Study On Delta Between Primes And Triangular Numbers

Marcin Barylski

Created: January 11, 2026
The last update: February 24, 2026

Abstract

There are several interesting properties of triangular numbers and research work devoted to them. One of the them is correlation between them and primes - there is hypothesis that between every two different triangular numbers >1 there is always a prime number. This paper is focused on detailed examination of difference, mainly between triangular numbers and their closest (smaller or greater) primes (this difference is called in this work delta, δ_{TP}), including its extreme values, also in spirit of finding effective test to search for prime numbers.

1 Introduction

Triangular numbers T [1] are a sequence of natural numbers representing the sum of consecutive integers from 1 to n, forming a system of points in the shape of an equilateral triangle. $i-th$ triangle number T_i can be calculated using the formula: $T_i = \frac{i(i+1)}{2}$ (which may be proven using mathematical induction). Prime numbers P are natural numbers >1 that are not a product of two smaller natural numbers >1.

Both T and P are infinite sets. This work is devoted to study differences between triangular numbers and the closest primes, also in spirit of selecting (potentially) triangular number as a starting point for effective prime number search. Framework [2] is used in all experiments.

2 Types of delta

Let's denote difference between triangular number T_i and its closest prime P as δ_{TP}. δ_{TP} can be positive ($\delta_{TP} = +N$, when the closest prime is bigger than triangular number), negative ($\delta_{TP} = -N$, when the closest prime is smaller than triangular number) or $\delta_{TP} = 0$ (triangular number is also a prime). Special case is when triangular number is an average of two primes and both are the closest - let's denote this case as $\delta_{TP} = \pm N$.

When comparing specific numbers T and P we can also write: $\delta_{T,P}$, where T and P will be exact integer values, for instance: $\delta_{1,3} = +2$, $\delta_{6,3} = -3$.

3 When triangular number is a prime

There is only one triangular number that is also a prime number: 3, thus $\delta_{TP} = 0$ happens only once: $\delta_{3,3} = 0$.

Lemma 1. *$\delta_{TP} = 0$ if and only if $T = 3$.*

Proof. $T_i = \frac{i(i+1)}{2}$ is the formula to calculate $i-th$ triangular number. It gives $T_2 = 3$ for $i = 2$ and 3 is a prime. Triangular numbers lesser than 3 are two: $T_0 = 0$ and $T_1 = 1$ - neither of them is prime, so let's focus on triangular numbers greater than 3 (i>2). i and $i+1$ are two consecutive integers, thus one of them is even, thus either i or $i+1$ is divisible by 2. If i>2, then half of even number (either i or $i+1$) is >1. This means that T_i, i>2 (based on its formula) is a multiplication of two integers >1 and cannot be prime by definition. □

4 Delta = ± 1

If $\delta_{TP} = \pm 1$, then T_i is odd and P is even, or vice-versa.

Let's analyze trivial cases first where the closest prime is 2. For $T_0 = 0$ the closest prime is 2: $\delta_{0,2} = 2$>1. For $T_1 = 1$ the closest prime is also 2: $\delta_{1,2} = +1$ and for $T_2 = 3$ the closest prime is also 2: $\delta_{3,2} = -1$. For $T_3 = 6$ and next triangular numbers, the closest prime is >2.

All primes greater than 2 are odd. In that case, if $\delta_{TP} = \pm 1$, then T_i needs to be even. Triangular numbers >0 are in infinite repeated sequence: "odd-odd-even-even", so 50% of candidates (odd) are eliminated by nature. Executed experiments show that ≈ 7% results are with delta ±1 (see Figure 1).

5 Delta = ± 2

If $\delta_{TP} = \pm 2$, then both T_i and P are either even or odd.

Let's analyze trivial case first. For $T_0 = 0$ we have $P = T_0 + 2 = 2$ and for $T_1 = 1$ we have $P = T_1 + 2 = 3$ - both 2 and 3 are primes and δ_{TP} cannot be negative here (because all primes $>T_1$).

All primes greater than 2 are odd, which leads to the non-trivial case: δ_{TP} can be -2 or 2 if and only if T_i is odd too. Triangular numbers >0 are in infinite repeated sequence: "odd-odd-even-even", so 50% of candidates (even) are eliminated by nature. Executed experiments show that ≈ 13% of results are with delta ±2 (see Figure 1).

6 In search of the largest delta

The largest delta between triangular number and the closest prime to it found so far is $\delta_{TP} = +202$ ($T_{1817553} = 3303502498845$, $P = T_{1817553} + 202 = 3\ 303\ 502\ 499\ 047$). Other notable findings are listed in Table 1.

Figure 1 is presenting frequency of $|\delta_{TP}|$ for the first 10^7 triangular numbers - it is clearly depicting that in most of the cases the δ_{TP} is small This observation is a foundation to the experimental Hypothesis 1.

Hypothesis 1. *Search for prime numbers is more effective around triangular number than a random number.*

Table 1: Big δ_{TP} found during experiments

T_i	δ_{TP}	Prime(s)
61 237 225 666	+157	61 237 225 823
395 550 422 641	±168	395 550 422 473, 395 550 422 809
429 137 662 096	+181	429 137 662 277
455 980 508 061	-170	455 980 507 891
479 887 920 721	+186	479 887 920 907
3 303 502 498 845	+202	3 303 502 499 047
13 626 817 955 751	+190	13 626 817 955 941
22 722 660 593 955	+188	22 722 660 594 143
25 477 724 120 586	-187	25 477 724 120 399
36 239 385 654 181	+196	36 239 385 654 377

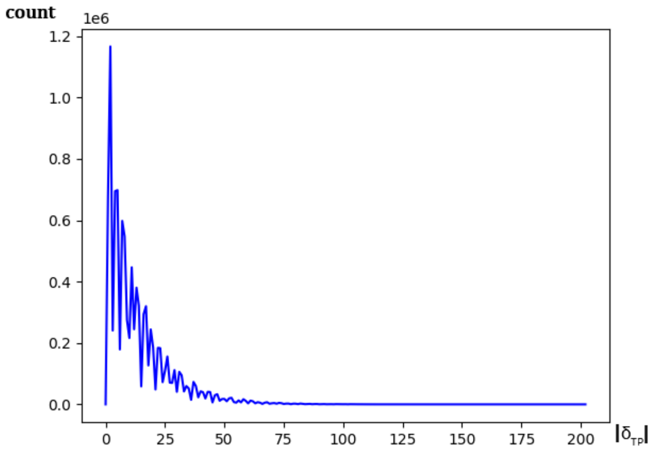

Figure 1: Frequency of $|\delta_{TP}|$ for the first 10^7 triangular numbers.

Figure 2 and Figure 3 show either positve or negative δ_{TP} recorded during experiments for consecutive T_i. Figure 4 is a concatenation of Figures 2 and 3 - it depicts that positive extreme δ_{TP} is rather more frequent than negative one (in other words, theer is more red than blue on top of the concatenated range).

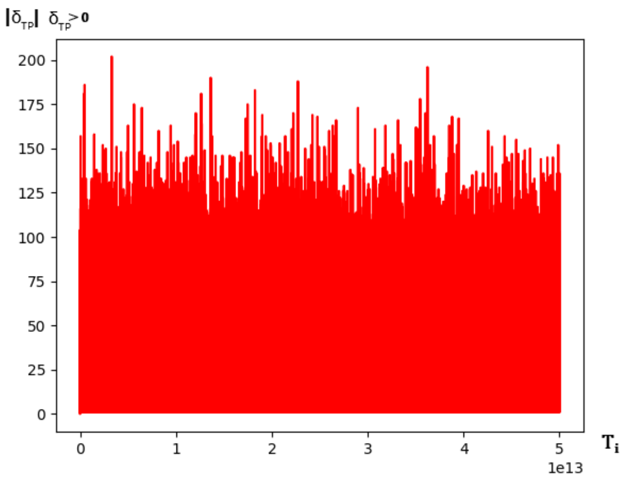

Figure 2: δ_{TP}>0 for the first 10^7 triangular numbers T_i.

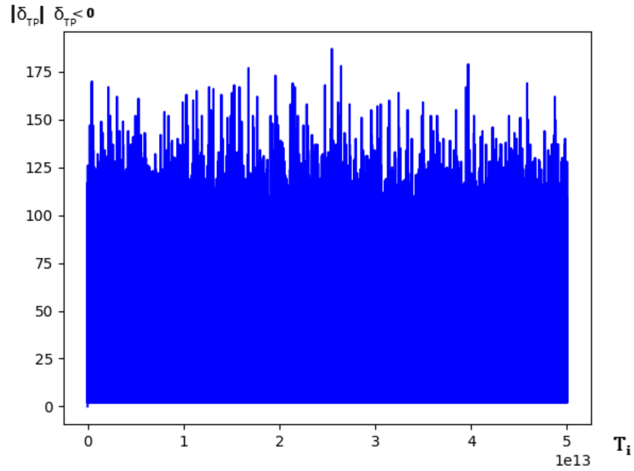

Figure 3: $\delta_{TP} < 0$ for the first 10^7 triangular numbers T_i.

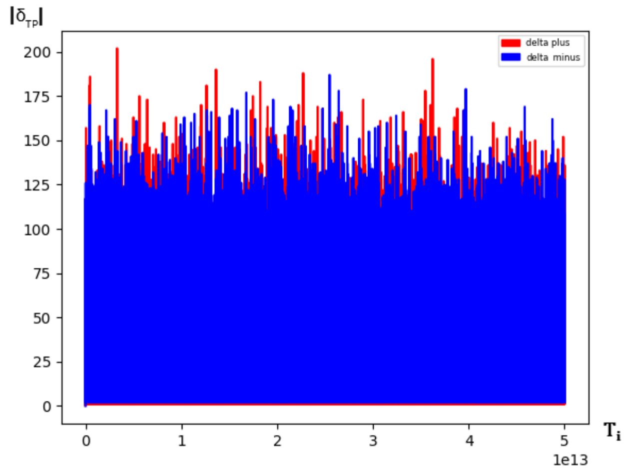

Figure 4: $|\delta_{TP}|$ for the first 10^7 triangular numbers T_i.

7 Experimental search for prime numbers

In order to verify Hypothesis 1 the following experiment was conducted: in examined interval $num_{min}; num_{max}$ it was compared how many iterations (±1) from the starting point was required to find a prime number. Search for closest prime for conducted around triangular numbers, and, for comparison, around pseudo-random numbers and numbers of form $6 \times n - 1$.

Experiments were run in 4 series. In each series, for each type of number, focus was to search for the delta between it and its closest prime. Figures 5-12 depict the results with the two types of graphs: 1) frequency of the given delta, and 2) delta over consecutive numbers/runs. Additionally, every series ends with calculation of arithmetic average of the recorded deltas.

- Series #1: $num_{min} = 2500000; num_{max} = 3000000$
- Series #2: $num_{min} = 4000000; num_{max} = 4500000$
- Series #3: $num_{min} = 5000000; num_{max} = 5500000$
- Series #4: $num_{min} = 6000000; num_{max} = 6500000$

Results of series #1 (see also Figure 5 and Figure 6):

- Average delta - prime to triangular: 13.183896
- Average delta - prime to random: 6.226838

- Average delta - prime to 6n-1: 5.649354

Results of series #2 (see also Figure 7 and Figure 8):

- Average delta - prime to triangular: 13.606476
- Average delta - prime to random: 6.3994
- Average delta - prime to 6n-1: 5.649354

Results of series #3 (see also Figure 9 and Figure 10):

- Average delta - prime to triangular: 13.79982
- Average delta - prime to random: 6.552896
- Average delta - prime to 6n-1: 5.649354

Results of series #4 (see also Figure 11 and Figure 12):

- Average delta - prime to triangular: 13.976484
- Average delta - prime to random: 6.648846
- Average delta - prime to 6n-1: 5.649354

Executed experiments visualized that Hypothesis 1 is most probably false - more effective is either selection of a pseudo-random or number of form $6 \times n - 1$ as a starting point rather than a triangular number. Surprisingly, average delta between the closest prime to 6n-1 converges to 5.649354 in all executed experiments.

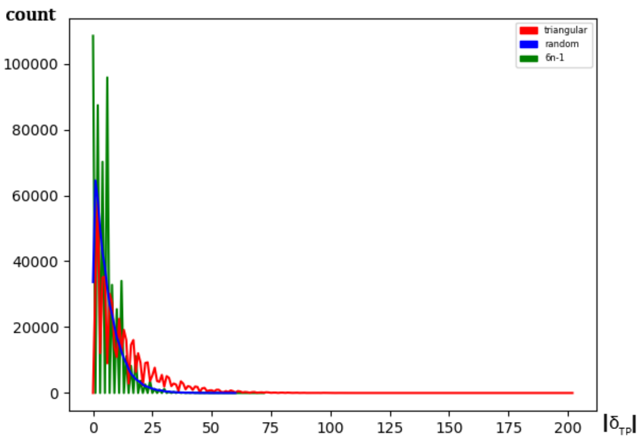

Figure 5: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 2500000; num_{max} = 3000000$.

8 Conclusions

The smallest possible δ_{TP} between triangular number and a prime is 0 but it happens only once. The largest delta between triangular number and the closest prime found during executed experiments is $\delta_{TP} = +202$. Based on executed experiments, triangular number is not the most effective starting point when looking for a prime using brute-force method (checking number ± 1, one-by-one) - much more effective is selection of pseudo-random or $6 \times n - 1$ number instead. Lastly, checking if average delta between $6 \times n - 1$ and the closest prime converges to some stable limit (is it really 5.649354?) may be the essence of next studies.

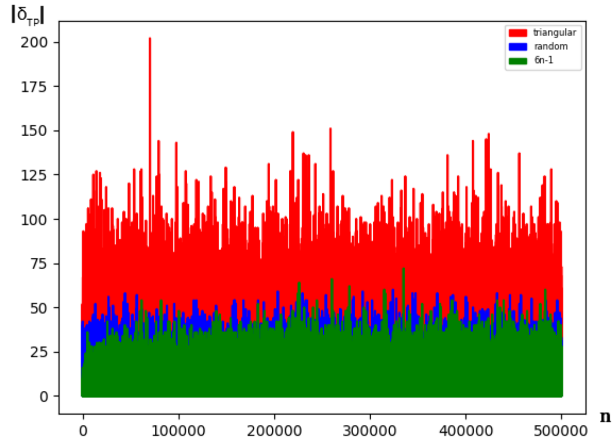

Figure 6: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 2500000; num_{max} = 3000000$.

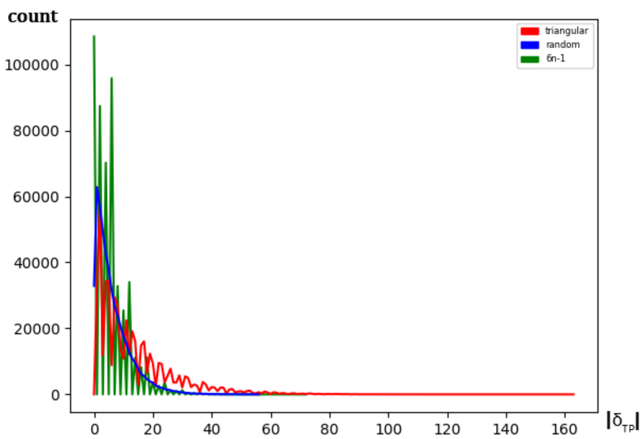

Figure 7: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 4000000; num_{max} = 4500000$.

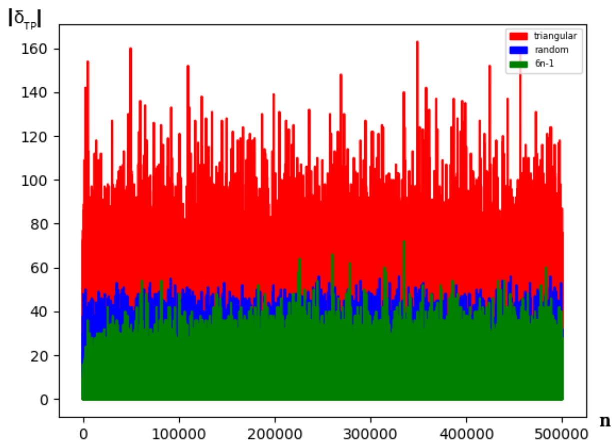

Figure 8: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 4000000; num_{max} = 4500000$.

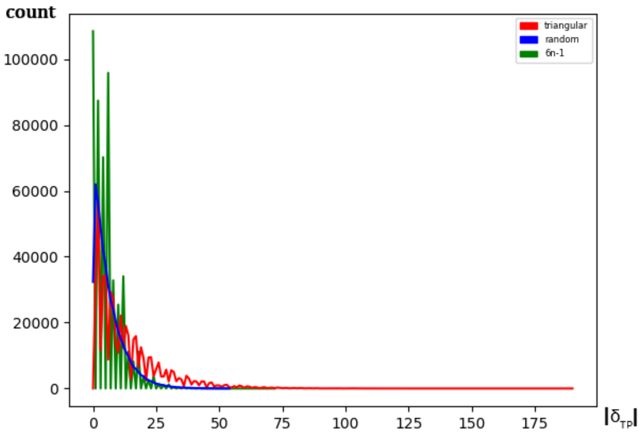

Figure 9: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 5000000; num_{max} = 5500000$.

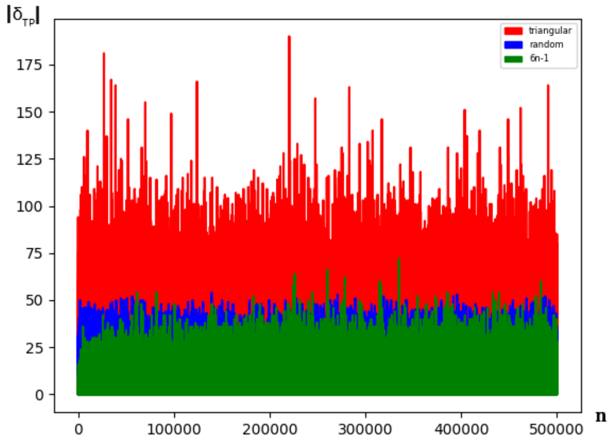

Figure 10: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 5000000; num_{max} = 5500000$.

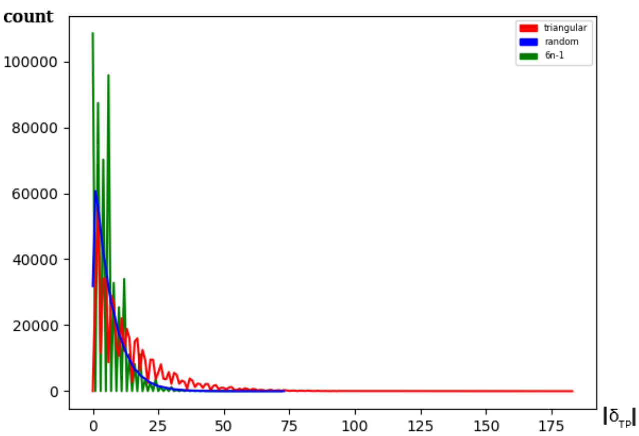

Figure 11: Comparison of frequency of $|\delta_{TP}|$ for three types of numbers (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 6000000; num_{max} = 6500000$.

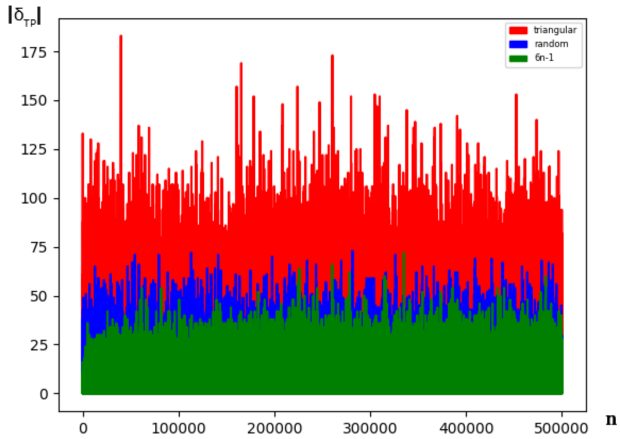

Figure 12: Comparison of $|\delta_{TP}|$ for three types of consecutive numbers n (triangular, pseudo-random, numbers of form $6 \times n - 1$) from a set: $num_{min} = 6000000; num_{max} = 6500000$.

References

[1] *Weisstein, Eric W.* "Triangular Number." From MathWorld – A Wolfram Resource. https://mathworld.wolfram.com/TriangularNumber.html

[2] *Library for various operations on primes.* https://github.com/mbarylsk/primes