

Emergent Gravitation and Quantum Wave Dynamics from a Bounded Vacuum

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Abstract

We present a framework in which gravitation, inertia, and wave dynamics emerge from the response of a vacuum endowed with finite potential capacity. The theory is formulated in terms of a scalar vacuum-potential field whose absolute normalization is fixed by relativistic considerations, such that the equilibrium value at infinity equals c^2 . Static relaxation of localized vacuum-potential deficits reproduces Newtonian gravity in the coarse-grained limit, while time-dependent redistribution generates propagating disturbances governed by a universal wave equation. Finite vacuum capacity implies intrinsic upper bounds on transmissible force and signal speed, yielding $F_{\max} = c^4/G$ and $v \leq c$ without invoking spacetime geometry or independent kinematic postulates. Vacuum microstructure further leads to a universal lattice dispersion relation with Planck-suppressed corrections,

$$\frac{\Delta v}{c} \simeq -\frac{1}{8} \left(\frac{E}{E_p} \right)^2,$$

consistent with current astrophysical and gravitational-wave constraints. Gravitational redshift, lensing, horizons, and quantum correlations arise as energetic consequences of bounded vacuum response. The vacuum is modelled as a Dynamical Planck Network (DPN), providing a conservative and internally consistent bridge between relativistic gravitation and quantum wave phenomena.

Keywords: Vacuum, Gravitation, Wave dynamics, Compton length, Dispersion

SECTION I — Introduction

Understanding the physical origin of gravitation, inertia, and quantum wave phenomena remains one of the central challenges of fundamental physics. General theory of relativity successfully describes gravity as the geometry of spacetime, while quantum theory governs matter and radiation through wave dynamics and discrete action. Despite their empirical success, these frameworks rely on conceptually distinct foundations, and a common microscopic origin for gravitational and quantum phenomena has yet to be established[1-3].

A recurring theme in attempts to bridge this gap is the idea that spacetime or the vacuum itself may possess an underlying structure with finite response capacity. Horizon thermodynamics, maximum-force arguments, and emergent-gravity approaches all point toward the existence of universal bounds—on force, tension, and signal propagation—that appear independent of detailed dynamics [4-7]. At the same time, lattice field theory and condensed-matter analogs demonstrate that wave propagation, dispersion, and effective geometry can arise from collective responses of discrete or capacity-limited media without assuming spacetime geometry a priori [8-10]. In general relativity, energy—not force—is the fundamental invariant, and the potential at spatial infinity is fixed by the requirement that clocks recover special-relativistic behavior far from gravitating sources [11-13]. This suggests that gravitational phenomena may be understood, at least in part, as consequences of spatial and temporal variations of a bounded vacuum-potential field rather than as primary geometric axioms.

Motivated by these observations, we introduce a framework in which gravitation, inertia, and wave dynamics emerge from the response of a vacuum endowed with finite, bounded potential capacity. The central object of the theory is a scalar vacuum-potential field Φ , normalized such that $\Phi_\infty = c^2$, whose variations encode both static and dynamical physical effects. No spacetime geometry, force laws, or canonical quantization are assumed a priori. Instead, gravitational attraction arises from static relaxation of localized vacuum-potential deficits, while wave propagation corresponds to time-dependent redistribution of the same field.

Within this approach, universal bounds on force, tension, and signal speed emerge naturally from the finite transmissibility of the vacuum. Horizons and black-hole-like behavior appear as limiting cases of vacuum-capacity saturation rather than as fundamental singularities, and Planck-scale corrections to wave propagation follow from the underlying vacuum response. Quantum correlations are interpreted as global constraints on vacuum configurations, consistent with relativistic causality.

The purpose of this paper is to develop the bounded-vacuum framework in a controlled and conservative manner. Section II establishes the relativistic normalization of the vacuum potential and its continuum representation. Sections III–V derive gravitational response, inertial effects, and universal force bounds from finite vacuum capacity. Sections VI–VIII analyze wave dynamics, dispersion, and observational consistency. The physical implications—including horizons, redshift, lensing, and quantum correlations—are summarized in later sections. Throughout, we emphasize that geometry, quantization, and relativistic structure emerge as effective descriptions of an underlying vacuum response rather than as independent postulates.

SECTION II — Vacuum Potential Field and Relativistic Normalization

This section establishes the continuum foundation of the bounded-vacuum framework. We introduce a scalar vacuum-potential field $\Phi(x)$, defined as energy per unit mass, fix its absolute normalization using theory of relativity, and formulate the notion of a self-consistent vacuum deficit. No assumptions regarding force laws, wave propagation, or microscopic discreteness are introduced at this stage; these features emerge in subsequent sections from the dynamical response of the bounded vacuum.

II.A. Vacuum Potential as Energy per Unit Mass

We describe the vacuum by a scalar field $\Phi(x)$ with dimensions of energy per unit mass. In relativistic physics, the invariant relation between mass and energy is expressed through the rest-energy formula [1, 2].

For a particle of inertial (test) mass m_t , special relativity gives

$$E_0 = m_t c^2. \quad (2.1)$$

Equation (2.1) identifies c^2 as the universal energy-per-mass scale associated with flat spacetime [1, 2, 14]. Motivated by this relation, we take the reference value of the vacuum potential in the absence of gravitational loading to be

$$\Phi_0 = c^2.$$

Physical gravitational or inertial effects are then described by deviations of $\Phi(x)$ from this baseline.

II.B. Relativistic Fixing of the Vacuum Reference Level

In Newtonian gravity, the potential is defined only up to an additive constant. In contrast, general relativity fixes the absolute normalization of energy through spacetime geometry [1, 2].

For a static weak gravitational field generated by a localized source, the Schwarzschild metric yields, to leading order [2, 15-18],

$$g_{00} \simeq -\left(1 + \frac{2\Phi_N}{c^2}\right), \quad |\Phi_N| \ll c^2, \quad (2.2)$$

where Φ_N denotes the Newtonian gravitational potential.

The total energy of a particle of test mass m_t at rest in this spacetime is [16, 18]

$$E = m_t c^2 \sqrt{-g_{00}}. \quad (2.3)$$

Expanding Eq. (2.3) using Eq. (2.2) in the weak-field limit gives

$$E \simeq m_t c^2 + m_t \Phi_N. \quad (2.4)$$

Defining the total potential as energy per unit test mass,

$$\Phi_{\text{tot}} \equiv \frac{E}{m_t}, \quad (2.5)$$

we obtain

$$\Phi_{\text{tot}}(r) = c^2 + \Phi_N(r). \quad (2.6)$$

Equation (2.6) shows that relativistic energy normalization naturally decomposes into:

- a universal relativistic baseline c^2 , and
- a deviation Φ_N determined by gravitational sources.

Thus c^2 is not an arbitrary additive constant; it is fixed by asymptotic Minkowski normalization [2, 18].

II.C. Asymptotic Flatness and Uniqueness of the Newtonian Constant

For a localized gravitating source of mass M , the Newtonian potential takes the form

$$\Phi_N(r) = -\frac{GM}{r} + C. \quad (2.7)$$

Asymptotic flatness requires that spacetime approach Minkowski form as $r \rightarrow \infty$ [2, 17, 18], implying

$$g_{00} \rightarrow -1 \Leftrightarrow \Phi_N(\infty) = 0. \quad (2.8)$$

This uniquely fixes

$$C = 0. \quad (2.9)$$

Hence,

$$\Phi_{\text{tot}}(\infty) = c^2. \quad (2.10)$$

The vacuum reference level c^2 is therefore uniquely fixed by relativistic normalization. The Newtonian potential represents deviations from this baseline [2, 18].

II.D. From External Sources to Self-Consistent Vacuum Deficits

Equations (2.7)–(2.10) are traditionally derived for a physical gravitating source of mass M . In this framework, however, the same coarse-grained potential form is employed more generally to represent a localized vacuum-potential deficit, irrespective of whether it originates from conventional material matter.

Accordingly, we introduce a parameter m , characterizing the total integrated vacuum deficit of a localized configuration. For a static, spherically symmetric configuration, the vacuum potential depends only on the radial coordinate $r = |\mathbf{x}|$. Replacing $M \rightarrow m$, the continuum vacuum-potential profile is written

$$\Phi(r) = c^2 - \frac{Gm}{r}. \quad (2.11)$$

In Eq. (2.11), the parameter m is not assumed a priori to represent a material mass. It parametrizes the strength of the vacuum deficit itself. The proportionality constant relating deficit strength to far-field response will be identified in Section IV through comparison with the Newtonian limit. Equation (2.11) is introduced purely as a structural representation of the scalar vacuum-potential field; it is not yet a force law or dynamical equation.

Scalar-potential formulations of gravitational phenomena, sometimes interpreted in terms of variations of c^2 , appear in early relativistic developments by Einstein [11] and later in Machian or variable-speed-of-light approaches [12, 19], although the physical interpretation adopted here is distinct.

II.E. Universal Capacity Constraint on Vacuum Loading

Because the vacuum potential is normalized relative to the finite reference value c^2 , any localized mass configuration produces a **deficit** in the vacuum potential relative to this baseline. For a deficit parameter m distributed over a characteristic length scale L , the magnitude of the induced deficit is of order

$$\Delta\Phi \sim \frac{Gm}{L}. \quad (2.12)$$

Here $\Delta\Phi$ represents the **vacuum potential deficit** relative to the reference level $\Phi = c^2$. From Eqs. (2.10)–(2.11), the vacuum potential must remain bounded within the interval $0 \leq \Phi \leq c^2$.

$$\frac{Gm}{L} \lesssim c^2. \quad (2.13)$$

Equivalently,

$$\frac{m}{L} \lesssim \frac{c^2}{G}. \quad (2.14)$$

This inequality expresses a universal capacity constraint on the vacuum potential deficit generated by localized mass configurations. A localized mass distribution imposes a **vacuum loading**, characterized by the mass-to-length ratio m/L , on the background vacuum capacity. The resulting deficit cannot exceed the finite reference capacity of the vacuum. This result follows directly from relativistic normalization and the finite reference value c^2 , without invoking force laws, inertia, wave dynamics, or spacetime discreteness.

II.F. Vacuum Field Response Postulate

We finally postulate that the vacuum responds to spatial and temporal variations of Φ through local redistribution of vacuum potential. Differences $\delta\Phi$ drive this response, while the finite reference capacity c^2 ensures bounded behavior.

All gravitational, inertial, and wave phenomena developed in later sections arise from this response mechanism.

SECTION III — Mass, Vacuum-Potential Saturation, Localization, and Inertia

In this section we analyze structural consequences of the vacuum-potential framework introduced in Sec. II. Finite vacuum capacity implies finite localization of vacuum loading, the existence of a collapse radius, the emergence of a distinguished minimal saturation threshold, and a structural interpretation of inertia.

III.A. Mass as Integrated Vacuum Loading

In Sec. II the vacuum was described by a scalar potential field $\Phi(r)$, normalized such that $\Phi \rightarrow c^2$ as $r \rightarrow \infty$, corresponding to asymptotically flat spacetime. For a static, spherically symmetric configuration, the coarse-grained asymptotic solution takes the form

$$\Phi(r) = c^2 - \frac{Gm}{r}. \quad (3.1)$$

The parameter m characterizes the total integrated vacuum deficit of the configuration. It is defined operationally through the Gauss-law relation

$$m = \frac{1}{4\pi G} \oint_{S_\infty} \nabla\Phi \cdot d\mathbf{S}, \quad (3.2)$$

where the surface integral is evaluated over a sphere at spatial infinity. Using Gauss's theorem, this may equivalently be written as a volume integral,

$$m = \frac{1}{4\pi G} \int_{\mathbb{R}^3} \nabla^2 \Phi(x) d^3x. \quad (3.3)$$

In Newtonian gravity, the gravitational potential Φ_N satisfies Poisson's equation $\nabla^2 \Phi_N = 4\pi G\rho$, so outside a localized source ($\rho = 0$) one has $\nabla^2 \Phi_N = 0$. In the exterior region, the potential satisfies Laplace's equation, whose spherically symmetric solution yields the universal $1/r$ behavior in Eq. (3.1).

Equations (3.2)–(3.3) show that m depends only on the global vacuum deficit and not on its detailed microscopic distribution. Different internal configurations that produce the same integrated loading yield identical asymptotic $1/r$ behavior and therefore correspond to the same value of m .

In this framework, mass is thus not postulated as an intrinsic pointlike substance but emerges as a global measure of vacuum deformation relative to the reference state $\Phi = c^2$. This relational interpretation is consistent with Machian and emergent viewpoints in which inertial properties arise from vacuum structure rather than from fundamental particles [19]. Inertial and gravitational properties are encoded in this integrated vacuum loading.

III.B. Capacity Constraint and Saturation

Section II established the universal capacity bound

$$\frac{m}{L} \lesssim \frac{c^2}{G}, \quad (3.4)$$

for a configuration characterized by loading m distributed over a characteristic length scale L , defined as $\mu = \frac{m}{L}$. When the inequality is strictly satisfied, the vacuum response remains unsaturated. Saturation occurs when

$$\frac{m}{L} = \frac{c^2}{G}. \quad (3.5)$$

We refer to this threshold as **vacuum-potential saturation**, corresponding to the onset of vacuum-capacity collapse.

Collapse in this framework does not correspond to geometric compression or curvature divergence. It signifies that the local deficit relative to the reference capacity c^2 has reached its maximal permitted magnitude. Beyond this point, further localization at fixed L cannot increase the deficit.

III.C. Finite Localization and Collapse Radius

Because the vacuum-potential field has finite capacity, a given loading m cannot be confined to arbitrarily small regions.

Define the collapse radius r_c as the scale at which the deficit relative to the reference value reaches order c^2 ,

$$c^2 - \Phi(r_c) \sim c^2. \quad (3.4)$$

Using the continuum profile (3.1),

$$c^2 - \Phi(r) = \frac{Gm}{r},$$

we obtain

$$r_c \sim \frac{Gm}{c^2}. \quad (3.5)$$

Thus any finite vacuum loading localizes over a finite region. Arbitrary compression to vanishing spatial extent is excluded solely by finite vacuum capacity, independent of additional field equations or geometric assumptions.

III.D. Distinguished Minimal Response Scale

If the vacuum admits a minimal response scale ℓ_* , below which the continuum description ceases to apply, a distinguished limiting case arises when

$$r_c \sim \ell_*. \quad (3.6)$$

Substituting Eq. (3.6) into Eq. (3.5) yields

$$m \sim \frac{c^2 \ell_*}{G}. \quad (3.7)$$

This defines a threshold mass corresponding to saturation at the minimal response scale. When ℓ_* is later identified via quantum action considerations with the Planck length (ℓ_P), Eq. (3.7) reproduces the Planck mass (m_P). Importantly, this threshold is not postulated here. It emerges as a limiting case of capacity saturation once a minimal response scale is independently introduced. For loadings m exceeding this threshold, saturation is already achieved at ℓ_* , and further localization is not possible.

III.E. Boundedness and Absence of Singular Behaviour

Because vacuum loading is capacity-limited, the scalar vacuum-potential field remains bounded for all physically realized configurations.

The continuum expression (3.1) does not extend to arbitrarily small r ; it ceases to apply once saturation is reached. Therefore no divergence of the vacuum-potential field is physically realized.

In this sense, pointlike singular behavior is replaced by finite saturation regions whose size is determined by total loading and vacuum bounded capacity. Similar regularization mechanisms appear in nonsingular black-hole models and quantum-gravity approaches derived from different principles [20-22]. In the present framework, boundedness follows directly from finite vacuum capacity.

III.F. Structural Origin of Inertia

Consider a configuration characterized by total loading m . Any displacement, deformation, or acceleration requires redistribution of the associated vacuum-potential deficit across the network. Because redistribution is local and capacity-limited, such reconfiguration incurs energetic cost. The larger the loading m , the greater the resistance to rapid redistribution.

Inertia therefore emerges as resistance of the vacuum to changes in the configuration of a given loading. At this stage this statement is structural rather than dynamical; no force law or equation of motion is assumed.

This interpretation is consistent with relational accounts of inertia proposed in earlier work [19, 23].

SECTION IV — Static Response and Emergent Gravitational Interaction

We now analyze the static response regime of the vacuum-potential field, in one dimensional response for spherically symmetric configuration, the potential depends only on the radial coordinate $r = |\mathbf{x}|$, $\Phi(x)$ introduced in Secs. II and III. In the bounded-vacuum framework, gravitation does not arise from a fundamental interaction law but from static relaxation of a bounded vacuum-potential deficit. The same scalar field that characterizes vacuum loading produces gravitational attraction when the configuration is time-independent, consistent with relational and vacuum-based perspectives on gravity and inertia [7, 19].

A localized vacuum loading m corresponds to a persistent deficit relative to the reference state $\Phi = c^2$. In the static regime, the vacuum field relaxes toward its maximal-capacity configuration far from any excitation,

$$\Phi(\mathbf{x}) \rightarrow c^2 \text{ as } |\mathbf{x}| \rightarrow \infty. \quad (4.1)$$

A test configuration characterized by loading m_t placed in this static background possesses a position-dependent vacuum-field energy

$$E(\mathbf{x}) = m_t \Phi(\mathbf{x}), \quad (4.2)$$

which follows directly from the interpretation of mass as integrated vacuum loading established in Sec. III. No additional coupling principle or interaction term is introduced; the energy arises from embedding the test loading within the vacuum-potential field.

In static systems, force is defined operationally as the spatial gradient of stored energy [24]. Accordingly,

$$\mathbf{F} \equiv -\nabla E = -m_t \nabla \Phi. \quad (4.3)$$

Force is therefore not fundamental in the framework; it emerges from spatial variation of the vacuum potential. Dividing by m_t gives the acceleration,

$$\mathbf{a} = -\nabla \Phi. \quad (4.4)$$

Acceleration is thus determined entirely by gradients of the vacuum field, and it is independent of the magnitude of the test loading m_t . The equality of inertial and gravitational mass therefore follows directly from the structure of the vacuum-potential energy $E = m_t \Phi$ and does not require an independent equivalence postulate. Universality of free fall emerges as a consequence of vacuum response, rather than as a fundamental principle imposed a priori.

We now determine the static profile $\Phi(r)$ associated with a spherically symmetric vacuum loading m . The solution must satisfy asymptotic relaxation $\Phi \rightarrow c^2$ as $r \rightarrow \infty$, isotropy, and the finite localization scale $r_c \sim Gm/c^2$ derived in Sec. III. At distances large compared with r_c , the leading-order coarse-grained relaxation profile consistent with these requirements takes the form

$$\Phi(r) = c^2 - \frac{Gm}{r}, r \gg r_c. \quad (4.5)$$

The constant G appearing in Eq. (4.5) is not introduced as a fundamental gravitational parameter but is fixed by matching the far-field relaxation profile to the observed Newtonian limit. It therefore acquires the interpretation of a transmissibility coefficient of vacuum-potential deficits, quantifying the response strength of the vacuum medium. Equation (4.5) is not postulated as a gravitational law but represents the asymptotic static relaxation profile of a bounded scalar field sourced by a localized vacuum loading. Analogous long-range equilibrium profiles arise in condensed-matter and analogue-gravity systems in which macroscopic fields emerge from relaxation of an underlying medium [7, 10].

Substituting Eq. (4.5) into Eq. (4.3) yields

$$\mathbf{F} = -m_t \nabla \left(c^2 - \frac{Gm}{r} \right) = -\frac{Gmm_t}{r^2} \hat{\mathbf{r}}, \quad (4.6)$$

recovering Newton's inverse-square law [10] as an emergent static response of the vacuum field. Gravitation in the bounded-vacuum framework is therefore interpreted as the tendency of the vacuum to reduce spatial gradients in the bounded scalar potential Φ , consistent with vacuum-based and emergent-gravity viewpoints [7, 10, 23].

The static gravitational response developed here and the time-dependent vacuum redistribution analyzed in subsequent sections arise from the same scalar field Φ . The distinction between gravitation and wave propagation is not one of field type but of response regime: time-independent relaxation versus time-dependent redistribution. Because both regimes are governed by the same finite vacuum capacity introduced in Sec. II, accelerations, forces, tensions, and signal speeds are constrained by the transmissibility of the vacuum field itself [7, 10]. This unified field-response perspective prepares the ground for Sec. V, where we show that bounded vacuum capacity implies a universal maximum transmissible tension and a universal signal-speed bound identified with c .

SECTION V — Emergent Force, Tension, and Signal-Speed Bounds

In this section we show that finite vacuum-potential capacity in the vacuum-space implies universal upper bounds on force, transmissible tension, and signal propagation speed. These bounds arise directly from vacuum loading constraints established in Secs. II–IV. No spacetime geometry, Lorentz symmetry, or independent kinematic postulate is assumed.

V.A. Vacuum Loading, Force, and Maximum Tension

From Sec. IV, a localized vacuum-potential deficit relative to the reference value $\Phi = c^2$ carries energy

$$\Delta E = m \Delta\Phi, \quad (5.1)$$

where m denotes the total vacuum loading and $\Delta\Phi$ the local deficit magnitude.

If this energy is redistributed across a characteristic separation L , the associated force scale is

$$F \sim \left| \frac{dE}{dr} \right| \sim \frac{\Delta E}{L}, \Delta E \equiv m \Delta\Phi \Rightarrow F \sim \frac{m \Delta\Phi}{L}. \quad (5.2)$$

Both factors in Eq. (5.2) are independently bounded.

First, the vacuum-potential deficit cannot exceed the reference magnitude,

$$\Delta\Phi \leq c^2, \quad (5.3)$$

as established in Sec. II.

Second, the vacuum loading ($\mu = \frac{m}{L}$) obeys the saturation bound derived in Sec. III,

$$\mu = \frac{m}{L} \leq \frac{c^2}{G}. \quad (5.4)$$

Combining Eqs. (5.2)–(5.4) yields a universal upper bound on force,

$$F \leq F_{\max} \equiv \frac{c^4}{G}. \quad (5.5)$$

This value coincides numerically with the maximum-force or maximum-tension scale discussed in gravitational contexts using horizon and thermodynamic arguments [4-6]. In the bounded-vacuum framework, however, it arises directly from finite vacuum capacity and bounded loading.

Because force transmission corresponds to tension along the vacuum network, Eq. (5.5) defines a maximum transmissible tension,

$$T_{\max} = \frac{c^4}{G}. \quad (5.6)$$

No larger force or tension can be supported without violating either vacuum capacity or the collapse condition derived in Sec. III.

It is worth noting that in general-relativistic discussions the maximum-force (or maximum-tension) scale is sometimes quoted with an additional numerical factor, e.g. $F_{\max} = c^4/(4G)$, depending on the geometric normalization adopted in horizon or surface-gravity arguments [4–6]. Such order-unity factors arise from specific metric conventions (e.g., the factor of 2 in the Schwarzschild radius and the associated surface gravity). In contrast, within the bounded-vacuum framework the bound $T_{\max} = c^4/G$ follows directly from the loading constraint $m/L \leq c^2/G$, independent of any particular horizon geometry. The precise numerical correspondence with GR therefore depends on whether spatial-curvature contributions are incorporated in the effective description.

V.B. Emergent Universal Signal–Speed Bound

In the bounded-vacuum framework the physical state of the vacuum is characterized by the scalar capacity field $\Phi(x, t)$. The homogeneous equilibrium vacuum corresponds to the maximal capacity state $\Phi_0 = c^2$. Dynamical signals correspond to small redistributions of this capacity about the equilibrium value. It is therefore convenient to introduce the deviation field

$$\phi(x, t) \equiv \Phi(x, t) - c^2. \quad (5.7)$$

With this definition the relaxed vacuum satisfies $\phi = 0$, while localized matter configurations correspond to negative deviations $\phi < 0$, reflecting the deficit of vacuum capacity relative to the asymptotic state. For the static configuration discussed in Sec. III the deviation reduces to $\phi(r) = -\frac{Gm}{r}$, which represents the local reduction of vacuum capacity produced by a localized loading m . In the present section we consider general dynamical perturbations $\phi(x, t)$.

At length scales large compared with the microscopic structure of the vacuum network, the dynamics of small perturbations is governed by the lowest-order local action consistent with translational symmetry and quadratic order in derivatives,

$$S = \int dt d^3x \left[\frac{1}{2} \mu (\partial_t \phi)^2 - \frac{1}{2} T (\nabla \phi)^2 \right], \quad (5.8)$$

where μ represents the effective inertial loading density of the vacuum and T is the transmissible vacuum tension. Variation of the action yields the Euler–Lagrange equation

$$\mu \partial_t^2 \phi = T \nabla^2 \phi. \quad (5.9)$$

For plane-wave solutions $\phi \propto e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$,

$$\omega^2 = \frac{T}{\mu} k^2, v \equiv \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}, \quad (5.10)$$

so that the characteristic propagation speed is determined by the stiffness-to-inertia ratio of the vacuum response.

Both T and μ are constrained by the same bounded vacuum-capacity structure. From Sec. V.A, the transmissible tension satisfies the universal upper bound

$$T \leq T_{\max} = \frac{c^4}{G}, \quad (5.11)$$

consistent with previously discussed maximum-force bounds in gravitational contexts [4, 5, 25]. Meanwhile, the effective inertial loading density cannot exceed the maximal vacuum loading per separation derived in Sec. III, implying

$$\mu \lesssim \frac{c^2}{G}. \quad (5.12)$$

Combining Eqs. (5.11)–(5.12) yields

$$\frac{T}{\mu} \leq c^2, \quad (5.13)$$

and therefore

$$v \leq c. \quad (5.14)$$

The invariant propagation bound thus follows directly from the finite stiffness-to-inertia ratio permitted by bounded vacuum capacity; no independent kinematic postulate is required. The ratio T/μ sets the intrinsic time scale of vacuum fluctuations, since it fixes the wave speed $v = \sqrt{T/\mu}$. The equality $v = c$ corresponds to the intrinsic unsaturated vacuum response for which the effective stiffness-to-inertia ratio attains its fundamental value $T/\mu = c^2$. Any additional vacuum loading increases the effective inertial response and thereby reduces the propagation speed below this bound. The massless sector therefore propagates at c , while loaded (massive) configurations exhibit subluminal dynamics.

This parallels the emergence of characteristic signal speeds in elastic continua and lattice field theories, where propagation velocity is fixed by stiffness-to-inertia ratios [26-28], while maximum-force bounds have been discussed independently in gravitational settings [4, 5, 25]. In the bounded-vacuum framework, both force and signal-speed limits arise from the same bounded vacuum-capacity structure. Planck-scale corrections and discrete realizations of this propagation bound are developed in Sec. VII.

SECTION VI — Action Quantization as a Vacuum Capacity Constraint and the Emergent Planck Scale

Quantum dynamics is governed by the action functional

$$S = \int L dt, \quad (6.1)$$

which enters transition amplitudes through the phase factor $e^{iS/\hbar}$ [29, 30]. Physical distinguishability arises only when variations in the action satisfy

$$\Delta S \sim \hbar, \quad (6.2)$$

so that \hbar defines the fundamental quantum of action[31]. Equation (6.2) introduces neither the gravitational constant G nor an intrinsic length scale.

In the bounded-vacuum framework established in Secs. II–V, the vacuum possesses a maximal potential reference level

$$\Phi_0 = c^2,$$

fixed by relativistic normalization. A localized vacuum loading m therefore carries a characteristic energy

$$E = mc^2, \quad (6.3)$$

as follows directly from the definition of vacuum loading in Sec. III.

Over a time interval Δt , the action associated with this redistribution is

$$S = E \Delta t. \quad (6.4)$$

Imposing the minimal quantum condition (6.2) gives

$$E \Delta t = \hbar, \quad (6.5)$$

or

$$mc^2 \Delta t = \hbar. \quad (6.6)$$

Because redistribution of the vacuum field cannot propagate faster than the universal bound c established in Sec. V, the associated spatial scale λ satisfies

$$\lambda = c \Delta t. \quad (6.7)$$

Combining Eqs. (6.6)–(6.7) yields

$$mc\lambda = \hbar, \quad (6.8)$$

or equivalently,

$$\lambda = \frac{\hbar}{mc}. \quad (6.9)$$

Equation (6.9) expresses a property of the vacuum medium: quantum action quantization enforces an inverse relation between vacuum loading m and the spatial scale λ over which redistribution occurs. No minimal length follows from quantum action alone, since both m and λ remain continuous.

Gravitational coupling introduces an independent localization constraint derived in Sec. III. Finite vacuum capacity implies a collapse radius

$$r_c \sim \frac{Gm}{c^2}, \quad (6.10)$$

which represents the maximal concentration of loading consistent with capacity saturation [2].

Self-consistency requires that quantum localization not exceed the gravitational saturation scale,

$$\lambda \gtrsim r_c. \quad (6.11)$$

At the threshold of simultaneous quantum and gravitational saturation, setting

$$\lambda = r_c \quad (6.12)$$

and substituting Eqs. (6.9) and (6.10) gives

$$\frac{\hbar}{mc} = \frac{Gm}{c^2}, \quad (6.13)$$

which implies

$$Gm^2 = \hbar c. \quad (6.14)$$

The critical loading is therefore

$$m = \sqrt{\frac{\hbar c}{G}} \equiv m_P, \quad (6.15)$$

and the corresponding localization scale becomes

$$\lambda = r_c = \sqrt{\frac{\hbar G}{c^3}} \equiv \ell_P. \quad (6.16)$$

The Planck scale thus emerges as the unique intersection of two independent vacuum-capacity constraints:

1. quantum action quantization (\hbar),
2. gravitational loading consistency (G),

both referenced to the invariant vacuum potential scale c^2 .

So, the Planck length ℓ_p arises as the saturation boundary of the vacuum medium at which quantum redistribution and gravitational capacity simultaneously reach their limiting values. At scales smaller than ℓ_p , the vacuum cannot consistently satisfy both constraints within the continuum description.

SECTION VII — Dispersion, Universality, and Observational Consistency

Sections II–VI established that the vacuum-potential field $\Phi(\mathbf{x}, t)$ possesses finite capacity $\Phi \leq c^2$, supports static relaxation profiles, and admits dynamical propagation with a universal signal-speed bound $v \leq c$. We now examine the ultraviolet structure implied by the existence of a minimal response scale. Rather than postulating fundamental discreteness as an axiom, we adopt the minimal microscopic realization compatible with bounded gradients and maximum transmissible tension: a scalar field defined on a lattice of spacing ℓ_* . Such discretizations are standard in lattice quantum field theory and in condensed-matter systems, where continuum behavior emerges at long wavelengths while ultraviolet modes are regulated by the underlying spacing [8, 9, 26].

Let $\phi_n(t) \equiv \Phi_n(t) - c^2$ denote small deviations from equilibrium at lattice site n . For a one-dimensional chain with nearest-neighbor coupling consistent with the continuum action of Sec. V, small-amplitude disturbances satisfy the discrete equation

$$\ddot{\phi}_n = \frac{c^2}{\ell_*^2} (\phi_{n+1} - 2\phi_n + \phi_{n-1}), \quad (7.1)$$

which reduces to the continuum wave equation in the long-wavelength limit $k\ell_* \ll 1$.

Seeking plane-wave solutions

$$\phi_n(t) = e^{i(kn\ell_* - \omega t)}, \quad (7.2)$$

yields the exact lattice dispersion relation

$$\omega(k) = \frac{2c}{\ell_*} \sin\left(\frac{k\ell_*}{2}\right), \quad |k| \leq \frac{\pi}{\ell_*}. \quad (7.3)$$

The corresponding group velocity is

$$v_g(k) = \frac{d\omega}{dk} = c \cos\left(\frac{k\ell_*}{2}\right), \quad (7.4)$$

which satisfies $0 \leq v_g \leq c$ for all modes. Thus the universal speed bound derived in Sec. V is automatically preserved in the microscopic realization. Causality is enforced dynamically by finite vacuum transmissibility rather than imposed kinematically, consistent with maximum-force arguments [4-6].

In the long-wavelength limit $k\ell_* \ll 1$, expanding Eq. (7.3) gives

$$\omega(k) = ck \left[1 - \frac{(k\ell_*)^2}{24} + \mathcal{O}((k\ell_*)^4) \right], \quad (7.5)$$

and therefore

$$v_g(k) = c \left[1 - \frac{(k\ell_*)^2}{8} + \mathcal{O}((k\ell_*)^4) \right]. \quad (7.6)$$

Relativistic dispersion is thus recovered universally at low energies, while deviations are quadratic and suppressed by the microscopic scale ℓ_* . The absence of linear corrections follows directly from parity symmetry of the discrete Laplacian and does not require additional symmetry assumptions [8, 26].

Using the standard identifications

$$E = \hbar\omega, p = \hbar k,$$

and defining the characteristic energy scale

$$E_* = \frac{\hbar c}{\ell_*}, \quad (7.7)$$

the fractional velocity deviation becomes

$$\frac{\Delta v}{c} \equiv \frac{v_g - c}{c} \simeq -\frac{1}{8} \left(\frac{E}{E_*} \right)^2. \quad (7.8)$$

From Sec. VI, simultaneous quantum and gravitational saturation uniquely fixed the minimal response scale to be

$$\ell_* = \ell_P = \sqrt{\frac{\hbar G}{c^3}}. \quad (7.9)$$

The corresponding energy scale becomes the Planck energy,

$$E_P = \frac{\hbar c}{\ell_P}. \quad (7.10)$$

Substituting Eq. (7.9) into Eq. (7.8) yields the parameter-free prediction

$$\frac{\Delta v}{c} \simeq -\frac{1}{8} \left(\frac{E}{E_P} \right)^2. \quad (7.11)$$

This quadratic, Planck-suppressed correction follows directly from bounded vacuum transmissibility and the minimal response scale derived in Sec. VI. The associated microscopic structure motivates the designation **Dynamical Planck Network (DPN)** for the underlying vacuum description. The correction depends only on energy and the fundamental constants \hbar, c, G , and is independent of particle species or interaction details. Similar infrared universality with ultraviolet lattice corrections appears in emergent-spacetime and analogue-gravity models [10, 23].

Observationally, high-energy astrophysical measurements place stringent bounds on leading-order (linear-in-energy) Lorentz-violating dispersion [32-34]. The present framework automatically satisfies these constraints because (i) no linear term arises in the dispersion relation and (ii) the leading modification is quadratic and suppressed by the Planck scale E_P [32-34]. Gravitational-wave observations likewise constrain deviations from luminal propagation [35]; quadratic, Planck-suppressed corrections of the form (7.11) remain well within current experimental limits.

The framework remains falsifiable. Any robust detection of linear dispersion, anisotropic propagation, or species-dependent velocity corrections would contradict the minimal symmetric vacuum model developed here.

In summary, once finite vacuum capacity and a minimal response scale are admitted, relativistic propagation emerges as a universal infrared limit, while Planck-suppressed quadratic dispersion arises as a controlled ultraviolet correction. The structure mirrors that of symmetric lattice field systems, yet here it follows from bounded vacuum transmissibility rather than from an imposed spacetime background.

SECTION VIII — Physical Implications and Mathematical Consequences of the bounded-vacuum framework

Sections II–VII established that the bounded-vacuum framework is governed by a single scalar vacuum-potential field $\Phi(\mathbf{x}, t)$, bounded by the universal capacity

$$0 \leq \Phi \leq c^2. \quad (8.1)$$

All physical phenomena in this framework arise from static or dynamical responses of this bounded field. We now summarize the principal physical implications and organize them into compact mathematical consequences.

VIII.A. Unified Static and Dynamic Vacuum Response

Two complementary response regimes follow directly from the behavior of Φ :

Static regime ($\partial_t \Phi = 0$)

Spatial relaxation of vacuum-potential deficits produces gravitational interaction. For a localized loading m , the coarse-grained profile derived in Secs. III–IV is

$$\Phi(r) = c^2 - \frac{Gm}{r}, r \gg r_c, \quad (8.2)$$

with collapse radius $r_c = Gm/c^2$. A test loading m_t experiences acceleration

$$\mathbf{a} = -\nabla\Phi, \quad (8.3)$$

recovering Newton's inverse-square law in the weak-field limit [2, 36].

Dynamic regime ($\partial_t\Phi \neq 0$)

Small deviations $\phi = \Phi - c^2$ satisfy the dynamical equation derived in Sec. V,

$$\partial_t^2\phi = \frac{T}{\mu}\nabla^2\phi, v^2 = \frac{T}{\mu} \leq c^2, \quad (8.4)$$

implying the universal propagation bound $v \leq c$.

Gravitation and wave propagation therefore represent two response modes of the same bounded vacuum field. No independent force law or kinematic postulate is introduced. This unified response structure parallels relational and vacuum-based approaches to gravity [7, 19].

VIII.B. Gravitational Redshift and Time Dilation

Physical oscillators embedded in the vacuum derive their characteristic energy scale from the local vacuum state. Since the energy of a test loading scales with the local vacuum potential, $E \propto \Phi(r)$, the intrinsic oscillation frequency of a local clock is likewise proportional to the local vacuum potential. The proper frequency of an oscillator at radius r therefore scales as

$$\omega_{\text{proper}}(r) = \omega_{\infty} \frac{\Phi(r)}{c^2}, \quad (8.5)$$

where ω_{∞} denotes the frequency measured in the unloaded vacuum at asymptotic infinity.

Because frequency is the inverse of proper time, $\omega_{\text{proper}} = d\theta/d\tau$, the local proper time measured by a clock at radius r is related to the asymptotic coordinate time t by

$$\frac{d\tau(r)}{dt} = \frac{\Phi(r)}{c^2}. \quad (8.6)$$

Using the static exterior solution derived earlier, $\Phi(r) = c^2 - \frac{GM}{r}$, one obtains

$$\frac{d\tau(r)}{dt} = 1 - \frac{GM}{rc^2}, \quad (8.7)$$

which is the leading weak-field gravitational time-dilation relation.

Thus proper time runs more slowly in regions where the vacuum capacity is locally depleted. Gravitational redshift therefore arises naturally as a consequence of position-dependent vacuum loading: clocks embedded in regions of reduced vacuum potential oscillate more slowly relative to clocks in the asymptotically relaxed vacuum where $\Phi = c^2$. In this description the familiar redshift effect reflects the renormalization of local clock rates by the vacuum state. The metric formulation of general relativity provides a macroscopic geometric representation of this lapse variation, while the bounded-vacuum framework interprets the same phenomenon as arising from redistribution of finite vacuum capacity rather than from spacetime curvature treated as a fundamental entity [36].

VIII.C. Light Deflection and Gravitational Lensing

The clock-rate relation derived in Sec. VIII.B also determines the coordinate propagation of massless disturbances in a loaded vacuum. Locally, propagation occurs at the invariant speed c with respect to proper time τ , $v_{\text{loc}} = \frac{d\ell}{d\tau} = c$. Using Eq. (8.6), $d\tau/dt = \Phi(r)/c^2$, the coordinate propagation speed with respect to the asymptotic time t becomes

$$v(r) = \frac{d\ell}{dt} = c \frac{\Phi(r)}{c^2} = \frac{\Phi(r)}{c}. \quad (8.8)$$

It is therefore convenient to introduce the effective refractive index

$$n(r) \equiv \frac{c}{v(r)} = \frac{c^2}{\Phi(r)}. \quad (8.9)$$

For the exterior vacuum solution $\Phi(r) = c^2 - GM/r$, the weak-field expansion gives

$$n(r) \simeq 1 + \frac{GM}{rc^2}. \quad (8.10)$$

Light propagation may thus be interpreted as refraction in an inhomogeneous vacuum medium. For a ray passing a mass M with impact parameter b , geometric optics yields the leading small-angle deflection

$$\delta\theta \simeq \int_{-\infty}^{\infty} \frac{\partial n}{\partial b} dz = \frac{GM}{c^2} \int_{-\infty}^{\infty} \frac{b dz}{(b^2 + z^2)^{3/2}} = \frac{2GM}{bc^2}. \quad (8.11)$$

Equation (8.11) represents the bending arising from the scalar (lapse) sector associated with the position-dependent vacuum capacity derived in Sec. VIII.B. In general relativity, an additional contribution arises from spatial curvature, yielding the well-known Schwarzschild result

$$\delta\theta_{\text{GR}} = \frac{4GM}{bc^2}. \quad (8.12)$$

Within the bounded-vacuum framework, gravitational lensing therefore appears as refraction in a medium, closely paralleling optical-metric and analogue-gravity formulations [10, 23], whose effective optical properties are determined by the local redistribution of vacuum capacity. Recovery of the full general-relativistic coefficient would require incorporating the spatial response of the vacuum network beyond the scalar sector considered here.

VIII.D. Horizons and Vacuum Saturation

Finite vacuum capacity implies the loading bound derived in Secs. III and V,

$$\frac{m}{r} \leq \frac{c^2}{G}. \quad (8.13)$$

Saturation of this bound defines a characteristic collapse scale,

$$r_c = \frac{Gm}{c^2}. \quad (8.14)$$

This radius follows directly from the bounded-capacity constraint and is therefore fixed within the scalar sector of the present framework. In general relativity the Schwarzschild horizon occurs at $r_s = \frac{2GM}{c^2}$, which differs by a factor of two [2]. The difference arises because the present model describes the weak-field vacuum response through the scalar (lapse) sector only, while the full general-relativistic result includes an additional contribution from spatial curvature.

Because the vacuum potential is bounded and localization cannot proceed below the Planck scale,

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}, \quad (8.15)$$

the vacuum response remains finite even as the loading approaches the collapse bound. In this framework, classical singularities are therefore replaced by finite saturation regions. A black-hole configuration corresponds to a state in which vacuum loading locally reaches the capacity limit, producing a collapse of available vacuum capacity while maintaining finite field amplitudes. This interpretation is consistent with regular black-hole constructions and quantum-gravity–motivated singularity-resolution scenarios [21, 22].

VIII.E. Vacuum Correlation and Entanglement

The vacuum field Φ is globally defined and capacity-limited. Localized loadings therefore modify the surrounding vacuum configuration and are not strictly independent.

Within this interpretation, entangled states correspond to correlated vacuum-potential configurations established during joint formation. Correlation does not require superluminal signal transfer, since the dynamical propagation bound $v \leq c$ remains intact. The bounded-vacuum framework provides a medium-based interpretation of quantum correlations without modifying standard quantum predictions [7].

VIII.F. Unified Capacity Bounds

All derived phenomena reduce to manifestations of finite vacuum transmissibility. The universal bounds are

$$\Phi \leq c^2, \quad (8.16)$$

$$\frac{m}{L} \leq \frac{c^2}{G}, \quad (8.17)$$

$$F \leq F_{\max} = \frac{c^4}{G}, \quad (8.18)$$

$$v \leq c. \quad (8.19)$$

Gravitational interaction, inertia, maximum force, invariant signal speed, horizon formation, and Planck-scale saturation are therefore not independent principles. They arise as different manifestations of a single structural property: finite vacuum-potential capacity.

VIII.G — Hierarchy and Planck-Scale Suppression of Gravitation

A central puzzle in fundamental physics is the extreme weakness of gravitation compared with gauge interactions. For example, the ratio of gravitational to electromagnetic forces between elementary particles is of order 10^{-39} [37]. In conventional theory, this disparity is encoded in Newton's constant G [11, 24], while the associated natural mass scale is the Planck mass $m_p = \sqrt{\hbar c/G}$.

Within the bounded-vacuum framework, the weakness of gravity follows from finite vacuum capacity and stiffness. A localized mass m induces the static relaxation profile

$$\delta\Phi(r) = -\frac{Gm}{r}, \quad (8.20)$$

consistent with the Newtonian limit of relativistic gravity [11, 24]. The physically relevant quantity is the fractional deformation of the bounded vacuum potential $\Phi_0 = c^2$,

$$\frac{|\delta\Phi|}{c^2} = \frac{Gm}{rc^2} = \frac{r_c}{r}, \quad (8.21)$$

where the collapse radius is

$$r_c = \frac{Gm}{c^2}. \quad (8.22)$$

For ordinary systems, $r \gg r_c$, so gravitation corresponds to a minute fractional redistribution of a vacuum potential whose natural scale is c^2 . This suppression reflects the extreme stiffness of the vacuum network, whose maximal transmissible tension is

$$F_{\max} = \frac{c^4}{G}, \quad (8.23)$$

a quantity related to relativistic upper force bounds [4].

Expressing the collapse scale in terms of the Planck mass,

$$m_p = \sqrt{\frac{\hbar c}{G}}, \quad (8.24)$$

the dimensionless gravitational strength of a localized excitation becomes

$$\frac{m}{m_p}. \quad (8.25)$$

Since all known elementary masses satisfy $m \ll m_p$, gravitational effects are parametrically suppressed by the Planck scale.

The apparent smallness of G is therefore reinterpreted as a consequence of Planck-scale vacuum stiffness: gravity measures the global fractional loading of a bounded medium, becoming non-perturbative only when $m \sim m_p$ or equivalently $r \sim r_c$.

SECTION IX — Conclusions and Outlook

We have presented a self-consistent framework in which gravitation, inertia, wave propagation, universal bounds, and Planck-scale structure emerge from the response of a vacuum endowed with finite, bounded potential capacity.

$$0 \leq \Phi \leq c^2.$$

Without assuming fundamental force laws, background spacetime geometry, or a priori spacetime discreteness, the theory reproduces Newtonian gravity in the static limit, relativistic wave propagation in the dynamical limit, a universal signal-speed bound $v \leq c$, and a maximum transmissible force

$$F_{\max} = \frac{c^4}{G}.$$

The central structural result is that gravitational and wave phenomena share a common origin: both are response modes of the same bounded vacuum-potential field. Static relaxation of vacuum loading yields gravitational interaction, while time-dependent redistribution yields propagating disturbances constrained by finite vacuum transmissibility. The invariant speed c emerges as the maximal propagation speed permitted by the stiffness-to-inertia ratio of the vacuum medium.

Combining quantum action quantization $\Delta S \sim \hbar$ with the gravitational loading bound $r_c \sim Gm/c^2$ yields a unique saturation scale characterized by the Planck mass $m_p = \sqrt{\hbar c/G}$ and Planck length $\ell_p = \sqrt{\hbar G/c^3}$. These scales arise not as primitive inputs but as intersection points of independent vacuum-capacity constraints.

Configurations approaching the loading bound correspond to vacuum-saturation regimes. In the coarse-grained limit these reproduce horizon-like behavior without requiring curvature singularities. Quantum

correlations may be interpreted as correlations of the globally defined vacuum field, consistent with the dynamical propagation bound $v \leq c$. In this framework, geometry appears as an effective macroscopic description of vacuum-potential variation rather than as a fundamental entity.

Several open directions remain. These include coupling the vacuum-potential field consistently to standard matter fields, exploring nonlinear and strong-loading regimes beyond the weak-field approximation, analyzing possible vacuum phase structure and transitions, and investigating cosmological evolution of the vacuum potential. Further study of Planck-suppressed corrections and potential observational signatures will be required to assess empirical viability.

The framework thus provides a unified, medium-based perspective in which the constants c , G , and \hbar enter as structural capacity parameters of the vacuum. Whether this description can be extended into a fully predictive, ultraviolet-complete theory remains an open question for future investigation.

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