

Beal's Conjecture

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Abstract: This idea gives a path to make new proofs using this method. Namely to prove Fermat's last theorem in simple and general way and it links the number theory and linear algebra.

1 Introduction

Beal's Conjecture is a conjecture in number theory formulated in 1993 while investigating generalizations of Fermat's Last theorem set forth in 1997 as a Price problem by the United States of America's Dallas, Texas number theory enthusiast and billionaire banker, Mr. Daniel Andrew Beal.

Beal's conjecture states that: The equation $A^x + B^y = C^z$ —(1)

if A, B, C, x, y, z are integers with all exponents (x, y, z) greater than 2 then the bases A, B, C must share a common prime factor. In other words if A, B, C are relatively prime then the equation (1) has no solution.

2 theorem

Statement: The equation $x^l + y^m = z^n$ has no solution when x, y, z are relatively prime and l, m, n are integers greater than 2.

Proof:— Assume, $ax - by = x$, $cx - dy = z$, $ex - fz = y$ then we go to write

these equations in the form of matrix as $A = \begin{pmatrix} 1 & b & -a \\ -c & d & 1 \\ -e & 1 & f \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $B =$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ then $AX = B$ —(2).

now, we get determinant of A we get zero. with simplification we obtain $df + bcf + ac = be + aed + 1$ —(3)

Consider

$$(az^2 - by^2)x^2 + (bx^2 - cz^2)y^2 = (ax^2 - cy^2)z^2$$

Here, take $az^2 - by^2 = x$, $bx^2 - cz^2 = y$ and $ax^2 - cy^2 = z$ then we get $x^3 + y^3 = z^3$ —(4) it has no solution in Euler's proof.

by our assumption, replace a, b, e, f, c, d as az, by, bx, cz, ax, cy then by equation (3) we get $c^2yz + a^2zx = b^2yx + 1$ —(5) by comparing with equation (4) we have no solution where x, y, z are relatively prime. If we replace y by y^r , z by z^s and x by x^t (in assumption replacement) where r, s, t are greater than 1 then by equation (5) we get no solution because it is still relatively prime with each other. so we conclude that $x^l + y^m = z^n$ has no solution where x, y, z are relatively prime and

for all integer l, m, n greater than 2.
Hence the proof.

3 Conclusion

This proof concludes that every beal's conjecture pattern conjectures can be solvable using matrix methods.

4 References

1. Zhang Yue's Fermat's last theorem for $n=3$. <https://www.mathematicaljournal.com>.
2. linear differential equation wikipedia. <https://en.wikipedia.org>