RELATIVISTIC ACTION AT A DISTANCE FORMULATION OF THE STRONG INTERACTION

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ABSTRACT

We demonstrate that the strong interaction can also be formulated as an action-at-a-distance theory. Starting from the nonlinear gluon wave equation, we derive an exact momentum-space propagator for the gluon with a dynamical mass determined by the color current. To make its momentum dependence analytically manageable, we also obtain an exact discrete spectral representation of the propagator, whose lower-order truncations reproduce the Refined and Very Refined Gribov-Zwanziger propagators used in confinement studies, but now with a dynamical origin for their mass scales. Ordinary Yang-Mills theory and the free gluon propagator are recovered in the ultraviolet regime, where confinement becomes irrelevant.

1 INTRODUCTION

A complete and consistent description of the fundamental interactions remains a central challenge in theoretical physics. Action-at-a-distance approaches offer technical and conceptual advantages over the contact-action approach used in field theory and general relativity: they are inherently many-body theories, whereas field theory is "*essentially a one-body theory*";[1] they employ physical masses and charges and so avoid unphysical bare quantities that must be removed by renormalization; they ensure conservation laws; they evade classical paradoxes such as the "*4/3 problem*"; they provide ontological clarity by eliminating fields as fundamental entities and thereby remove gauge-dependent surplus degrees of freedom; and they are computationally efficient by dispensing with infinite field degrees of freedom.

These strengths motivated Wheeler and Feynman to reformulate classical electrodynamics as an action-at-a-distance theory, though their program faced challenges such as the need for a cosmological absorber condition to explain temporal asymmetry and radiative phenomena, and the complete absence of a Hamiltonian formulation. Hoyle and Narlikar extended these ideas toward quantum electrodynamics and explored the application to the weak interaction.[2,3]

The action-at-distance model also appears in constituent-quark models, which describe hadronic phenomena with a many-body Hamiltonian. A common relativistic form is

$$H = \sum_{i}^{N} \sqrt{m_i^2 c^4 + \boldsymbol{p}_i^2 c^2} + \mathcal{V}^{\text{strong}}, \tag{1}$$

where $\sqrt{m_i^2 c^4 + \boldsymbol{p}_i^2 c^2}$ is the energy of quark i, with mass m_i and momentum $\boldsymbol{p}_i$, and the potential $\mathcal{V}^{\text{strong}}$ describes direct interactions between quarks. A widely used choice is the Cornell potential, combining a Coulombic term with a linear confinement term[4,5]

$$\mathcal{V}^{\text{strong}} = -\sum_{ij}^{N} \frac{\alpha}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} - \beta|\boldsymbol{r}_i - \boldsymbol{r}_j|, \tag{2}$$

where α and β are adjustable parameters. More sophisticated potentials, including spin and relativistic corrections, are available.[6,7] Despite the empirical success of such Hamiltonians, they do not constitute a complete action-at-a-distance formulation of the strong interaction. A recent proposal to construct an action-at-a-distance description of Yang-Mills interactions[8] amounts only to a reinterpretation of the color current in the ordinary Yang-Mills equations and does not modify the underlying local field-theoretic structure.

In this article we construct a genuine action-at-a-distance formulation of the strong interaction, showing how non-Abelian color dynamics, perturbative limits, and confinement phenomenology can be encoded in direct interparticle kernels for quark-quark interactions.

2 THE ELECTROMAGNETIC INTERACTION

Before developing the new formalism, we briefly review the action-at-a-distance reformulation of quantum electrodynamics to establish the foundations for our treatment of interactions. As noted in the introduction, Hoyle and Narlikar showed that quantum electromagnetic phenomena can be described in terms of charges acting directly, without introducing an independent electromagnetic field.[2] Our approach differs in important technical respects.

The Hamiltonian for a system of N interacting charges is

$$H = \sum_{i}^{N} \sqrt{m_i^2 c^4 + \boldsymbol{p}_i^2 c^2} + \mathcal{V}^{\mathrm{em}}, \tag{3}$$

m_i and $\boldsymbol{p}_i$ are the mass and momentum of charge i, respectively, c is the speed of light, and the electromagnetic interaction energy $\mathcal{V}^{\mathrm{em}}$ is

$$\mathcal{V}^{\mathrm{em}} = \frac{\kappa^{\mathrm{em}}}{2} \sum_{ij}^{N} \int \mathrm{d}t' \mathrm{d}r \mathrm{d}r' \boldsymbol{J}_i^{\mu}(\boldsymbol{r}, t) \mathcal{D}_{\mu\nu} \boldsymbol{J}_j^{\nu}(\boldsymbol{r}', t'). \tag{4}$$

Here $\boldsymbol{J}_i^{\mu}(\boldsymbol{r}, t)$ the electromagnetic four-current of particle i, and the coupling constant is $\kappa^{\mathrm{em}} = 1/(8\pi\epsilon_0 c^2)$, with ϵ_0 the vacuum permittivity. The photon propagator $\mathcal{D}_{\mu\nu}$ may be written as

$$\mathcal{D}_{\mu\nu} = -\hbar^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{\exp[-i\boldsymbol{q}_{\alpha}(\boldsymbol{r}^{\alpha} - \boldsymbol{r}'^{\alpha})]/\hbar}{\boldsymbol{q}_{\alpha}\boldsymbol{q}^{\alpha} + \mathrm{i}0} \eta_{\mu\nu}. \tag{5}$$

Here $\hbar$ is the reduced Planck constant, $\eta_{\alpha\beta} = \mathrm{diag}(+, -, -, -)$ is the flat spacetime metric, $\boldsymbol{q}^{\alpha}$ is the four-momentum of the photon, and the i0 prescription enforces the usual causal regularization.

It is more convenient to work in momentum space, where the propagator takes the form

$$\mathcal{D}(\boldsymbol{q}, \boldsymbol{q}^0) = \mathcal{D}(\boldsymbol{q}^\alpha) = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + \mathrm{i}0}. \tag{6}$$

Defining the electromagnetic four-potential $\boldsymbol{A}_\mu = \boldsymbol{A}_\mu(\boldsymbol{r}, t)$ by

$$\boldsymbol{A}_\mu = 1/(4\pi\epsilon_0 c^2) \int \mathrm{d}t' \mathrm{d}r' \mathcal{D}_{\mu\nu} \boldsymbol{J}^\nu, \tag{7}$$

we can obtain the Maxwell equations in potential form

$$\Box \boldsymbol{A}_\mu = 1/(4\pi\epsilon_0 c^2) \boldsymbol{J}_\mu, \tag{8}$$

with $\Box = \partial_\alpha \partial^\alpha$ the d'Alembert operator. The close relationship between the propagator and the wave equation will be important when we construct an analogous Hamiltonian for the strong interaction.

3 THE GLUON PROPAGATOR

The Hamiltonian for a system of quarks is analogous to (3), but m_i and $\boldsymbol{p}_i$ denote now the mass and momentum of quark i

$$H = \sum_i^N \sqrt{m_i^2 c^4 + \boldsymbol{p}_i^2 c^2} + \mathcal{V}^{\mathrm{sn}}. \tag{9}$$

The strong interaction energy $\mathcal{V}^{\mathrm{sn}}$ is given by

$$\mathcal{V}^{\mathrm{sn}} = \frac{\kappa^{\mathrm{sn}}}{2} \sum_{ij}^N \int \mathrm{d}t' \mathrm{d}r \mathrm{d}r' \boldsymbol{J}_{ia}^\mu(\boldsymbol{r}, t) \mathcal{D}_{\mu\nu}^{ab} \boldsymbol{J}_{jb}^\nu(\boldsymbol{r}', t'). \tag{10}$$

Here κ^{sn} is the strong coupling constant and $\boldsymbol{J}_{ia}^\mu(\boldsymbol{r}, t)$ the color four-current of quark i (with color index a), and $\mathcal{D}_{\mu\nu}^{ab}$ is the gluon propagator. Although quarks also interact electromagnetically and thorough the weak nuclear force, in this work we focus exclusively on the strong interaction.

The main difficulty lies in the form of the gluon propagator.[9] Despite its importance, its nonperturbative structure remains largely unknown even within standard quantum chromodynamics. Nevertheless, modern lattice simulations support the Refined Gribov-Zwanziger form in the infrared regime.[10]

Unlike the photon, which carries no electric charge, the gluon carries color charge and acts as a source for the chromodynamic potential A^μ_a, which satisfies the nonlinear wave equation (in the Lorenz gauge)

$$\Box \boldsymbol{A}^\mu_a = \boldsymbol{J}^\mu_a - 2g f_{abc} \boldsymbol{A}_{b\nu} \partial^\nu \boldsymbol{A}^\mu_c + g f_{abc} \boldsymbol{A}_{b\nu} \partial^\mu \boldsymbol{A}^\nu_c - g^2 f_{abc} f_{cde} \boldsymbol{A}_{b\nu} \boldsymbol{A}^\nu_d \boldsymbol{A}^\mu_e. \quad (11)$$

Note: We adopt the convention in which the gauge covariant derivative in the fundamental representation acts as $D_\mu = \partial_\mu + igA^a_\mu T^a$, and the Lie algebra generators satisfy $[T^a, T^b] = if^{abc}T^c$, with totally antisymmetric structure constants f^{abc}.

Solving this wave equation to obtain the gluon propagator is a formidable task, since standard perturbative techniques are not applicable in this regime.

As a first step towards a solution, we formally write the equation as

$$\Box \boldsymbol{A}^\mu_a = \boldsymbol{J}^\mu_a + \frac{M^2 c^2}{\hbar^2} \boldsymbol{A}^\mu_a, \quad (12)$$

where M is a dynamical mass. This equation can be formally solved, yielding the momentum-space propagator

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + M(\boldsymbol{q}^\alpha)^2 c^2 + i0}, \quad (13)$$

where the dynamical mass $M(\boldsymbol{q}^\alpha)$ is a complicated function of the gluon four-momentum. If $M = 0$, the propagator reduces to (6). The non-zero mass encapsulates a fundamental difference between gluons and photons and is essential for describing strong interaction phenomena such as confinement.

Expression (13) is exact and the key step that allows us to reinterpret quantum chromodynamics as a theory of direct interparticle interactions by means of Hamiltonian (9). However, (13) can be only solved numerically. To make the momentum dependence analytically tractable, we partition four-momentum space into L small regions labeled by s, within each of which the dynamical mass is approximately constant: $M \simeq m_s$.

The propagator in region s then takes the form of a fixed-mass propagator:

$$\mathcal{D}(\boldsymbol{q}^\alpha) \simeq \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0}. \tag{14}$$

This expression for the propagator is only valid within a small region of four-momentum. Outside that region, the exact propagator can be writen as the sum of (14) and a correction term $\Delta_s \mathcal{D}$

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0} + \Delta_s \mathcal{D}. \tag{15}$$

Repeating this procedure for each of the L regions,

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_1^2 c^2 + \mathrm{i}0} + \Delta_1 \mathcal{D} \tag{16}$$

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_2^2 c^2 + \mathrm{i}0} + \Delta_2 \mathcal{D} \tag{17}$$

$$\vdots$$

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_L^2 c^2 + \mathrm{i}0} + \Delta_L \mathcal{D}, \tag{18}$$

and summing the L equations, we obtain

$$L\, \mathcal{D}(\boldsymbol{q}^\alpha) = \sum_s^L \left(\frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0} + \Delta_s \mathcal{D} \right), \tag{19}$$

which we rewrite as

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \sum_s^L \frac{W_s(\boldsymbol{q}^\alpha)}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0}, \tag{20}$$

with $W_s(\boldsymbol{q}^\alpha) = (1/L) + \Delta_s \mathcal{D}(\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0)/L$. The correction term is

$$\Delta_s \mathcal{D} = \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + M^2 c^2 + \mathrm{i}0} - \frac{1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0}, \tag{21}$$

which yields

$$W_s(\boldsymbol{q}^\alpha) = \frac{1}{L}\left[1 + \frac{m_s^2 c^2 - M^2 c^2}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + M^2 c^2 + \mathrm{i}0}\right]. \tag{22}$$

Thus the momentum dependence of the dynamical mass $M(\boldsymbol{q}^\alpha)$ is absorbed into the weight functions $W_s(\boldsymbol{q}^\alpha)$. The keypoint is that in the limit of a very large number of regions, these functions become constants: $\partial W_s/\partial \boldsymbol{q}^\alpha = 0$ when $L \to \infty$. In this limit we obtain

$$\mathcal{D}(\boldsymbol{q}^\alpha) = \sum_s^\infty \frac{W_s}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_s^2 c^2 + \mathrm{i}0} \tag{23}$$

with W_s a set of constants satisfying $\sum_s W_s = 1$, as can be deduced from the ultraviolet limit $\mathcal{D}(\boldsymbol{q}^\alpha) \to 1/(\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + \mathrm{i}0)$.

Note: In the limit of an arbitrarily fine partition of momentum space, each region becomes infinitesimal and the weight functions $W_s(\boldsymbol{q}^\alpha)$ take a single value within each region. In this limit, $W_s(\boldsymbol{q}^\alpha)$ is therefore replaced by a constant W_s, in direct analogy with the Riemann-sum construction of an integral, where a function becomes effectively constant on each infinitesimal interval. This leads to the finite spectral representation (23), with the only approximation arising when the infinite sum is truncated to a finite number of terms.

Thus, the gluon propagator can be expressed as a weighted superposition of fixed-mass modes, from which the Refined-Gribov-Zwanziger (RGZ) and Very Refined-Gribov-Zwanziger (VRGZ) propagators arise as low-order truncations of the spectral sum. Retaining only two terms in (23) yield the RGZ form:

$$\begin{aligned}\mathcal{D}(\boldsymbol{q}^{\alpha}) &\approx \frac{W_1}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_1^2 c^2 + \mathrm{i}0} + \frac{W_2}{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + m_2^2 c^2 + \mathrm{i}0} \\ &= \frac{\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha + (W_1 m_2^2 c^2 + W_2 m_1^2 c^2)}{(\boldsymbol{q}_\alpha \boldsymbol{q}^\alpha)^2 + \boldsymbol{q}_\alpha \boldsymbol{q}^\alpha (m_1^2 c^2 + m_2^2 c^2) + (m_1^2 m_2^2 c^4) + \mathrm{i}0}\end{aligned}, \tag{24}$$

while retaining three terms produces the VRGZ form.

Numerical studies show that the RGZ form captures the infrared behavior of the gluon remarkably well,[10] while the VRGZ form provides an improved description in the intermediate-momentum region.

In the RGZ and VRGZ approaches, the gluon propagator is modeled as a finite sum of Yukawa terms, with the corresponding mass scales and residues determined phenomenologically from lattice data. These mass scales originate from three dimension-two condensates, $\langle \boldsymbol{A}^2 \rangle$, $\langle \bar{\varphi}\varphi \rangle$, and $\langle \bar{\omega}\omega \rangle$, introduced in the extended Gribov-Zwanziger action.

In contrast, in the present framework the same functional structure emerges from an exact variable-mass propagator, and the effective masses m_s are, in principle, determined by the dynamical mass function $M(\boldsymbol{q}^{\alpha})$. Truncations of the resulting discrete spectral representation thus reproduce the RGZ and VRGZ propagators while providing a dynamical origin for their parameters and a natural generalization to higher-order truncations. In the infrared limit, the dynamical mass behaves as $M(\boldsymbol{q}^{\alpha}) \propto \boldsymbol{A}^2$, paralleling the gluonic condensate $m^2 \sim \langle \boldsymbol{A}^2 \rangle$. Unlike RGZ, where the condensates are introduced phenomenologically, here the same structure arises directly from the nonlinear dynamics. These results complete our construction of the gluon propagator within the present framework.

4 CONCLUDING REMARKS

It is widely assumed that the strong interaction can only be formulated within the framework of quantum field theory. In contrast, we have shown that an action-at-a-distance description can account for confinement in a natural way, without relying on the traditional Faddeev-Popov quantization of Yang-Mills theory and its required nonperturbative extensions, such as those introduced by the Gribov-Zwanziger approach.

The gluon propagator is intrinsically complex, since its dynamical mass exhibits a highly nontrivial dependence on the potential generated by the color current. Nevertheless, low-order truncations of the spectral representation like the RGZ form, together with numerical determinations of the characteristic gluon mass scales, have proved remarkably effective for capturing nonperturbative phenomena. These approximations have been successfully applied to heavy-quark dynamics, to the modeling of parton energy loss in the quark-gluon plasma, and to several other contexts where infrared physics plays a central role.

While the Hamiltonian (3) for interacting electric charges reduces to the standard Dirac-Maxwell Hamiltonian of quantum electrodynamics, the Hamiltonian (9) does not reduce to the Dirac-Yang-Mills Hamiltonian except in the ultraviolet regime, where the propagator (13) approaches that of a massless free particle. This deviation from the standard Dirac-Yang-Mills structure in the infrared seems to be a necessary price for achieving a nonperturbative description of confinement.

This work shows that the action-at-a-distance framework offers a coherent and nonperturbative alternative perspective on strong-interaction dynamics, suggesting that confinement can be understood without invoking a local gauge field as a fundamental degree of freedom.

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