A STATEMENT OF THE COSMOLOGICAL CONSTANT PROBLEM AND AN EFFECT OF THE REDUCING OF VACUUM BY MATTER BASED ON UNCERTAINTY RELATIONS

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ABSTRACT

A problem of the connection of cosmology with elementary particle physics is shown on the level of uncertainty relations. At the scales about 10^{-2} m the contribution of one single type virtual elementary particles in the lower boundary of vacuum energy is considered. The observed value of vacuum energy or energy density on the large scale of the Universe corresponds only to this scale. This is the energy about 3.34 GeV per each one cubic meter. The minimal high energy physics scale achieved by experiments at present is considered. The lower boundary of the energy is generated by the quantum vacuum of empty space and the quantum vacuum limited by matter in the Universe mainly at scales down to 10^{-15} m and more much are not in agreement with the observed value, as that is established. These lower limits for the energies of the vacuum are considered in the model of estimating where they generate by the presence of virtual particles in free space and the virtual particles which are limited by matter and exist together with matter in the Universe. The numerical values of the boundary energies are obtained using the computer algorithm.

INTRODUCTION

The Heisenberg uncertainty principle is the fundamental principle of the quantum mechanics. Written down for any pair of conjugate quantities uncertainty relation follows from the commutator of these quantities. Uncertainty relations can be used for the assessment of quantities not having certain values in this quantum state via the determination of classical quantities in this state. The description of the motion of quantum particles in such a way becomes possible due to wave-particle duality or wave-corpuscle dualism. Such assessment has been given in [1] for the energy of the particle in the first Bohr orbit and the radius of the first Bohr orbit expressed via momentum of the particle which does not have certain value. This approach has been applied to hydrogen atom, and the formula gained from the uncertainty relations coincided with the formula for the energy of the particle in the Bohr atom theory.

In turn, the Bohr atom theory can exist due to the existence of wave-particle duality. The assessment of some quantities on uncertainty relations has to give the correct order coinciding with the conclusions of the theory of a quantum system, for example, Bohr atom theory or Schrödinger theory. Consideration of fundamental laws in quantum theory on uncertainty relations for specific quantities in the order of the magnitude of these quantities is justified. The wave-particle duality allows an using the classical laws of motion for particles at the quantum level. Quantum effects taking place at the quantum level remain at this level due to interaction is at this level. This interaction leads to the spatial limitation of quantum particles happening, usually, in the electric or/and the magnetic field(s) of other particles which make small in amplitude finite motion in comparison with the considered particle. The described situation is implemented in an atom where the considered particle is the moving electron, and the second particle is nucleus, in the field of which the electron exists. Thus, its state in the simplest case of hydrogen atom implements as motion on the circular stationary orbit with the value of the main quantum number. Also, the wave-particle duality gives the possibility for the existence of the description of particles' scattering known as Rutherford's theory, where quantum particles are considered as point-like particles moving on classical trajectories.

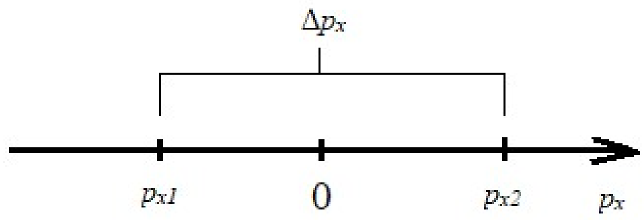

fig. 1 – Uncertainty of the *x* component of the momentum and the average value of the *x* component of the momentum (median) for vacuum virtual particle

1. Uncertainty Relations As An Approach To Assess The Observed Energy Of The Universe And The Modern Achieved Threshold Of Energy

The represented approach is acceptable when quantum particles are considered, but nevertheless the classical idea of particles is necessary. By means of such theory one can assess some quantities appearing in "right theory". This theory containing at the same time classical and quantum description is applicable like, for example, the Bohr's theory for hydrogen atom, and is true due to the existing of the wave-particle duality. Consider the uncertainty relations for momentum and coordinate

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}. \tag{1.1}$$

The relation (1.1) is applicable for one particle so that the uncertainty of the coordinates of particle is the width of the wave packet along the axis x, and the corresponding uncertainty of momentum is the width of the wave packet in momentum space along the same axis [2]. This relation can be applied for real particle or virtual particle. Virtual particles, according quantum field theory (QFT), fill the vacuum everywhere and always. An approach considering in this paper employs only the quantum vacuum of the quantum electrodynamics (QED), where it is taken virtual electron-positron pairs and quantum photon vacuum is outside of the consideration. The vacuums, generated by other matter particles and fields of other fundamental interactions are to be the part of consequent work. If the vacuum is filled by virtual particles, one can imagine one single virtual particle in the sizes of its, limited by interaction on the quantum level, wave packet, then expanding the model, according the concept of vacuum, on all space of the Universe by translation this cell for the all directions and the arbitrary distances of space, it can be gained the simple approximate model of the fraction of real vacuum. Thus, this uncertainty relation as correct for one particle will not lose force in the case of the large number of virtual particles in the vacuum. So, one can use this relation for the vacuum, then, the first, it is needed to care that the average value of every component of momentum (and energy) must be zero:

$$\langle p_i \rangle = 0, \tag{1.2}$$

$$\langle E \rangle = 0, \tag{1.3}$$

that is justified for vacuum. This can be achieved by placing the middle of the corresponding interval exactly at the point zero on its axis (see fig. 1). If that, the value $\mathbf{p}_2$ for the momentum can be used to express the uncertainty interval of the momentum through this value

$$\Delta \mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1, \tag{1.4}$$

where $\mathbf{p}_1 = -\mathbf{p}_2$ and one can get

$$\Delta \mathbf{p} = 2\mathbf{p}_2 . \tag{1.5}$$

The second, it is necessary to choose the detecting values of the uncertainty intervals, because the interval of uncertainty does not imply the certain value of corresponding quantity. One needs to give a value to the quantity which can give a value within the interval owing to the detecting or the measurement in the general sense of word, according the wave-particle duality, applying to this case and according the limit transition to the classical/quasiclassical physics. Let will be chosen the maximum values of the uncertainty intervals in momentum space as detecting values to get the maximal possible energy of the vacuum, and these intervals in such a way can be expressed via this value (see fig. 1). This will show an upper limit for the energy exists. For vacuum is essential energy but not momentum, therefore one can pass from the maximal value of momentum according to (1.1) to the maximal value of energy corresponding this momentum on the formula of special theory of relativity

$$E_2^2 = p_2^2 c^2 + m^2 c^4 \tag{1.6}$$

in the inequality expressing the values $\mathbf{p}_2$ from (1.1), according (1.5) and collecting them as it is required in (1.6). At that as it is accepted in such assessments, one doesn't need to use virtuality for virtual particles to be taken into account, they are thought like real particles. It is also going to use the equality condition of the momentum components. As a result of simple applying the similar inequalities to each other one can receive the uncertainty relation for the energy and the all location uncertainties along each of 3 space axes

$$E^2 \ge \frac{\hbar^2 c^2}{16}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + m^2 c^4, \tag{1.7}$$

or

$$\sqrt{\frac{\hbar^2 c^2}{16}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + m^2 c^4} \le E \le -\sqrt{\frac{\hbar^2 c^2}{16}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + m^2 c^4} . \tag{1.8}$$

Passing from energy to energy density

$$\left(\Delta x \Delta y \Delta z\right)^{-1} \sqrt{\frac{\hbar^2 c^2}{16}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + m^2 c^4} \le w \le -\left(\Delta x \Delta y \Delta z\right)^{-1} \times \tag{1.9}$$

$$\times \sqrt{\frac{\hbar^2 c^2}{16}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) + m^2 c^4} .$$

At this stage it is needed to choose a form of the virtual particle wave packet. Now the wave packet has the form of rectangular parallelepiped. As was considered above this uncertainty relation can describe not only one particle and the vacuum as whole. This is actually for the inequality for the energy density. From here it can be determined the scale of the observed cosmological energy density. For this put

$$\Delta x = \Delta y = \Delta z , \tag{1.10}$$

so it was chosen the cubic form of the wave packet or cell of the vacuum. As the vacuum energy according observation is positive, one should take the inequality with positive energy

$$w_\Lambda \ge \Delta x^{-3} \sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{\Delta x^2} + m^2 c^4} . \tag{1.11}$$

The energy density here is the observed cosmological energy density, which follows from the cosmological constant [3] on the formula [4] and has the value in SI

$$w_\Lambda = \frac{c^4}{8\pi G}\Lambda = 5.34\cdot 10^{-10}\,\frac{\mathrm{J}}{\mathrm{m}^3} = 3.34\frac{\mathrm{GeV}}{\mathrm{m}^3}\,. \tag{1.12}$$

Then, it can be gained the following inequality of eighth order

$$\Delta x^8 - w_\Lambda^{-2} m^2 c^4 \Delta x^2 - \frac{3}{16}\hbar^2 c^2 w_\Lambda^{-2} \geq 0\,, \tag{1.13}$$

(for one single type of particles) which must be reduced to the number inequality, and must be chosen acceptable solution (real, positive). Further it is laid $\Delta x \equiv l$. Thus, one gains this scale

$$l \geq 0.054\mathrm{m}\,. \tag{1.14}$$

.

Then the minimal energy of such virtual particle, i.e. particle having this lower limit of linear uncertainty one can gain having multiplied the volume of this scale by the energy density (1.12):

$$E_\Lambda = l^3 w_\Lambda = 8.41\cdot 10^{-14}\,\mathrm{J} = 0.53\mathrm{MeV}\,. \tag{1.15}$$

It is well-known that the modern high energy physics is gone down to 10^{-18} meters scale [5] (for instance, one can take $6.18\times 10^{-18}\,\mathrm{m}$), which according the assessment

$$w_{HEP} \geq l_{HEP}^{-3}\sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{l_{HEP}^2} + m^2 c^4} \tag{1.16}$$

('HEP' is 'High Energy Physics') gives the energy density only at this scale and only for one single type of particles, a number

$$w_{HEP} \geq 10^{43}\,\frac{\mathrm{J}}{\mathrm{m}^3}\,. \tag{1.17}$$

It leads to the conclusion that the contribution of the quantum vacuum in the cosmological energy is insignificant. One can get relative error between these numbers and calculate that the energy density of real electron-positron vacuum on the 53 orders of magnitude lower then it must be on the confirmed in experiment theory. This fact as was shown in manner of this article is yet known as the cosmological constant problem [6]. An understanding for a resolution of this problem might express in that the cosmological energy is not the energy of only ordinal matter and interactions have already found in nature and also other undiscovered essence.

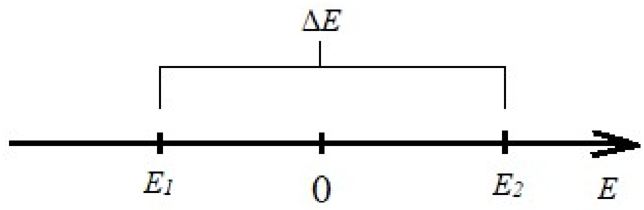

fig. 2 – Uncertainty of the energy and the average value of the energy (median) for vacuum virtual particle

2. Uncertainty Relations As An Approach To Assess The Energy Of The Free Quantum Vacuum And The Energy Of The Vacuum In Presence Of Matter

2.1. The General Consideration

Considering structure of the vacuum, the mechanism of the processes in which was provided by P. Dirac, the problem of description an interaction any type of vacuum with matter (not vice versa in this context) comes down to the problem of the limitation of vacuum virtual particles. The problem can be addressed, as the author offers, like the problem of the limitation of the real electron in an atom, by consideration the virtual electron of the electron-positron pair in an atom, i.e. in the electric field of atomic nucleus. Atomic nuclei are enough massive, so that them location uncertainties, therefore, must be small; as the vacuum polarization effect and the vacuum limitation by matter effect must act simultaneously. The vacuum limitation by matter effect or simply the vacuum limitation effect is the name for a new effect of the reducing of sizes of wave packets vacuum virtual particles creating near strong condensed matter, and can be found as yet one phenomenon of QFT from the qualitative analysis. This effect is explained by the quantum nature of the virtual particles of vacuum. The virtual particle is described by wave function, maximum(s) of which can be localized, like in the case of the electron in atom. Such matter field or wave function has much strength with non-zero strength gradient.

The elementary particles of matter are also offered as such natural limiting structures, when they are localized. For example, the electron can limit vacuum energy density by its own electric field. If to consider the naked electron (i.e. the point without the field), according the formula for the strength of the electric field of the electron in its coordinates: $E = \frac{ke}{r^2}$, $r = 0$, the strength of the field will be equal to infinity $E = \infty$. Thus, the strength of the field in the coordinates of the electron limits the vacuum, so that the energy density of the vacuum is equal to infinity as well as the energy density of the electric field of the electron: $w = \frac{\varepsilon_0 E^2}{2} = \frac{\varepsilon_0}{2}\frac{k^2 e^2}{r^4}$, $r = 0$ and $w = \infty$.

The electron in its coordinates limits the wave function of the virtual particle of the vacuum to the zero sizes, so that the vacuum energy density is equal in this point to infinity. And it is not surprising, the electron in the modern physics is considered as the point-like particle, is the singularity with infinite space-time curvature and the characteristics of density (the energy of field, mass). Also and that is important, the electron limits the wave functions of the virtual particles of the vacuum at any distance from its coordinates. But, apparently, the greatest contribution in the energy density is made by the vacuum particles limited near the electron.

Any particle, for example, proton or atomic nucleus can limit vacuum particles, and not only electrons.

The calculation of the scale for the observed energy density of the vacuum which is performed above is not accurate, as matter, which limits the vacuum everywhere, is not considered. It is necessary for the correct calculation to consider the density (and quantity) of matter in the Universe which limits the vacuum. The formula (1.8) or equivalently (1.9) does not change, but on one's representations, gained from the qualitative analysis of the problem, matter will lead to the effective reducing of the considered scale on the value Δl making up the new scale on which the energy/energy density is set without matter. Though matter has small location uncertainties, it is not localized with absolute accuracy, singularities in the coordinates of particles do not exist, they are cut out from the picture of matter, in which they can asymptotically be for the simplicity of the model.

So, the lower assessment of the energy density existing due to the virtual particles on the scale l in the free vacuum gives by the formula

$$w_v(l) \geq l^{-3}\sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{l^2} + m^2 c^4}\ , \tag{2.1.1}$$

'w_v' has brackets to mark the dependence of the assessment. And in the vacuum containing matter the scale l must be effectively reduced, so that the assessment will give by the formula

$$w_v(l-\Delta l) = w_{v+m}(l) \geq (l-\Delta l)^{-3}\sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{(l-\Delta l)^2} + m^2 c^4}\ , \tag{2.1.2}$$

'v' means 'vacuum' and 'm' means 'matter'.

There exists in the quantum mechanics the uncertainty relation for energy and time, which is suitable for this reasoning. It has the view

$$\Delta E \Delta t \geq \frac{\hbar}{2}, \quad (2.1.3)$$

here ΔE is the uncertainty of the energy, which a quantum system can have and Δt is the lifetime of virtual particle-antiparticle pair before annihilation and vanishing. From this point of view the pair can have any energy which hits the arbitrary uncertainty interval of the energy, if the temporal interval is enough short. Concretely, one can arbitrary increase the energy interval decreasing the lifetime and vice versa. Note, that the relation (2.1.1) is correct only for virtual particles, as it is written for energy density, and the relation (1.1) is correct for virtual and real particles.

The uncertainty interval for energy according (1.3) is arranged like the momentum intervals (see fig. 2), therefore it is correct

$$\Delta E = 2E_2, \quad (2.1.4)$$

and it is implied that the value of the energy E_2, the maximum value of the uncertainty interval is the detecting value.

As was mentioned above the vacuum polarization effect and the vacuum limitation effect act simultaneously, but both these effects act much more considerably when the virtual pair electron-positron creates exactly on the axis, on which the nucleus and this pair are lying so, that the virtual electron is closer to the nucleus and the virtual positron is far from the nucleus. The energy E_2 in (2.1.4) and in (1.6), (1.7) and (1.8) is the same energy. Hence, one can employ (2.1.3) for the theory.

If Δt is the lifetime of the pair, it must approximately be correct

$$v\Delta t = 2s, \quad (2.1.5)$$

where v is the velocity of the electron, s is the half of the passed distance by the electron. This formula is approximate unless to think that the velocity of the electron must really be variable, so that its motion is accelerated. However, in this approximation one can think that the virtual electron moves in average uniformly. A challenge is determination an effective reducing of the scale, caused by presence of matter in space. This can be done using simple 2D geometry. The first, it is needed to place reference matter – the proton or another nucleus, as they are heavy and have the small sizes of the wave packets in location space, in the picture of this phenomenon (see fig. 3). Near the nucleus the pair electron-positron creates exactly on the axis or the line connecting the proton, the electron and the positron. The electron is located near the proton and the positron is far from the proton. This creates a model which is justified due to that in real electron-positron vacuum virtual pairs create in all directions, i.e. along any axes, but for the quantum polarization effect are significant only dipoles which are located on the axes or the rays going from the charged center, i.e. the particle, and other directions are not instantly contributed in the effect. The directions whose projections on the rays going from the center are not equal to zero contribute as dipoles of least dipole moment. And for the vacuum limitation effect everything is effectively the same as for the vacuum polarization effect, the first, the rays directions are preferable for the effect with the condition that electrons should be created near the proton or nucleus and virtual positrons – far from the proton. Otherwise the pair is going to annihilate much more quickly than in the case of the right orientation of the pair. The second, if the direction has zeroth projection on the ray, such pair is going to be rotated from this orientation owing to the electrical force, acting on the electron to

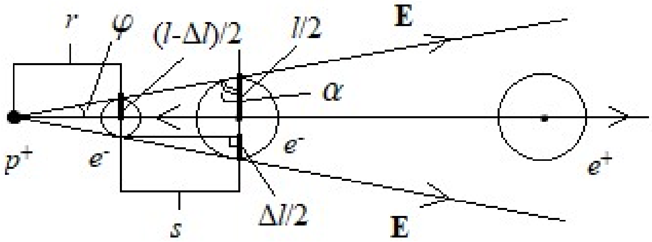

fig. 3 – Geometrical scheme of the vacuum limitation effect

attract it and on the positron to push away it from the particle. Thus, as it is seen, this case is going to be transformed to the intermediate case with nonzero projections on the rays or even to the case which is implied the pair, lying exactly on the ray. These resulted cases are justified only on the enough distance from the proton, for the pair must be able to rotate, it must have enough free space before the pair vanishes. The third, if the pairs create not exactly on the rays, this is intermediate case, they contribute as dipoles of least dipole moment in dependence on the angle between the corresponding ray and the axis on which the pair lies. Also in this case they are going to rotate to the rays directions as far as they could do it for them lifetime. In any case a model which the author offers is the first approximation and it is justified.

When the electron lying on the axis approaches to the proton in the time of scattering at once after the creation, the wave packet of the electron begins to move between the strength lines of the static electrical field of the proton coming nearer to the charge center. Now the dispersion of the wave packets of the virtual particles is not taken into account. In the process of that motion the electron's wave packet is reducing and it moves only between the same electrical field strength lines (see fig. 3). At that even if one takes into account the dispersion of the virtual matter waves, the positron moving away from the proton can increase the sizes of its wave packet and by that could decrease the vacuum energy. And the electron considering at an account of the dispersion nevertheless is going to reduce the sizes of its wave packet, so the dispersion will be suppressed by the tightening of the electron into the considering singularity between the same strength lines, i.e. actually by the vacuum limitation effect. The electron will move in such a way up to the certain limit of distance and it will turn back to the positron to annihilate. This limit from the center of the electron's wave packet to the center of the proton is designated as r. The electron goes the full way $2s$, i.e. in the one direction from the positron to the proton and in the opposite direction to the positron and from the proton for its lifetime Δt. In the initial state the electron has the linear sizes of its wave packet l, in the point of the maximum removal from the creation point it has linear sizes $l-\Delta l$, the corresponding angles are seen on the Figure 3, and the reducing of the linear sizes of the electron's wave packet is designated as Δl, and this is the required effective reducing of the scale generated by matter.

So, basing on the picture and using (2.1.5) one can gain the formula

$$\Delta l = v\Delta t \tan(\varphi). \qquad (2.1.6)$$

Also the following formula can be gained using Figure 3

$$\tan(\varphi) = \frac{l-\Delta l}{2r}. \qquad (2.1.7)$$

Combining these two formulae, one gets the formula

$$\Delta l = \frac{l}{1+\frac{2r}{v\Delta t}}. \qquad (2.1.8)$$

Expressing the lifetime from (2.1.3) in the form of inequality and using (2.1.4), simultaneously expressing the lifetime from (2.1.8), and collecting this all in the one single inequality, one can gain the inequality

$$v \le \frac{8E_2 r}{\hbar\left(\frac{l}{\Delta l}-1\right)}. \qquad (2.1.9)$$

Further it is laid

$$E_2 = E, \qquad (2.1.10)$$

for the energy in (2.1.4) and in (1.6); energy entering in (1.7) and in (1.8) is the same. The following formula of special theory of relativity will be needed

$$\mathbf{p} = \frac{\mathbf{v}E}{c^2}. \tag{2.1.11}$$

Multiplying the inequality (2.1.9) by the energy E and dividing it into the squared speed of light one can gain the inequality

$$\frac{vE}{c^2} \le \frac{8E^2 r}{\hbar c^2 \left(\frac{l}{\Delta l} - 1\right)}, \tag{2.1.12}$$

where it is thought $E > 0$. Using (1.5) without index 2 in (1.1) and the similar inequalities for the other space components, expressing the momentum from these inequalities, equating the right hand side of (2.1.11) and the gained vector expression, one can take normalization operation in these inequalities and can gain the inequality

$$\frac{vE}{c^2} \ge \frac{\sqrt{3}}{4}\frac{\hbar}{\Delta x}. \tag{2.1.13}$$

Coupling (2.1.12) and (2.1.13) one gets

$$\frac{\sqrt{3}}{4}\frac{\hbar}{l} \le \frac{vE}{c^2} \le \frac{8E^2 r}{\hbar c^2 \left(\frac{l}{\Delta l} - 1\right)}. \tag{2.1.14}$$

Expressing Δt from (2.1.8)

$$\Delta t = \frac{2r}{v\left(\frac{l}{\Delta l} - 1\right)}, \tag{2.1.15}$$

it is seen, for it must be $\Delta t > 0$, the following inequality must be executed

$$l > \Delta l. \tag{2.1.16}$$

Taking the positive part of (1.8) and taking into account (1.10) it can be gained the assessment

$$E \ge \sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{l^2} + m^2 c^4}, \tag{2.1.17}$$

also using (2.1.15) in (2.1.3) at account (2.1.4) it can be gained the assessment

$$E \ge \frac{\hbar}{8}\frac{v}{r}\left(\frac{l}{\Delta l} - 1\right). \tag{2.1.18}$$

It is needed to suppose the equality between (2.1.17) and (2.1.18). Equating them, one can solve this algebraic equation for the velocity of the particle

$$v = \frac{8r}{\hbar}\left(\frac{l}{\Delta l} - 1\right)^{-1} \sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{l^2} + m^2 c^4}. \tag{2.1.19}$$

Now the velocity v is expressed via l, Δl and r variables. The using (2.1.19) in (2.1.13) can give yet one assessment for the energy

$$E \ge \frac{\sqrt{3}}{32}\frac{\hbar^2 c^2}{rl}\left(\frac{l}{\Delta l} - 1\right)\left(\frac{\hbar^2 c^2}{16}\frac{3}{l^2} + m^2 c^4\right)^{-\frac{1}{2}}. \tag{2.1.20}$$

Also one should suppose the equality between (2.1.17) and (2.1.20). Then, solving the gained equation for the scale reducing, one gains

$$\Delta l = \frac{\sqrt{3} l^2}{\sqrt{3} l + 6r + 32 r l^2 \frac{m^2 c^2}{\hbar^2}}. \tag{2.1.21}$$

Having gained the result (2.1.21) for the effective vacuum scale reducing by matter one can check the inequality (2.1.16), and it is seen from here that this inequality-condition is executed.

The result (2.1.21) must be substituted in (2.1.2) for one single scale l and distance r to the proton or nucleus or much localized heavy particle. It is obvious that in reality all the scales exist and all they contribute to the vacuum energy/energy density. If to take into account this fact and that as was described above about any orientation of the axis of the pair's scattering, one needs, the first, to sum each the right hand side of (2.1.2) inequality for its own scale or linear size of the electron's wave packet l, to take into account all the scales. Further it is needed to insert some important remarks. One considers only the 'right' orientation of the electron-positron pairs, namely, negative charge must be in the hemisphere turned to the positive-charged matter considered. Also one considers only an increasing of the vacuum energy, which happens when the virtual electron attracts to the proton, decreasing sizes of its wave packet, and the moving away positron, as was said above, decreases or at least does not change the vacuum energy, when it increases sizes of its wave packet. So, the moving away positron once after the creation of the pair might have the negligible addition to the vacuum energy (it is possible) like the coming (to the electron) positron for annihilation. The coming positron decreases sizes of its wave packet only to its former sizes, when it was just created, and the electron, which has been so in detail considering in this paper, when it goes back to the annihilation point, is increasing the sizes of its wave packet that might slightly increase of the vacuum energy (it is possible). This is because the electron yet had the reduced sizes of its wave packet when it was scattering together with the positron from the creation point. Such increasing is smaller than the former sizes of the electron's wave packet. Of course, this all must be taken into account in the accurate calculation, which will be done in consequent work. However, the positron in this situation like the electron in empty space (without matter) contributes to the vacuum energy that was seen from the Section 1. Further the previous will be discussed. The second, because the axes can have arbitrary orientation remaining in the right hemisphere, they have any projection on the corresponding ray, so that the projection can vary from zero to maximum value $2s$ lying exactly on the ray, and this is the dipole without taking into account the rotation of real dipoles in the static electrical field of the nucleus. Thus, this dipole-projection has, like the ray dipoles, the distance to the nucleus, i.e. it is on the distance from the nucleus to the center masses of the dipole. So, as in this model all the dipoles lay exactly on the rays, to gain the full energy density one should sum every component of the sum in the first step varying the distance r from the least value at which the considering model is applicable to the maximum value at which the model is still working.

As well-known 3D space in the usual physics is continuous, therefore the scale l varies continuously, and the sum, about which was said in previous subparagraph actually must be replaced with an integral. But if one does that, a problem is appeared, because the integral is actually a sum with differential that changes the dimensionality of the quantity and adds this unnecessary difference. It is needed only the sum, but the sum must be continuous. This problem can be resolved by consideration of the definition of integral

$$\int_a^b f(x)\,dx = \lim_{N\to\infty}\sum_{i=1}^{N} f(x_i)\,\Delta x_i\,, \qquad (2.1.22)$$

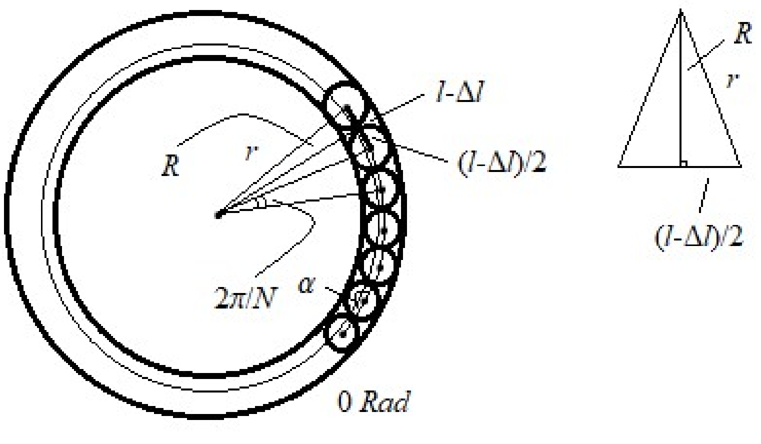

fig. 4 – Meridian slash of the one layer of the concentric ball layers packaging

where $\Delta x_i = \frac{b-a}{N}$. The last has the additional dimensionality x due to the difference $b-a$. The segment Δx_i can be taken out from the brackets of the sum and it in limit is infinite small, therefore one should divide the integral into the step Δl to restore the right dimensionality and only the sum to be left. But a new problem is appeared, namely, in continuous space the step Δl must go to zero, so that one gains the infinite expression and correspondingly the infinite vacuum energy. Thus, on this stage it is needed to suppose not continuous, and discrete physical space with minimal in average in all the directions (because the discrete structure of space implies anisotropy on fundamental scale) length, which one chose the Planck length l_P. The Planck length is determined by fundamental physical constants

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \tag{2.1.23}$$

and has the value

$$l_P \approx 1.62 \cdot 10^{-35}\,\text{m}\,. \tag{2.1.24}$$

Thus, the continuous sum can no more be continuous, and represents the discrete sum equal approximately to the continuous integral divided into the Planck length. The integral on scale l can be restricted as

$$l \in \left[l_{\min}, l_{\max}\right] \tag{2.1.25}$$

or if integrand decreases rapidly enough

$$l \in \left[l_{\min}, \infty\right), \tag{2.1.26}$$

that is not meaning the Universe must surely be infinite. And one has the restriction

$$r \in \left[r_{\min}, r_{\max}\right] \tag{2.1.27}$$

for a while. As it is seen one gets an integral with at least one infinite limit.

Above it was described one virtual pair and, correspondingly, one virtual electron. The found lower estimation (2.1.2) is assessment for the energy density $w_{v+m}(l)$ which can describe one single wave packet or all a universe consisting of such wave packets in each cubic cell of space. It is clear that this can't describe the real Universe. At this stage one needs to describe location or spatial packaging of the virtual particles. One must consider one single atom or heavy and well-localized particle with certain vicinity. Within this atom one considers spatial cells in which virtual particles are. The cubic structure of the vacuum virtual particles in space is not completely acceptable. Actually, the most convenient for the representation and description of particles packaging is a concentric ball layers. So that, particles can be in this layer up to full fill it for each layer of arbitrary thickness and distance from the center of spheres. A shape of the particles' wave packets is thought a spherical or almost spherical. In the Figure 4 is pictured such one flat layer with the balls of the particles' wave packets on the zeroth meridian of the ball. One needs to package all the balls of the wave packets in each one and all ones these layers as much as they are gone in each such layer. This must be calculated geometrically. As well-known the side of regular tangential polygon is given by the formula

$$b_N = 2R\tan\left(\frac{\pi}{N}\right). \tag{2.1.28}$$

As, the perimeter of such polygon

$$L = Nb_N = 2NR\tan\left(\frac{\pi}{N}\right). \tag{2.1.29}$$

Also according the Figure 4, the perimeter is equal

$$L = N\left(l - \Delta l\right). \tag{2.1.30}$$

Equating (2.1.29) and (2.1.30), one gains

$$l - \Delta l = 2R\tan\left(\frac{\pi}{N}\right), \tag{2.1.31}$$

where the radius of the inscribed circle can be calculated from the Pythagoras' theorem, using Figure 4

$$R = \sqrt{r^2 - \frac{1}{4}\left(l - \Delta l\right)^2}\ . \tag{2.1.32}$$

Thus, N is the number of the sides or the number of the vertices of the polygon and it is the number of the balls, because the center of each ball is located exactly on the vertex of the polygon. It is gained the equation for N

$$l - \Delta l = 2\tan\left(\frac{\pi}{N}\right)\sqrt{r^2 - \frac{1}{4}\left(l - \Delta l\right)^2}\ . \tag{2.1.33}$$

From here

$$N = \frac{\pi}{\arctan\left(\dfrac{l - \Delta l}{2\sqrt{r^2 - 0.25\left(l - \Delta l\right)^2}}\right)}. \tag{2.1.34}$$

On the Figure 4 is pictured 2D projection on the meridian plane of the considering real 3D model of the virtual particles packaging. One needs to understand how many balls are going to go in one single arbitrary concentric ball layer. To understand this, it is needed and sufficient to consider an equatorial slash of the 3D picture, pictured on the Figure 5. The meridian section is included, then, at changing the azimuthal angle all the sections, the flat circles are summed without the polar balls which were considered in the zeroth meridian flat circle. The balls on the equator go in twice, that is why one needs the

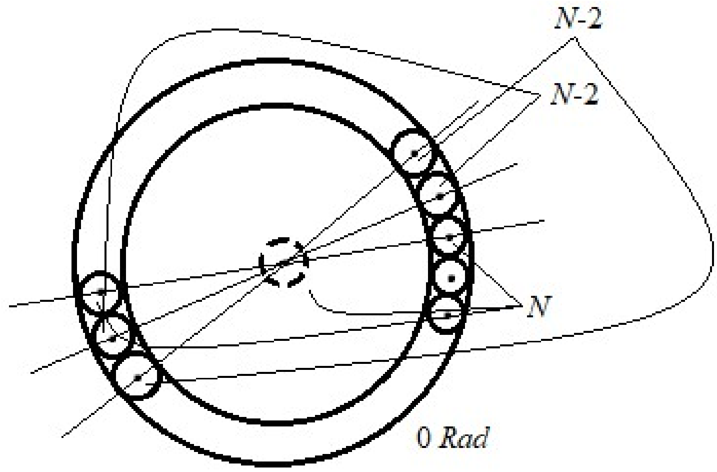

fig. 5 – Equatorial slash of the one layer of the concentric ball layers space around a particle is parted on

multiplier ½. So, it is needed to sum $N-2$ balls as many times without considering in projection on the equator two meridian balls and without duplication, i.e. using the one-half multiplier as they go in. Therefore, one gains the full number of the balls (wave packets) going in the one single arbitrary ball layer

$$A = N + \frac{(N-2)^2}{2}. \quad (2.1.35)$$

It is useful to simplify this result

$$A = \frac{N^2}{2} - N + 2. \quad (2.1.36)$$

At that the thickness of the ball layer is

$$\delta = l - \Delta l, \quad (2.1.37)$$

the radius of the most sphere and the radius of the least sphere correspondingly

$$\rho_2 = r + \frac{1}{2}(l - \Delta l), \quad (2.1.38)$$

$$\rho_1 = r - \frac{1}{2}(l - \Delta l). \quad (2.1.39)$$

2.2. The Theoretical Vacuum Energy In Presence Of Matter, The Theoretical Free Vacuum Energy And The Observed Vacuum Energy

If $w_{v+m}(l)$ is multiplied by $(l-\Delta l)^3$, it can be gained the energy E_{v+m} in this cell of space with the side $l-\Delta l$, i.e.

$$E_{v+m} = w_{v+m}(l - \Delta l)^3. \quad (2.2.1)$$

Let it be designated

$$E_{v+m} := E_{v+m}^{(cell)}. \quad (2.2.2)$$

If l is fixed and r is arbitrary, then, according the considered geometrical model of the vacuum near matter, its energy will be equal

$$E_{v+m} = N_a A E_{v+m}^{(cell)}, \quad (2.2.3)$$

where N_a is the number of atoms in the Universe, and here each ball is now thought being in a cube with side $l-\Delta l$. The energy (2.2.2) like the energy satisfied positive part of (1.8) at condition (1.10) not necessarily can relate to the cubic wave packet of the virtual electron, it can correspond to a ball-shaped wave packet as well. In the model described in this work it is thought that the wave functions of virtual particles at the constant scale l are not overlapped. Therefore the distance r must change discretely, to do not allow the wave functions are overlapped. As was noted yet, radii the most and the least spheres are set by formulae (2.1.38) and (2.1.39); with the substitution $r = r_{\min}$ and holding $l = const$ that is related to the first ball layer, where $r_{\min} = const$ and this radius is the introduced value. The next distances can be calculated by consistent solving of the system of the algebraic equations

$$\rho_1(r_1, l) = \rho_2(r_{\min}, l),\ \rho_2(r_1, l), \quad (2.2.4)$$

$$\rho_1(r_2, l) = \rho_2(r_1, l),\ \rho_2(r_2, l), \quad (2.2.5)$$

$$\rho_1(r_3, l) = \rho_2(r_2, l),\ \rho_2(r_3, l), \quad (2.2.6)$$

and so on, for r_1, r_2, r_3 and so on. Now l can be variable. Range of l is (2.1.25) or (2.1.26), where $l_{\min} > 0$ means the ultraviolet cutoff. The full vacuum energy near matter for one single type of particles (particles with them own antiparticles, the pairs) according to the all said above now takes the form

$$E_{v+m}^{(0)} = \frac{1}{l_P} \int_{l_{\min}}^{\infty} \sum_{n=0}^{\infty} E_{v+m}\left(l, r_n\right) dl, \qquad (2.2.7)$$

where $r_0 = r_{\min}$. Then, the estimation for the full vacuum energy near matter takes the form

$$E_{v+m}^{(0)} \geq \frac{1}{l_P} \int_{l_{\min}}^{\infty} \sum_{n=0}^{\infty} N_a \left(\frac{N\left(l, r_n\right)^2}{2} - N\left(l, r_n\right) + 2 \right) \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{\left(l - \Delta l\left(l, r_n\right)\right)^2} + m^2 c^4} dl, \qquad (2.2.8)$$

where $N\left(l, r_n\right)$ is defined by (2.1.34), $\Delta l\left(l, r_n\right)$ is defined by (2.1.21) and discretization of r_n is defined by the system of equations like (2.2.4) – (2.2.6). The infinite sum in (2.2.8) can be reduced to an approximate finite sum if one takes into account the sizes of real atoms. The experimentally determined sizes (radii) of atoms have values in the range 30 pm – 300 pm ±5 pm [7], where 1 pm is the picometer, as $1\text{pm} = 1 \cdot 10^{-12}\,\text{m}$, the minimal value has hydrogen atom 25 pm and the maximum value has cesium atom 260 pm, and the most heavy atoms existing in the Universe such as uranium and plutonium have a value 175 pm with accuracy ±5 pm [7] (1 Å = 100 pm).

Thus, basing on the Section 1 and the current section of this paper, the full vacuum energy in the Universe with one single type of virtual particles (with them own antiparticles) taking into account matter existing in the Universe and empty space without matter takes the form

$$E_0 = E_v^{(0)} + E_{v+m}^{(0)}, \qquad (2.2.9)$$

where $E_v^{(0)}$ is the full vacuum energy in empty space of the Universe. The energy $E_{v+m}^{(0)}$ is already found, to find the energy $E_v^{(0)}$ it is needed to multiply the minimal value of (2.1.1) by the volume of one single cell, in which the energy is; and one finds the energy in one single cell of empty space at fixed l

$$E_v = l^3 w_v\left(l\right). \qquad (2.2.10)$$

Analogously to (2.2.2), let it be designated

$$E_v := E_v^{(cell)}. \qquad (2.2.11)$$

The energy density $w_v\left(l\right)$ in the inequality (2.1.1) describes one single cell of space or all empty universe at fixed scale l. To take into account all the scales, one needs, like it was done in (2.2.7), to integrate the energy density entering in (2.2.11) over all the scales, and, as it was discussed above in the relation of the definition (2.1.22), one needs to divide this integral into the Planck length l_P to gain the approximate result (see the text above related to (2.1.22)), i.e.

$$w_v^{(0)} = \frac{1}{l_P} \int_{l_{\min}}^{\infty} w_v dl, \qquad (2.2.12)$$

the range of l can be taken from (2.1.25) or, as it was done here, from (2.1.26); and the spatial ultraviolet cutoff $l_{\min}$ was used. It is needed to note that here the space is thought discrete as well. Thus, now it is thought that the energy densities w_v and $w_v^{(0)}$ describe all empty space of a universe. To gain the full vacuum energy of empty space of the Universe (the part of all the space), one should multiply the energy density (2.2.12) by the volume of empty space of the Universe

$$E_v^{(0)} = w_v^{(0)} V_s. \qquad (2.2.13)$$

Then, the assessment (2.1.1) for the full vacuum energy of empty space of the Universe will be

$$E_v^{(0)} \geq \frac{1}{l_P} V_s \int\limits_{l_{\min}}^{\infty} l^{-3} \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{l^2} + m^2 c^4} dl . \qquad (2.2.14)$$

Basing on (2.2.9) the estimation for the full vacuum energy can be gained and takes the form

$$E_0 \geq \frac{1}{l_P} \int\limits_{l_{\min}}^{\infty} \sum_{n=0}^{\infty} N_a \left(\frac{N(l,r_n)^2}{2} - N(l,r_n) + 2 \right) \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{\left(l - \Delta l(l,r_n)\right)^2} + m^2 c^4} dl + \qquad (2.2.15)$$

$$+ \frac{1}{l_P} V_s \int\limits_{l_{\min}}^{\infty} l^{-3} \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{l^2} + m^2 c^4} dl .$$

Now in the formulae above the energy density of the space cell relates to the cubic wave packet of the virtual particle that is not natural enough. Not only energy can describe the both types of wave packets, are cubic and ball-shaped, energy density can also be transformed allowing it to correspond to ball-shaped wave packets as well. Any energy density in this work consists of energy divided into volume in which this energy is distributed. As already was said, energy is invariant to changing between cubic and ball wave packets, this means that it is needed to transform only volume into which energy is divided. Now, one has the volume of a cube with side l or $l - \Delta l$, to consider the volume of a ball one should multiply the volume of a cube by $\pi/6$ in all the formulae for energy density.

In this paper the vacuum energy distributed only in average on chemical elements existing in the Universe has been estimating. The accurate calculation considering the vacuum energy distributed on the quantities of chemical elements existing in the Universe will be the part of consequent work.

One defines the average number of particles in the following way

$$N_a = \frac{\rho_m V_U}{m_{\text{av}}}, \qquad (2.2.16)$$

where ρ_m is the average density of matter in the Universe; V_U is the volume of the Universe; m_{av} is the mass of one average particle in the Universe. Each particle gives rise the spherical vicinity around itself. The volume of vicinity is the volume of atom, thus, the volume of all matter in the Universe can be calculated on the formula

$$V_A N_a = V_M . \qquad (2.2.17)$$

As well-known the average density of matter in the Universe is equal to the critical density (the error makes up about 1%) [3, 4]

$$\rho_m = \rho_c , \qquad (2.2.18)$$

$$\rho_c = \frac{3H^2}{8\pi G}, \qquad (2.2.19)$$

where H is the Hubble constant [8]. It is required to calculate the mass of the average particle. If that how many each sort of atoms are in the Universe is known, then the number of all heavy well-localized particles and atoms is known as well, to find the mass of the average particle, one must sum the multiplications of the masses of the particles of each sort by the numbers of the particles of these sorts to find the full mass of the Universe (the mass of all matter in the Universe) and one must divide this total mass into the number of all particles, as a result the mass of the average particle will be found; for atoms it will have the view

$$m_{\text{av}} = \frac{\sum_{k=1}^{93} m_k N_k}{\sum_{k=1}^{93} N_k}, \tag{2.2.20}$$

where m_k is the mass of k-th nucleus of this chemical element, N_k are the numbers of atoms in the Universe numbered by index k, 93 is the number of the natural chemical elements.

One receives the expression for the estimation (2.2.14) of the vacuum energy density related to the part of the Universe without matter

$$E_v^{(0)} \geq \frac{1}{l_P} V_s \frac{mc^2}{36\hbar^2 l_{\min}^2}\left(\left(3\hbar^2 + 16m^2c^2l_{\min}^2\right)\sqrt{16 + \frac{3\hbar^2}{m^2c^2l_{\min}^2}} - 64m^2c^2l_{\min}^2\right). \tag{2.2.21}$$

It is possible to suppose

$$l_{\min} = l_P, \tag{2.2.22}$$

if one does that, the assessment (2.2.21) takes the form

$$E_v^{(0)} \geq V_s \frac{mc^2}{36\hbar^2 l_P^3}\left(\left(3\hbar^2 + 16m^2c^2l_P^2\right)\sqrt{16 + \frac{3\hbar^2}{m^2c^2l_P^2}} - 64m^2c^2l_P^2\right). \tag{2.2.23}$$

Actually, the volume of the Universe is

$$V_U = V_s + V_M. \tag{2.2.24}$$

Hence, if V_s is not known (not observed), the volume of all empty space can be expressed from here

$$V_s = V_U - V_M, \tag{2.2.25}$$

where V_U is the volume of the observed part of the Universe, which can be assessed from observations. The volume of matter also is known due to that the full number of atoms in the Universe has been assessed as well. If one accepts hypothesis (2.2.22), substituting (2.2.25) in (2.2.23), considering the values SI for m, c, $\hbar$; l_P is (2.1.24), $V_U = 3.5\cdot10^{80}\,\text{m}^3$ [9], $N_a\left({}_1^1\text{H}\right) = 10^{80}$ [10] (if to think that the all atoms are hydrogen atoms), $V_A\left({}_1^1\text{H}\right) = 6.55\cdot10^{-32}\,\text{m}^3$ (diameter $d\left({}_1^1\text{H}\right) = 2\cdot25\text{pm} = 5\cdot10^{-11}\,\text{m}$), the lower number estimation

$$E_v^{(0)} \geq 2.32\cdot10^{193}\,\text{J}. \tag{2.2.26}$$

This value is absolutely far from the observable value

$$E_v^{(0)\text{obs}} = 1.87\cdot10^{71}\,\text{J}, \tag{2.2.27}$$

calculated on the value (1.12) on the formula

$$E_v^{(0)\text{obs}} = \left(V_U - V_M\right)w_\Lambda. \tag{2.2.28}$$

It is seen that only in free space the divergence is 122 orders of magnitude, this is the cosmological constant problem, with the hypothesis of discrete space.

2.3. Features Related To The Model

In the concept of discrete space is not well-understood how matter moves through this space. If one considers one point particle, that it can move by jumps from one point to another divided by the Planck length in the all three directions. At that the fundamental quanta of space is the cube with the Planck length side. For point particle time interval between jumps does not exist, and particle can be in this point, the vertex of the cube or be in other vertex, that particle can do jumps instantaneously. This may not be said for macroscopic bodies, they can move through that discrete space with varies velocities. Every point in such body (the point of the wave functions) does jump from one to the next point of space but they delay in each point of space for the period Δt (here it is not the lifetime of the particle) of infinitely divisible time. It is alike the moving between two points, as that if the particle-point started from this point at the moment of time t, in the next point of space it must delay for the period $t_1 - t = \Delta t$, having jumped between these two points instantaneously. Thus, doing such jumps with delays, this from large, macroscopic scale is going to see as a motion with seeming arbitrary velocity. But on the fundamental level for each quanta of space this 'motion' is alike the continuous real motion between each two points of space, i.e. along the Planck length l_P and the delay is going to seem the period of time for which the real continuous motion would happen. Thus, the instant velocity of motion in discrete non-infinite divisible space (constrained by the fundamental scale which is the Planck length) happening for infinite divisible and continuous time is to be

$$v_f = \frac{l_P}{\Delta t} \tag{2.3.1}$$

for each linear quanta of space l_P. Indeed in such a way the motion in discrete space must be justified. This article presumes the discrete nature of real physical space. Also in the model described in the Section 2 of this paper it is neglected with anisotropy on the fundamental level, which occurs due to the discreteness of space. One can easily imagine what really is a fundamental wave packet or a wave packet reduced down to the fundamental level: due to the discreteness of space the Planck-sized wave packets of particles are eight-point, on the number of the vertices – the points of a cube with the side l_P.

The following consideration relates to the Planck scale and the vacuum near matter. It can be for the free vacuum

$$l_{\min} = l_P, \tag{2.3.2}$$

that means the minimal length is the Planck length. The vacuum near matter can be reduced, in such case (2.3.2) is justified at maximum degree of reducing. Or (2.3.2) is not right for the free vacuum and it is right in such case

$$l_{\min} = l_P + \delta l,\ \delta l > 0, \tag{2.3.3}$$

so that for the vacuum near matter (2.3.2) is right at maximum limiting, i.e. the reducing of overplanck scale to the Planck scale happens. The author states that in free space (2.3.2) is justified as it is the most possible, and near matter does happen the reducing of the Planck scale.

2.4. The Final Results Of The Calculation

The estimation (2.2.8) can be calculated on the computer in the first place for a universe consisting only of hydrogen atoms. It is needed to say that the Universe often considers as such universe [10] on the reason that in the Universe the hydrogen is prevailed. For hydrogen it is important to note the existence of the values of the reducing of the Planck scale. These values describe the Planck-sized vacuum wave packets for the hydrogen atom and can be obtained on (2.1.21) by substituting $l \to l_P$, $r \to r_{\min}$ and $r \to r_{\max}$. Here $r_{\min}$ is the minimal distance to the proton which can still think a point and $r_{\max}$ is the radius of the atom. One obtains

$$\Delta l\left(l_P, r_{\min}\right) = \frac{\sqrt{3} l_P^2}{\sqrt{3} l_P + 6 r_{\min} + 32 r_{\min} l_P^2 \dfrac{m^2 c^2}{\hbar^2}} \approx 7.58 \cdot 10^{-60}\,\mathrm{m}, \tag{2.4.1}$$

$$\Delta l\left(l_P, r_{\max}\right) = \frac{\sqrt{3} l_P^2}{\sqrt{3} l_P + 6 r_{\max} + 32 r_{\max} l_P^2 \dfrac{m^2 c^2}{\hbar^2}} \approx 2.53 \cdot 10^{-60}\,\mathrm{m}. \qquad (2.4.2)$$

The Planck-sized wave packets of the vacuum virtual particles do not actually contribute in the theory because if the Planck length is small, the reducings (2.4.1) and (2.4.2) are yet smaller, so that one can neglect them in calculation. For example, this relates to the number of particles on the zeroth meridian (2.1.34) and the energy density (2.1.2). Thus, these formulae take the forms

$$N = \frac{\pi}{\arctan\left(\dfrac{l}{2\sqrt{r^2 - 0.25 l^2}}\right)}, \qquad (2.4.3)$$

$$w_{v+m}\left(l\right) \geq l^{-3} \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{l^2} + m^2 c^4} \qquad (2.4.4)$$

for the Planck-sized wave packets. Also for the computation it is needed to have in the view the constraint following from (2.1.34), namely

$$r^2 - \frac{1}{4}\left(l - \Delta l\right)^2 \geq 0, \qquad (2.4.5)$$

which thanks to the known conditions $r > 0$ and $l > \Delta l$ can be led to the form

$$r_n \geq \frac{1}{2}\left(l - \Delta l_n\right) \text{ for all } n. \qquad (2.4.6)$$

That the range (2.1.26) for l is not applicable but the range (2.1.25) is correct follows from (2.4.6) and from that the origin of r_n is in the system of algebraic equations (2.2.4) – (2.2.6) and so on. In such a way one at once gains the limitation for scales in the task of the computation of the estimation for the vacuum energy near matter which follows from (2.4.6), (2.1.21) and the dependence $r_n\left(l\right)$ as was said above. In nature this limitation can be caused by the spatial limitation of the acting of the interaction fields which matter itself has.

Also it must be understandable that in the real practical calculation one should consider limited atom, an object having finite sizes in space, therefore the number n of the radii r_n must be limited by the maximum value. Thus, it must be computed

$$E_{v+m}^{(0)} \geq \frac{1}{l_P} \int_{l_{\min}}^{l_{\max}} \sum_{k=0}^{n} N_a \left(\frac{N\left(l, r_k\right)^2}{2} - N\left(l, r_k\right) + 2 \right) \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{\left(l - \Delta l\left(l, r_k\right)\right)^2} + m^2 c^4}\, dl. \qquad (2.4.7)$$

It was accepted in the computation that hydrogen atom has values: $r_{\min} = 10^{-11}\,\mathrm{m}$, $l_{\min} = 10^{20} l_P$, $r_{\max} = 3 \cdot 10^{-11}\,\mathrm{m}$. It can be obtained from (2.4.6) that $l_{\max} = 2.00008 \cdot 10^{-11}\,\mathrm{m}$ by substituting $r_n \to r_{\min}$, and all other values r_n including $r_{\max}$ give true conditions if the previous conditions are accepted. Solving the system of the algebraic equations (2.2.4) – (2.2.6) analytically, one can quickly understand that the volume of the expressions increases very rapidly. In fact the second substitution for number $n = 2$ already gives a large final result. However, one could notice that the first, the second and the third results can be calculated analytically on the computer and have approximately straight lines plots. This does give a resolution of the problem. As one, n-th result must be substituted in the next having number n on unit more, the author came to the conclusion that all $r_n\left(l\right)$ are approximately straight lines. However, this conclusion must be verified on each step of the computation. For this all interval from $l_{\min}$ to $l_{\max}$ was divided into ten segments with eleven points including the start and the end points and in each point values of the function gained from exact solution for the first step were compared with the points calculated from approximate linear representation of this exact solution. And this comparison was done for each value of n. This has been done by the written program by the author, the listing of this program written in the "Wolfram language®" is applied below the main text. Thus, the computed value at the all applied above conditions in the right hand side of the estimation (2.2.8) allows one to write down the numerical assessment

$$E_{v+m}^{(0)} \geq 1.31797 \cdot 10^{104}\,\mathrm{J}. \tag{2.4.8}$$

This final result has been gained by the computer program which is done $n = 12347$ iterations for 9300 seconds with the rest. The general assessment (2.2.15) for the full vacuum energy, i.e. the vacuum energy of empty space of the Universe and the vacuum energy in the space near matter takes the exact quantitative form

$$E_0 \geq 2.32 \cdot 10^{193} + 1.32 \cdot 10^{104}\,\mathrm{J}. \tag{2.4.9}$$

The gained data testifies in sake of the cosmological constant problem, it really takes place and the lower boundary for the full vacuum energy exceeds the observed value over 122 orders of magnitude.

At the next stage it is needed to introduce a wrapping vacuum coefficient to show that matter really reduces vacuum leading to the increasing of vacuum energy for the entire Universe. If to assess the energy of an empty space in each hydrogen atom taking into account admissible for the atom scale interval provided in the previous computation, also taking into account that there is no reducing of the wave packets of the virtual particles in empty space, one should compute the fraction

$$\sigma \geq \frac{1.31797 \cdot 10^{104}}{\frac{1}{l_P} N_a V_A \int\limits_{l_{\min}}^{l_{\max}} \frac{1}{l^3} \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{l^2} + m^2 c^4}\, dl}, \tag{2.4.10}$$

where $\sigma > 1$ is the wrapping vacuum coefficient; with more σ the wrapping of the vacuum by matter in the entire Universe is more. The approximate value of the working volume of the atom

$$V_A = \frac{4}{3}\pi r_{\max}^3 \approx 1.13 \cdot 10^{-31}\,\mathrm{m}^3 \tag{2.4.11}$$

at the all discussed above conditions and the values of the quantities leads to the numerical inequality for the coefficient

$$\sigma \geq 176.055. \tag{2.4.12}$$

The more accurate estimation of the working volume of the hydrogen atom, i.e. the volume occupying the reduced/not reduced wave packets of the virtual particles according this model, namely

$$V_A = \frac{4}{3}\pi r_{\max}^3 - \frac{4}{3}\pi r_{\min}^3 \approx 1.09 \cdot 10^{-31}\,\mathrm{m}^3 \tag{2.4.13}$$

leads to the lower limitation for the wrapping vacuum coefficient

$$\sigma \geq 182.515. \tag{2.4.14}$$

As become obvious for the reader the lower edge of the vacuum energy in all the hydrogen atoms volumes at the absence of them nuclei, the protons that means there is no the reducing of the vacuum virtual particles can be calculated by assuming $\Delta l(r_n, l) = 0$ for all n. That is in this method of calculation one should compute

$$\frac{1}{l_P} \int\limits_{l_{\min}}^{l_{\max}} \sum_{k=0}^{n-1} N_a \left(\frac{N(l, r_k)^2}{2} - N(l, r_k) + 2 \right) \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{l^2} + m^2 c^4}\, dl, \tag{2.4.15}$$

where for N must be used (2.4.3) and at $k = 0$ must be $r = r_{\min}$. For this case the scale l is no more restricted with the maximum value, because the inequality (2.4.6) now gets

$$r_n \geq \frac{1}{2} l . \qquad (2.4.16)$$

Also assuming $\Delta l(r_n, l) = 0$, the system of the equations (2.2.4) – (2.2.6) can be solved by simple way, as, one obtains

$$r_n = nl + r_{\min} . \qquad (2.4.17)$$

Substituting (2.4.17) in (2.4.16), it can obtain

$$l \geq -\frac{r_{\min}}{n - 0.5} , \qquad (2.4.18)$$

but $l > 0$, hence, (2.4.18) is always true. Thus, in this case l must be restricted on top by the value $l_{\max}$ from the previous calculation. The number $n-1$ up to which the sum in (2.4.15) must be summed one can explain that solving the algebraic equations up to the condition $r_n \leq r_{\max}$, the value r_n, when n is maximal, is a little exceeded the value $r_{\max}$. So, computing (2.4.15) and making up the fraction like (2.4.10) with (2.4.15) in the denominator and the value from (2.4.9) in the numerator, one could gain the numerical condition

$$\sigma \geq 77.5007 . \qquad (2.4.19)$$

The conclusion is that the matter in the Universe really limits or wraps (the author introduces such term) the vacuum, at that the vacuum energy increases.

And finally, purely qualitative it can be shown that the lower assessment for the vacuum energy is able to be calculated without any treatment to the concept of individual atoms. If one would consider a one single atom with the volume of all empty space of the entire Universe in the task of calculation of the vacuum energy being in empty space or the free vacuum energy, this method has never yet been considered, then it must be applied

$$r_{\min} = \frac{1}{2} l , \qquad (2.4.20)$$

the maximum value of the radius of such imagined atom must be calculated from the condition

$$V_s = \frac{4}{3} \pi r_{\max}^3 , \qquad (2.4.21)$$

it takes the form

$$r_{\max} = \left(\frac{3}{4\pi} V_s \right)^{\frac{1}{3}} . \qquad (2.4.22)$$

The condition on the range for scales is to be (2.1.26). The minimal number of balls on the zeroth meridian at the condition (2.4.20) can be found out by substituting (2.4.20) in (2.4.3) and taking the limit

$$\lim_{x \to 0} \left(\arctan\left(\frac{l}{x} \right) \right) = \frac{\pi}{2} , \qquad (2.4.23)$$

thus, $N_0 \equiv N_{\min} = 2$. This is the formal number. The substitution (2.4.17) in (2.4.3) and taking into account (2.4.20) leads to

$$N = \frac{\pi}{\arctan\left(\frac{1}{2} \frac{1}{\sqrt{k(k+1)}} \right)} , \qquad (2.4.24)$$

where $k \in [1, n]$ and n is to be set from iterating (2.4.17) at the condition (2.4.20) up to that the condition $r_n \leq r_{\max}$ fails to give true. Thus, the inequality for the free vacuum energy calculated by this method takes the qualitative form

$$E_v^{(0)} \geq \frac{1}{l_P} \int_{l_{\min}}^{\infty} \sum_{k=0}^{n-1} \left(\frac{N(k)^2}{2} - N(k) + 2 \right) \sqrt{\frac{\hbar^2 c^2}{16} \frac{3}{l^2} + m^2 c^4} \, dl \, . \tag{2.4.25}$$

This method has a feature in packaging of the wave packets vacuum virtual particles. The inequality (2.4.25) has the qualitative role and cannot be computed because the value (2.4.22) is too much, namely

$$r_{\max} = 4.37 \cdot 10^{26} \, \mathrm{m} \, , \tag{2.4.26}$$

so that, it must be done too much number of the iterations to find all r_n and the final n.

$VU = 3.5 \times 10^{80};$

$Na = 10^{80};$

$VA = 8.18 \times 10^{-33};$

$m = 9.1 \times 10^{-31};$

$c = 3 \times 10^{8};$

$h = 6.62 \times 10^{-34};$

$\hbar = \frac{h}{2\pi};$

$r_{min} := 10^{-11}$

$lmin := 10^{20}\, lP$

$lP := 1.62 \times 10^{-35}$

$r_{max} := 3 \times 10^{-11}$

$lmax := 2.00008 \times 10^{-11}$

$a1 := 9.99998 \times 10^{-12}$

$k1 := 0.999976$

$a2 := 9.99996 \times 10^{-12}$

$k2 := 1.99996$

$r_1 := a1 + k1\, l$

$r_2 := a2 + k2\, l$

$$P_2 := 1.00032 \times 10^{-11} + 3.99976 \times 10^{-11} \frac{l - 1.62 \times 10^{-15}}{2.00008 \times 10^{-11} - 1.62 \times 10^{-15}}$$

$l1 := 2.00154 \times 10^{-12}$

$l2 := 4.00146 \times 10^{-12}$

$l3 := 6.00138 \times 10^{-12}$

$l4 := 8.0013 \times 10^{-12}$

$l5 := 1.00012 \times 10^{-11}$

$l6 := 1.20011 \times 10^{-11}$

$l7 := 1.40011 \times 10^{-11}$

$l8 := 1.6001 \times 10^{-11}$

$l9 := 1.80009 \times 10^{-11}$

Inoculating values:

$a_2 := 9.99996 \times 10^{-12}$

$k_2 := 1.99996$

$$\Delta l_{min} := \frac{\sqrt{3}\ l^2}{\sqrt{3}\ l + 6\, r_{min} + 32\, r_{min}\, l^2 \times \frac{m^2 c^2}{\hbar^2}}$$

$\Delta l_1 := \Delta l_{min}\ /.\ r_{min} \to r_1$

$\Delta l_2 := \Delta l_1\ /.\ r_1 \to r_2$

For[n = 2;
цикл ДЛЯ

$l = lmin,\ P_n \le r_{max},\ n{+}{+},\ l =.;$

$m =.;$

$c =.;$

$\hbar =.;$

$a2 =.;$

$k2 =.;$

$a_n =.;$

$k_n =.;$

$\Delta l_n =.;$

$$\Delta l_{n+1} = \frac{\sqrt{3}\ l^2}{\sqrt{3}\ l + 6\, R_{n+1} + 32\, R_{n+1}\, l^2 \times \frac{m^2 c^2}{\hbar^2}};$$

$$\Delta l_n = \frac{\sqrt{3}\ l^2}{\sqrt{3}\ l + 6\, r_n + 32\, r_n\, l^2 \times \frac{m^2 c^2}{\hbar^2}};$$

$$P_{n+1} = R_{n+1}\ /.\ \text{Flatten}\left[\text{Solve}\left[R_{n+1} - \frac{1}{2}\,(l - \Delta l_{n+1}) == r_n + \frac{1}{2}\,(l - \Delta l_n),\ R_{n+1}\right]\right][[1]];$$

уплостить решить уравнения

$l = lmin;$

$m = 9.1 \times 10^{-31};$

$c = 3 \times 10^{8};$

$\hbar = 1.05361 \times 10^{-34};$

$a2 = 9.99996 \times 10^{-12};$

$k2 = 1.99998;$

$$a_n = y_{1,n} - (y_{2,n} - y_{1,n}) \times \frac{lmin}{l1 - lmin};$$

$$k_n = \frac{y_{2,n} - y_{1,n}}{l1 - lmin};$$

If[$P_{n+1} > 0,\ l =.;$
условный оператор

$m =.;$

$c =.;$

$\hbar =.;$

$a2 =.;$

$k2 =.;$

$a_n =.;$

$k_n =.;$

```
r_{n+1} = R_{n+1} /. Flatten[Solve[R_{n+1} - 1/2 (l - Δl_{n+1}) == r_n + 1/2 (l - Δl_n), R_{n+1}]][[1]];
  уплостить  решить уравнения

l = lmin;
m = 9.1 × 10^-31;
c = 3 × 10^8;
ℏ = 1.05361 × 10^-34;
a2 = 9.99996 × 10^-12;
k2 = 1.99998;
a_n = y_{1,n} - (y_{2,n} - y_{1,n}) × lmin/(l1 - lmin);
k_n = (y_{2,n} - y_{1,n})/(l1 - lmin);
y_{1,n+1} = r_{n+1};
l =.;
l = l1;
y_{2,n+1} = r_{n+1};
d_{n+1} = y_{1,n+1} + (y_{2,n+1} - y_{1,n+1}) × (x - lmin)/(l1 - lmin);
x = l2;
l =.;
l = l2;
If[Abs[r_{n+1} - d_{n+1}] < 10^-3, l =.;
 ⋯  абсолютное значение

 x =.;
 l = l3;
 x = l3;
 If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
  ⋯  абсолютное значение

  x =.;
  l = l4;
  x = l4;
  If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
   ⋯  абсолютное значение

   x =.;
   l = l5;
   x = l5;
   If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
    ⋯  абсолютное значение

    x =.;
    l = l6;
    x = l6;
    If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
     ⋯  абсолютное значение

     x =.;
     l = l7;
     x = l7;
     If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
      ⋯  абсолютное значение
```

```
x =.;
l = 18;
x = 18;
If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
 абсолютное значение

 x =.;
 l = 19;
 x = 19;
 If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6, l =.;
  абсолютное значение

  x =.;
  l = lmax;
  x = lmax;
  If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6 , l =.;
   абсолютное значение

   x =.;
   r_{n+1} =.;
   a_n =.;
   k_n =.;
   r_{n+1} = a_{n+1} + k_{n+1} l;
   P_{n+1} =.;
   P_{n+1} = a_{n+1} + k_{n+1} l;
   a_{n+1} = y_{1,n+1} - (y_{2,n+1} - y_{1,n+1}) × lmin/(l1 - lmin);
   k_{n+1} = (y_{2,n+1} - y_{1,n+1})/(l1 - lmin);
   l = lmin, Null];, Null];, Null];, Null];, Null];, Null];, Null];, Null];,
 Null];, P_{n+1} =.;

l =.;
m =.;
c =.;
ħ =.;
a2 =.;
k2 =.;
a_n =.;
k_n =.;
P_{n+1} = R_{n+1} /. Flatten[Solve[R_{n+1} - 1/2 (l - Δl_{n+1}) == r_n + 1/2 (l - Δl_n), R_{n+1}]][[2]];
   уплостить решить уравнения

r_{n+1} = R_{n+1} /. Flatten[Solve[R_{n+1} - 1/2 (l - Δl_{n+1}) == r_n + 1/2 (l - Δl_n), R_{n+1}]][[2]];
   уплостить решить уравнения

l = lmin;
m = 9.1 × 10^-31;
c = 3 × 10^8;
ħ = 1.05361 × 10^-34;
```

```
a2 = 9.99996 × 10^-12;
k2 = 1.99998;
a_n = y_{1,n} - (y_{2,n} - y_{1,n}) × lmin/(l1 - lmin);
k_n = (y_{2,n} - y_{1,n})/(l1 - lmin);
y_{1,n+1} = r_{n+1};
l =.;
l = l1;
y_{2,n+1} = r_{n+1};
d_{n+1} = y_{1,n+1} + (y_{2,n+1} - y_{1,n+1}) × (x - lmin)/(l1 - lmin);
x = l2;
l =.;
l = l2;
If[Abs[r_{n+1} - d_{n+1}] < 10^-3, l =.;
  абсолютное значение

 x =.;
 l = l3;
 x = l3;
 If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
   абсолютное значение

  x =.;
  l = l4;
  x = l4;
  If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
    абсолютное значение

   x =.;
   l = l5;
   x = l5;
   If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
     абсолютное значение

    x =.;
    l = l6;
    x = l6;
    If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
      абсолютное значение

     x =.;
     l = l7;
     x = l7;
     If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
       абсолютное значение

      x =.;
      l = l8;
      x = l8;
      If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
        абсолютное значение

       x =.;
```

```
l = 19;
x = 19;
If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6, l =.;
   абсолютное значение
 x =.;
 l = lmax;
 x = lmax;
 If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6 , l =.;
    абсолютное значение
  x =.;
  r_{n+1} =.;
  a_n =.;
  k_n =.;
  r_{n+1} = a_{n+1} + k_{n+1} l;
  P_{n+1} =.;
  P_{n+1} = a_{n+1} + k_{n+1} l;
```

$$a_{n+1} = y_{1,n+1} - (y_{2,n+1} - y_{1,n+1}) \times \frac{lmin}{l1 - lmin};$$

$$k_{n+1} = \frac{y_{2,n+1} - y_{1,n+1}}{l1 - lmin};$$

```
  l = lmin, Null];, Null];, Null];, Null];, Null];, Null];, Null];, Null];,
            пустой  пустой  пустой  пустой  пустой  пустой  пустой  пустой
Null];];] // Timing
пустой      затраченное время
```

{9300.937500, Null}

n

12 347

l =.

$$\text{Table}\left[a_k = y_{1,k} - (y_{2,k} - y_{1,k}) \times \frac{lmin}{l1 - lmin}, \{k, 3, n\}\right];$$

таблица значений

$$\text{Table}\left[k_i = \frac{y_{2,i} - y_{1,i}}{l1 - lmin}, \{i, 3, n\}\right];$$

таблица значений

$$\text{Table}[r_j = a_j + k_j\, l, \{j, 3, n\}];$$

таблица значений

$$\text{Table}\left[\Delta l_k = \frac{\sqrt{3}\, l^2}{\sqrt{3}\, l + 6\, r_k + 32\, r_k\, l^2 \times \frac{m^2 c^2}{\hbar^2}}, \{k, 3, n\}\right];$$

таблица значений

$$N1_{min} := \frac{\pi}{\text{ArcTan}\left[\frac{l - \Delta l_{min}}{2\sqrt{r_{min}^2 - \frac{1}{4}(l - \Delta l_{min})^2}}\right]}$$

$$\text{Table}\left[N1_i = \frac{\pi}{\text{ArcTan}\left[\frac{l-\Delta l_i}{2\sqrt{r_i^2-\frac{1}{4}(l-\Delta l_i)^2}}\right]}, \{i, 1, n\}\right];$$

таблица значений

$$\text{Reduce}\left[r_n \geq \frac{1}{2}\left(l - \frac{\sqrt{3}\ l^2}{\sqrt{3\ l + 6\, r_n + 32\, r_n\, l^2 \times \frac{m^2 c^2}{\hbar^2}}}\right), l\right]$$

привести

Reduce::ratnz : Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

$-8.09885\times10^{-16} \leq l \leq -8.09871\times10^{-16} \;||\; l > -8.09852\times10^{-16}$

$$\text{Reduce}\left[r_{min} \geq \frac{1}{2}\left(l - \frac{\sqrt{3}\ l^2}{\sqrt{3\ l + 6\, r_{min} + 32\, r_{min}\, l^2 \times \frac{m^2 c^2}{\hbar^2}}}\right), l\right]$$

привести

Reduce::ratnz : Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

$l \leq 2.00008\times10^{-11}$

```
l = lmax;
```

$N1_{min}$

2.001

```
l =.
```

```
l = lmin;
```

$N1_{min}$

38 786.9

```
l =.
```

$$\text{Int} = \text{NIntegrate}\left[\left(\frac{N1_{min}^2}{2} - N1_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{(l-\Delta l_{min})^2} + m^2 c^4} + \sum_{j=1}^{n}\left(\left(\frac{N1_j^2}{2} - N1_j + 2\right)\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{(l-\Delta l_j)^2} + m^2 c^4}\right), \{l, lmin, lmax\}\right]$$

квадратурное интегрирование

2.13511×10^{-11}

$$N\left[\frac{1}{lP}\ \text{Na Int}\right]$$

численное приближение

1.31797×10^{104}

$$\text{NIntegrate}\left[\left(\frac{N1_{min}^2}{2} - N1_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{(l-\Delta l_{min})^2} + m^2 c^4}, \{l, lmin, lmax\}\right]$$

квадратурное интегрирование

5.15089×10^{-18}

VA =.

$N\left[\frac{4}{3}\pi r_{max}^3\right]$
численное приближение

1.13097×10^{-31}

$VA := 1.13\times10^{-31}$

$N\left[\frac{1.31797\times10^{104}}{\frac{1}{lP}\,Na\,VA\int_{lmin}^{lmax}\frac{1}{l^3}\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{l^2}+m^2c^4}\,dl}\right]$
численное приближение

176.055

VA =.

$N\left[\frac{4}{3}\pi r_{max}^3-\frac{4}{3}\pi r_{min}^3\right]$
численное приближение

1.08909×10^{-31}

$VA := 1.09\times10^{-31}$

More precise value

$N\left[\frac{1.31797\times10^{104}}{\frac{1}{lP}\,Na\,VA\int_{lmin}^{lmax}\frac{1}{l^3}\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{l^2}+m^2c^4}\,dl}\right]$
численное приближение

182.515

r_1 =.

n =.

$r_1 := l + r_{min}$

Table[r_n =., {n, 2, 12 347}];
таблица значений

For[n = 1; l = lmin, $r_n \le r_{max}$, n++, $r_{n+1} = (n+1)\,l + r_{min}$]
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n

12 346

r_n

3.00005×10^{-11}

r_{n-1}

2.99989×10^{-11}

l =.

$$N2_{min} := \frac{\pi}{\text{ArcTan}\left[\frac{1}{2\sqrt{r_{min}^2 - \frac{1}{4} l^2}}\right]}$$

$$\text{Table}\left[N2_i = \frac{\pi}{\text{ArcTan}\left[\frac{1}{2\sqrt{r_i^2 - \frac{1}{4} l^2}}\right]}, \{i, 1, n\}\right];$$
таблица значений

Int =.

$$\text{Int} = \text{NIntegrate}\left[\left(\frac{N2_{min}^2}{2} - N2_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{l^2} + m^2 c^4} + \sum_{j=1}^{n-1}\left(\left(\frac{N2_j^2}{2} - N2_j + 2\right)\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{l^2} + m^2 c^4}\right), \{l, lmin, lmax\}\right]$$
квадратурное интегрирование

2.75495×10^{-13}

$$N\left[\frac{1}{lP} \text{Na Int}\right]$$
численное приближение

1.70059×10^{102}

$$\text{NIntegrate}\left[\left(\frac{N2_{min}^2}{2} - N2_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{l^2} + m^2 c^4}, \{l, lmin, lmax\}\right]$$
квадратурное интегрирование

5.14944×10^{-18}

$$\frac{1.31797 \times 10^{104}}{1.70059 \times 10^{102}}$$

77.5007

r_1 =.

r_{max} =.

$$r_1 := \frac{3}{2} l$$

VM := Na VA

Vs := VU − VM

$$r_{max} = \left(\frac{3}{4\pi} \text{Vs}\right)^{\frac{1}{3}}$$

4.3718×10^{26}

Table[r_k =., {k, 2, n}];
таблица значений

n =.

$\text{For}\left[n = 1;\ l = \text{lmin},\ r_n \le r_{max},\ n{+}{+},\ r_{n+1} = (n+1)\ l + \frac{1}{2}\ l\right]$
цикл ДЛЯ

$Aborted

n

321 572 413

r_n

5.20947×10^{-7}

$\text{Limit}\left[\text{ArcTan}\left[\frac{l}{x}\right],\ x \to 0\right]$
предел арктангенс

$$\frac{\sqrt{l^2}\ \pi}{2\ l}$$

$N3_{min} := 2$

$\text{Table}\left[N3_i = \frac{\pi}{\text{ArcTan}\left[\frac{1}{2\sqrt{i\ (i+1)}}\right]},\ \{i,\ 1,\ n\}\right];$
таблица значений

Int =.

$$\text{Int} = \text{NIntegrate}\left[\left(\frac{N3_{min}^2}{2} - N3_{min} + 2\right)\sqrt{\frac{\hbar^2\ c^2}{16} \times \frac{3}{l^2} + m^2\ c^4} + \sum_{j=1}^{n-1}\left(\left(\frac{N3_j^2}{2} - N3_j + 2\right)\sqrt{\frac{\hbar^2\ c^2}{16} \times \frac{3}{l^2} + m^2\ c^4}\right),\ \{l,\ \text{lmin},\ \infty\}\right]$$
квадратурное интегрирование

$\text{N}\left[\frac{1}{lP}\ \text{Int}\right]$
численное приближение

CONCLUSION

In this work the observed value for the vacuum energy density in the Universe was compared with the lower boundary for the vacuum energy density corresponding to the verified high energy physics scale that is the least space scale and the divergence in the 53 orders of magnitude was found. In the work the cosmological constant problem was stated on the data of the calculated observed full vacuum energy of the entire Universe and has been calculated the lower value for the free vacuum energy, i.e. the vacuum energy of all empty space of the Universe. The carried out research testifies that the cosmological constant problem really takes place. It was considered the part of the quantum electrodynamics vacuum, namely, the electron-positron quantum vacuum. Also the estimation for the energy of the vacuum near the matter has been calculated. The lower value of the energy has been found below the lower boundary of the free vacuum energy on the 89 orders of magnitude. The main purpose of this work was to consider the interaction of the vacuum with matter. In the result, the minimal energy of the vacuum near matter has been obtained. The total lower assessment for the full vacuum energy including the free vacuum and the vacuum interacting with matter confirmed the approximate known value for the vacuum energy, and the found in this work value exceeds the observed value mainly on the 122 orders of magnitude. This result confirmed the previous number for the free vacuum as the approximate estimation for the minimal value of the vacuum energy. Note, that the known excess of the observed value is the 120 orders of magnitude. Thus, the found value is approximately relevant to the well-known and accepted value on the order of magnitude. These assessments for the all Universe have been done in the model where all atoms of the Universe are thought with hydrogen atoms. In consequent work it is supposed the considering all types of the atoms existing in the Universe. In this work has been considered only the motion of the virtual particles that leads to essential increasing of the vacuum energy. That is why also in consequent work the consideration of the full motion of virtual particles in the pair and all the types of vacuums respective to all the types of the particles of matter is supposed. The aforementioned calculations except the calculation of the energy density on the achieved by the high energy physics scale have been done at the assumption of discrete space. Actually, these calculations can approximately be executed only in the hypothesis of discrete space. According the approach of this work space must be discrete; otherwise any vacuum energy would be infinite.

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