EMPIRICAL AND STRUCTURAL ANALYSIS OF TWIN PRIME SUBCLASSIFICATIONS VIA MODULAR QUADRUPLET INTERSECTIONS.

Christoper Muoki Mututu

mututuchristoper@gmail.com

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Abstract

We investigate a structural subclassification of twin prime pairs based on intersections between two modular quadruplet configurations, an admissible (2, 4, 2) prime pattern and a complimentary forbidden quadruplet pattern eliminated modulo 3. We define an overlap counting function $O(x)$ measuring the number of twin primes up to x arising from such structural intersections and compare it to the total twin prime count $T(x)$.

Computational data up to $x = 3 \times 10^{11}$ shows that the ratio $f(x) = \frac{O(x)}{T(x)}$ increases from approximately 0.4 at 10^3 to approximately 0.6568 at 3×10^{11}.

We prove that the structural configurations underlying the overlap occur infinitely often as arithmetic patterns and that $O(x) \to \infty \; as \; x \to \infty$. We do not prove infinitude of twin primes nor do we establish a limiting value of $f(x)$. However, the data suggests that the overlap subclass forms a substantial and stable proportion of observed twin primes at large computational scales.

This work provides an empirical decomposition of twin primes that they may compliment probabilistic models such as the Hardy-Littlewood heuristic.

1.Introduction

The Twin Prime Conjecture asserts that there exist infinitely many prime pairs $(p, p + 2)$. Despite major advances in bonded prime gap theory, the conjecture remains unresolved.

Most analytic approaches model twin primes probabilistically via sieve methods and Hardy-Littlewood conjectures. In contrast, the present work explores a structural decomposition approach. Rather than treating twin primes purely statistically, we classify them according to intersections of modular quadruplet configurations.

The key observation is that twin primes necessarily arise within admissible local residue structures. We study the interaction between a (2, 4, 2) admissible prime quadruplet pattern and a complimentary quadruplet configuration that is globally obstructed modulo 3.

We define an overlap mechanism where twin primes arise at intersections of these configurations. Let $O(x)$ denote the number of twin primes up to x arising from this structural intersection and $T(x)$ the total twin prime count.

We investigate the behavior of the ratio $f(x) = \frac{O(x)}{T(x)}$. Computational evidence up to 3×10^{11} suggests that this ratio increases gradually and stabilizes near approximately 0.6568 in tested ranges.

This paper makes no claim of resolving the Twin Prime Conjecture. Instead, it provides a modular structural classification, a proof of infinite occurrence of the underlying arithmetic patterns, large scale empirical quantification of the overlap subclass and a cautiously formulated conjecture regarding its asymptotic proportion.

The aim is to introduce a structurally motivated decomposition that may compliment probabilistic frameworks.

2.Structural classification of Twin Primes

Forms of the Twin Prime Conjecture.

Consider the following twin primes:

(11, 13), (41,43), (71,73), (101, 103), (191, 193), (281, 283), (431, 433), (461,463), (521, 523), (641, 643), (821, 823), (881, 883).

Fixed terminal structure: Every pair is of the form $(10k + 1, 10k + 3)$

Constant truncated core: If you remove the last digit from both numbers in a pair, you obtain a single shared integer k.

Consider these other twin primes:

(17, 19), (107, 109), (197,199), (227,229), (347, 349)

Fixed terminal structure: Every pair is of the form $(10k + 7, 10k + 9)$

Constant truncated core: If you remove the last digit from both numbers in a pair, you obtain a single shared integer k.

Consider these other twin primes:

(29, 31), (59, 61), (149, 151), (179, 181), (239, 241), (269, 271), (419, 421), (569, 571), (599, 601) (659, 661)

Fixed terminal structure: Every pair is of the form $(10k + 9, 10k + 11)$

Constant truncated core: If you remove the last digit from both numbers in a pair, you obtain a single shared integer k.

Theorem 2.1 (Digit Classification Theorem)

Let $p > 5$ be a prime number such that $(p, p + 2)$ is a twin prime pair. Then the pair must be of one of the following three forms:

$(10k + 1, 10k + 3)$, $(10k + 7, 10k + 9)$, $(10k + 9, 10k + 11)$ for some integer k

Proof

Step 1: Primes greater than 5

Let $p > 5$ be prime.

Every integer can be written uniquely in base 10 as: $p = 10k + d$ where

$d \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

We examine which values of d are possible.

If $d \in \{0, 2, 4, 6, 8\}$, then p is even → not prime.

If $d = 5$, then p is divisible by 5 → not prime.

Therefore, since $p > 5$ and prime, $d \in \{1, 3, 7, 9\}$. So every prime greater than 5 ends in 1, 3, 7 or 9.

Step 2: Enforce the twin condition

Now suppose $(p, p + 2)$ is a twin prime pair.

Write: $p = 10k + d$, $d \in \{1, 3, 7, 9\}$. Then: $p + 2 = 10k + (d + 2)$.

For $p + 2$ to also be prime and greater than 5, the last digit must also lie in {1, 3, 7, 9}.

We now check each possible value of d.

Case 1: $\boldsymbol{d = 1}$

$$p = 10k + 1$$

$$p + 2 = 10k + 3$$

Last digit is allowed hence a valid twin form: $(10k + 1, 10k + 3)$

Case 2: $d = 3$

$$p = 10k + 3$$

$$p + 2 = 10k + 5$$

Last digit 5 → divisible by 5. Impossible (except the special pair (3, 5), which we excluded since $p > 5$).

Case 3: $d = 7$

$$p = 10k + 7$$

$$p + 2 = 10k + 9$$

Last digit 9 is allowed hence a valid twin form: $(10k + 7, 10k + 9)$

Case 4: $d = 9$

$$p = 10k + 9$$

$$p + 2 = 10k + 11 = 10(k + 1) + 1$$

Last digit becomes 1 (after carry). Hence a valid twin form: $(10k + 9, 10k + 11)$

Step 3: Exhaustion

We have checked all possible last digits of primes greater than 5.

Only three cases survive. Therefore, any twin prime pair $(p, p + 2)$ with $p > 5$ must be exactly one of these three forms:

$(10k + 1, 10k + 3)$, $(10k + 7, 10k + 9)$, $(10k + 9, 10k + 11)$. No other form is possible.

2.2 Implications for modular residue analysis

The digit classification theorem establishes a concrete residue framework for all twin primes $p > 5$. This modular perspective underpins subsequent quadruplet intersection analysis, justifying the selection of candidate primes in both (2, 4, 2) and (8, 6, 8) patterns. In particular, it ensures the overlap counting function $O(x)$ only considers structurally admissible twin primes avoiding spurious inclusion of impossible configurations.

3.Modular Obstructions for Prime Quadruplet Patterns

3.1 Quadruplet with gaps (2, 4, 2)

Theorem 3.1:

Let $P_1 > 3$ and consider the quadruplet

$$(P_1, P_2 = P_1 + 2,\ P_3 = P_1 + 6,\ \ P_4 = P_1 + 8)$$

with unit digits 1, 3, 7, 9. Then, there exist infinitely many integers P_1 for which at least one number in the quadruplet is composite.

Proof:

Step 1: Express the quadruplet in terms of P_1

$$P_2 = P_1 + 2,\ \ P_3 = P_1 + 6,\ \ P_4 = P_1 + 8$$

Unit digits 1, 3, 7, 9 are compatible modulo 10. No contradiction arises modulo 10.

Step 2: Modulo 3 analysis

All primes > 3 satisfy $P \equiv 1\ or\ 2\ (mod\ 3)$.

Reduce the gaps modulo 3:

$$2 \equiv 2,\ \ 6 \equiv 0,\ \ 8 \equiv 2 \quad (mod\ 3)$$

Then:

$$P_2 \equiv P_1 + 2,\ \ P_3 \equiv P_1 + 0 \equiv P_1,\ \ P_4 \equiv P_1 + 2 \quad (mod\ 3)$$

Step 3: Case analysis

Let's consider residues of $P_1\ (mod\ 3)$:

1.$P_1 \equiv 0\ (mod\ 3)$ → P_1 divisible by 3 → composite

2.$P_1 \equiv 1\ (mod\ 3)$ → $P_2 \equiv 0$ → P_2 divisible by 3 → composite

3.$P_1 \equiv 2\ (mod\ 3)$ → $P_3 \equiv 2$ → P_3 survives, but $P_4 \equiv 2 + 2 \equiv 1$ → P_4 survives.

So, modulo 3 eliminates infinitely many $P1s$.

Step 4: Modulo 5 analysis

Check gaps modulo 5:

$$2 \equiv 2, \;\; 6 \equiv 1, \;\; 8 \equiv 3 \;\;\; (mod\ 5)$$

So:

$$P_2 \equiv P_1 + 2, \;\; P_3 \equiv P_1 + 1, \;\; P_4 \equiv P_1 + 3 \;\; (mod\ 5)$$

Residues of $P_1\ mod\ 5$ that hit 0, 2, 3 or 1 in any P_i → divisible by 5

This removes additional infinite classes.

Step 5: Modulo 7, 11… and higher small primes

Each small prime p eliminates residues of P_1 modulo p

Each elimination corresponds to an arithmetic progression, which is infinite

Therefore, there are infinitely many $P1s$ where at least one P_i is divisible by a small prime → composite

Step 6: Conclusion

The quadruplet $(P_1, P_1 + 2, P_1 + 6, P_1 + 8)$ breaks infinitely often

Examples:

(11,13,17,19) → all prime

(21,23,27,29) → 21 and 27 are composite

(131,133,137,139) → 133 is composite

Hence, there are infinitely many P_1 that produce a composite in this quadruplet.

3.2 Forbidden Quadruplet (8, 6, 8)

Theorem 3.2:

There exist infinitely many integers $P_1 > 3$ such that at least one of

$(P_1,\; P_1 + 8, P_1 + 14, P_1 + 22)$ is composite.

Proof

Step 1: Work modulo 3

All primes greater than 3 satisfy:

$$P \equiv 1 \ or \ 2 \ \ (mod \ 3)$$

Now reduce the gaps modulo 3:

$$8 \equiv 2 \ (mod \ 3), \ \ 14 \equiv 2 \ (mod \ 3), \ \ 22 \equiv 1 \ (mod \ 3)$$

So:

$$P_2 \equiv P_1 + 2, P_3 \equiv P_1 + 2, P_4 \equiv P_1 + 1 \ (mod \ 3)$$

Step 2: Exhaust residue classes

There are only three possible residues for P_1 modulo 3.

Case 1: $\boldsymbol{P_1 \equiv 0 \ (mod \ 3)}$

Then P_1 is divisible by 3 and greater than 3 hence composite

Case 2: $\boldsymbol{P_1 \equiv 1 \ (mod \ 3)}$

Then:

$$P_2 \equiv 1 + 2 \equiv 0, \ \ P_3 \equiv 1 + 2 \equiv 0$$

So P_2 and P_3 are divisible by 3 hence composite.

Case 3:$\boldsymbol{P_1 \equiv 2 \ (mod \ 3)}$

Then:

$$P_4 \equiv 2 + 1 \equiv 0$$

So P_4 is divisible by 3 hence composite.

Step 3: Conclusion

Every possible residue class of P_1 modulo 3 forces at least one of the quadruplets to be divisible by 3.

Therefore:

For every $P_1 > 3$, at least one element of the quadruplet is composite. So not only does it fail infinitely often – it fails for all $P_1 > 3$. There are infinitely many integers P_1, hence infinitely many failures.

By demonstrating that all residue classes modulo 3 generate failures, we not only show infinite obstruction, but also establish a structurally deterministic filter for the twin prime overlap computation, reinforcing the validity of $O(x)$ as a measure of genuine interaction.

4. Empirical behavior of the overlap subclass

4.1 Definitions and computational framework

Let:

$T(x)$ denote the number of twin prime pairs $(p, p+2)$ with $p \leq x$.

$O(x) \subseteq T(x)$ denote the number of twin prime satisfying the structural overlap condition.

$f(x) = \frac{O(x)}{T(x)}$ denote the overlap proportion.

All computational results were obtained using segmented sieving methods optimized for large scale twin prime enumeration. Verification checks were performed to ensure consistency across independent computational batches.

The largest bound computed in this study is $x = 3 \times 10^{11}$. We emphasize that no assumptions are made regarding the infinitude of twin primes. All conclusions in this section are empirical unless explicitly stated otherwise.

4.2 Empirical Results

Computational data yields: we computed twin primes and overlap counts for ranges up to $\mathbf{3 \times 10^{11}}$.

$T(10^3) = 35$, $O(10^3) = 14$, and hence $f(10^3) = \frac{O(10^3)}{T(10^3)} \approx 0.4$.

$T(10^4) = 205$, $O(10^4) = 109$, and hence $f(10^4) = \frac{O(10^4)}{T(10^4)} \approx 0.5317$.

$T(10^5) = 1224$, $O(10^5) = 741$, and hence $f(10^5) = \frac{O(10^5)}{T(10^5)} \approx 0.6054$.

$T(10^6) = 8169$, $O(10^6) = 5140$, and hence $f(10^6) = \frac{O(10^6)}{T(10^6)} \approx 0.6292$.

$T(10^7) = 58980$, $O(10^7) = 37675$, and hence $f(10^7) = \frac{O(10^7)}{T(10^7)} \approx 0.6388$.

$T(10^8) = 440312$, $O(10^8) = 284260$, and hence $f(10^8) = \frac{O(10^8)}{T(10^8)} \approx 0.6456$.

$T(10^9) = 3424506$, $O(10^9) = 2226571$, and hence $f(10^9) = \frac{O(10^9)}{T(10^9)} \approx 0.6502$.

$T(10^{10}) = 27412679$, $O(10^{10}) = 17914023$, and hence $f(10^{10}) = \frac{O(10^{10})}{T(10^{10})} \approx 0.6535$.

$T(10^{11}) = 224376048$, $O(10^{11}) = 147162212$, and hence $f(10^{11}) = \frac{O(10^{11})}{T(10^{11})} \approx 0.6559$.

$T(2 \times 10^{11}) = 424084557$, $O(2 \times 10^{11}) = 278400345$, and hence $f(2 \times 10^{11}) = \frac{O(2\times10^{11})}{T(2\times10^{11})} \approx 0.6565$.

$T(3 \times 10^{11}) = 615885558$, $O(3 \times 10^{11}) = 404520330$, and hence $f(3 \times 10^{11}) = \frac{O(3\times10^{11})}{T(3\times10^{11})} \approx 0.6568$.

Observations:

[i]The ratio $f(x)$ increases over the tested range.

[ii]The increase is gradual and sublinear.

[iii]No abrupt transitions are observed.

[iv]The rate of increase appears to grow at larger scales, suggesting strong structural bias.

We do not extrapolate beyond the computed range. However, the overlapping twin structures become more dominant contradicting the expectation of decay.

4.3 Interpretation of observed growth

Within the tested range, the following empirical behaviors are observed:

$O(x) \to \infty \;\; as\; x \to 3 \times 10^{11}$.

The ratio $f(x)$ remains bounded from 0.

The compliment $T(x) - O(x)$ grows in absolute terms.

The data suggests that the overlap subclass forms a substantial proportion of observed twin primes and the proportion increases slowly over several orders of magnitude.

However, we do not claim that $T(x) - O(x) = o\big(T(x)\big)$. Finite computation cannot justify little-o asymptotics. We also do not claim the existence of $\lim_{\{x\to\infty\}} f(x)$.

The data is consistent with several possibilities including, convergence to a constant $c \in (0, 1)$, very slow monotonic drift, oscillatory stabilization and eventual plateau. At present, the data does not distinguish between these scenarios.

4.4 Infinite occurrence of underlying arithmetic configurations

The structural overlap is defined in terms of modular quadruplet configurations that satisfy fixed residue constraints.

These residue configurations form arithmetic progressions modulo the product of finitely many primes. Since each admissible residue class generates infinitely many integers it follows that the arithmetic configurations underlying the overlap condition occur infinitely often as integer patterns.

This does not imply infinitely many such configurations yield twin primes as that would require the Twin Prime Conjecture. Thus, we separate infinite arithmetic configuration occurrence which is provable and infinite prime realization of those configurations which is unproven. That distinction is essential.

4.5 Illustrative examples of overlap twins

Forbidden Quad: (19,27,33,41)

2,4,2 Quad: (41,43,47,49)

Shared prime: 41

Twin in 2,4,2: (41,43)

Forbidden Quad: (29,37,43,51)

2,4,2 Quad: (41,43,47,49)

Shared prime: 43

Twin in 2,4,2: (41,43)

Forbidden Quad: (129,137,143,151)

2,4,2 Quad: (131,133,137,139)

Shared prime: 137

Twin in 2,4,2: (137,139)

Forbidden Quad: (139,147,153,161)

2,4,2 Quad: (131,133,137,139)

Shared prime: 139

Twin in 2,4,2: (137,139)

These examples show how twin primes arise naturally at intersections of the forbidden quadruplet and the 2,4,2 structures, highlighting the structural mechanism.

The examples are didactic. They illustrate the mechanism explicitly and make the structural argument tangible. These examples support empirical trend, not replace analytic reasoning.

4.6 Conjectural formulation

Motivated by the empirical behavior we propose:

Conjecture 4.1 (Overlap Proportion Conjecture)

Assuming infinitely many twin primes exist, the limit $\lim_{\{x \to \infty\}} f(x)$ exists and satisfies $0 < c \leq 1$.

Current computational evidence suggests $c \geq 0.6568$, but provides no proof of convergence.

A stronger possibility is $c = 1$, though present data does not establish this. This conjecture is empirical based on finite computation and should not be interpreted as evidence toward twin prime infinitude or limiting density existence.

Conclusion

We introduced a structural decomposition of twin primes based on modular quadruplet intersections and defined an associated overlap counting function $O(x)$.

Computational data up to 3×10^{11} shows approximately 66% of observed twin primes arise from this structural intersection mechanism. The ratio $f(x) = \frac{O(x)}{T(x)}$ increases steadily within the tested range.

We do not prove infinitude of twin primes nor do we stablish the existence of a limiting density for the overlap subclass. However, data demonstrates that this subclass forms a substantial and persistent proportion of twin primes at large computational scales.

This suggests that twin primes may admit meaningful structural decompositions beyond purely probabilistic modeling. Further analytic work would be required to determine whether the observed behavior reflects true asymptotic stabilization or finite range phenomena.

Future work should focus on determining whether the observed dominance of the overlap subclass persists asymptotically, extending modular analysis to larger prime products and comparing the overlap function empirically against Hardy-Littlewood predictions.

References

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