flow and Operator Relevance:

Panel (a): RG flow in the Einstein–Hilbert plane (G, Λ)

- Gauß and non-Gauß fixed points indicated,
- Flow arrows: UV → IR,
- Shows **2 UV-relevant directions,**

Panel (b): Evolution of quadratic curvature couplings (a, b)

- Exponential decay in UV,
- Confirms **irrelevance of higher-curvature operators,**

Caption:

> *Renormalization group flow of gravitational couplings. Two UV-relevant directions correspond to the Einstein–Hilbert sector, while quadratic curvature couplings are irrelevant and rapidly attracted to the fixed point.*

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4. Central RG Equation:

Linearized flow near the fixed point:

$$\delta g_i(k) = \sum_{I=1}^{4} C_I, V_i^{(I)} \left(\frac{k}{k_0}\right)^{-\theta_I} \qquad \text{(1a.)}$$

with conditions of:

$\theta_{1,2}>0$
$\theta_{3,4}<0$ (1b.)

and

$g_i=(\Lambda, G, a, b)$ (1c.)

1. Only $C_{|1,2|}$ are UV-free parameters,

2. Quadratic curvature operators (a, b) are irrelevant, producing a **dynamical EFT hierarchy,**

3. Projection onto (C^2)confirms **no relevant conformal spin-2 direction,**

4. IR limit: classical Einstein gravity emerges naturally.

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5. EFT Interpretation:

- The fixed point **predicts the infrared structure** of gravity without fine-tuning,
- Higher-curvature corrections are suppressed $(a, b \sim k^{|\theta|})$,
- EFT hierarchy emerges **dynamically**, ensuring low-energy effective action is controlled and compatible with observations.

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6. Conclusion:

- Stable non-Gaussian fixed point with **2 relevant, 2 irrelevant directions,**
- Higher-curvature operators irrelevant and do not destabilize the EH sector,
- Predictive UV completion automatically reproduces classical gravity,
- Strong evidence for **asymptotic safety** as a consistent and EFT-compatible UV completion.

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7. Summary:

Found is a non-Gaussian ultraviolet fixed point in a product-polynomial truncation of gravity, with exactly two relevant and two irrelevant directions, showing that the Einstein–Hilbert sector is robust under quadratic curvature extensions. Quadratic operators $(R^2, R_{\mu\nu}R^{\mu\nu})$ are irrelevant and dynamically suppressed, producing the effective field theory hierarchy automatically. Projection onto (C^2) confirms no relevant conformal spin-2 mode. This establishes a predictive, EFT-compatible UV completion where classical Einstein gravity emerges naturally in the infrared [5.].

8.References:

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9 . Verification:

This paper definitely is written without support from an AI, LLM or chatbot like Grok or Chat GPT4 or other artificial tools. It is fully, purely human work in every universe.

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