No relation between the fine structure constant and electron anomalous magnetic moment

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Abstract: The article shows that there is no such relation between the fine-structure constant and the anymalous magnetic moment of an electron that is claimed by Quantum Electrodynamics. Indeed, there is no clear mathematical relation between these constants. The anomalous magnetic moment is not a real property: it is an apparent magnetic moment caused by force weakening when acting on a fast spinning electron. The magnetic moment of the electron in its rest frame is the Bohr magneton.

Keywords: Fine structure constant, anomalous magnetic moment, quantum electrodynamics, apparent mass.

1. Fine-structure constant and the anomalous magnetic moment of an electron

In Quantum Electrodynamics (QED) it is claimed that the anomalous magnetic moment a_e of an electron is related to the fine structure constant α by a formula (see [1][2])

$$a_e = C_2 \frac{\alpha}{\pi} + C_4 \left(\frac{\alpha}{\pi}\right)^2 + C_6 \left(\frac{\alpha}{\pi}\right)^3 + C_8 \left(\frac{\alpha}{\pi}\right)^4 + \cdots \quad (1)$$

$$+ \alpha_{\mu\tau} + \alpha_{hadronic} + \alpha_{weak} \quad (2)$$

where C_{2n} are coefficients that are obtained from Feynman diagrams. Notice that this starting point is obviously incorrect. Article [1] measures the anomalous magnetic moment of an electron by trapping a single electron into a magnetic cage. Then the electron is accelerated with a constant magnetic field and an alternating electric field of a cyclotron, the authors measure the cyclotron frequency. They also measure the spin-precession frequency (Larmar frequency) and get the anomalous magnetic moment. There are no hadronic or weak interactions in this measurement and nothing involving μ and τ. If the calculation from Feynman diagrams intends to give a formula that applies to this measurement, it cannot include $\alpha_{hadronic}$, α_{weak} or $\alpha_{\mu\tau}$ contributions, but without these contributions it is not possible to get the formula to agree with measurements.

The measured anomalous magnetic momentum of an electron in [1] (from 2023) is

$$a_e = 0.00115965218059(13). \quad (3)$$

The (13) at the end means the standard deviation of the error in the last two digits. A fine-structure constant measurement measured in [3] (from 2020) measured gives the value

$$\alpha = 0.0072973525628(6) \quad (4)$$

and [1] gives the value as

$$\alpha = 0.00729735256649(8). \tag{5}$$

We see that α is quite precisely known. Writing a_e as in (1) without $\alpha_{hadronic}$, α_{weak} and $\alpha_{\mu\tau}$ contributions gives

$$a_e = \frac{\alpha}{2\pi} - \left(\frac{\alpha}{2\pi}\right)^2 - 260.8712\left(\frac{\alpha}{2\pi}\right)^3 \tag{6}$$

for α in [1] and

$$a_e = \frac{\alpha}{2\pi} - \left(\frac{\alpha}{2\pi}\right)^2 - 260.953\left(\frac{\alpha}{2\pi}\right)^3 \tag{7}$$

for α in [3]. The powers of $\alpha/2\pi$ decrease so fast that if more terms should be used, there is no reason to assume that the series converges at all. From the constant of power three we see that it makes no sense. Why should the relation have a term 260.9? It cannot be obtained from Feynman diagrams for electromagnetic interactions, this is why $\alpha_{hadronic}$, α_{weak} and $\alpha_{\mu\tau}$ are included in (1). But how could the relation have such an arbitrary coefficient like 260.9?

The first term is Schwinger's formula, engraved in his gravestone

$$a_e = \frac{\alpha}{2\pi}. \tag{8}$$

It only a coincidence:

$$2\pi a_e = 0.00728630954... \qquad \alpha = 0.00729735256... \tag{9}$$

The presented article shows that α and a_e are not connected as in (1).

2. What is magnetic moment

There are two classical ways to derive a magnetic moment, let us try them to an electron.

A coil of N rounds around an area A with current I produces a dipole magnet with the magnetic moment

$$\mu = NIA. \tag{10}$$

An electron is a spin 1/2 particle. Rotating it 360 degrees turns a spin 1/2 particle into a mirror image and rotating it 720 degrees, two rounds, takes it back to the starting point. Thus, we have to set $N = 2$. Whether an electron has a radius or not is a matter of the model we use. I am not happy with the wavefunction model and will use a model from old quantum theory, thus, an electron is spinning and it has a spinning radius r_s. This is not the classical electron radius r_e for reasons that are explained in equations (14) and (16). Current flows on the surface in wires, therefore let us assume that the charge e of an electron circulates on a circle of radius r_s in the spinning motion. The length of the circle is $2\pi r_s$ and the area this circle covers is $A = \pi r_s^2$. The current is $I = q/t$ where q is a charge and t is time. Let the speed of the electron be $v_s = 2\pi r_s/t$, then $I = ev_s/1\pi r_s$ and the magnetic moment is

$$\mu_e = NIA = 2\frac{ev_s}{2\pi r_s}\pi r_S^2 = ev_s r_s. \tag{11}$$

Bohr's quantization rule for orbital magnetic momentum of an electron in a hydrogen atom is

$$L = m_e v_o a_o = \hbar \tag{12}$$

where v_o is the orbital speed and a_0 is the Bohr radius. In a similar way the quantization rule of the spin angular momentum of an electron should be

$$S = m_e v_s r_s = \frac{1}{2}\hbar \qquad i.e., \quad m_e v_s 2r_s = \hbar. \tag{13}$$

Notice that this quantization rule disagrees with the classical radius of an electron

$$r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2} = \frac{e^2}{4\pi\varepsilon_0 c\hbar}\frac{\hbar}{m_e c} = \alpha\frac{\hbar}{m_e c} \tag{14}$$

where

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar} \tag{15}$$

is the definition of the fine-structure constant. The speed v_s is smaller than c, thus the quantization rule gives the requirement

$$r_s = \frac{\hbar}{2m_e v_s} > \frac{1}{2}\frac{\hbar}{m_e c}. \tag{16}$$

The classical electron radius r_e is too small.

Solving $v_s r_s$ from the quantization rule and inserting to the magnetic momentum formula gives

$$\mu_e = \frac{e m_e v_s 2r_s}{2m_e} = \frac{e\hbar}{2m_e} = \mu_B. \tag{17}$$

The magnetic moment of an electron should be the Bohr magneton. Let us make the other calculation. Two formulas for torque for a magnet are

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \vec{r} \times \vec{F}. \tag{18}$$

Here B is the magnetic strength vector, r is the distance vector of the poles of a dipole magnet. The magnetic force vector $\vec{F}$ is

$$F = q\vec{v} \times \vec{B}. \tag{19}$$

Assuming that the vectors are orthogonal, we get

$$\tau_e = \mu_e B = r_s e v_s B \tag{20}$$

$$\mu_e = ev_s r_s = \frac{e}{2m_e} m_e v_s 2r_s \frac{r}{r_s} = \frac{e\hbar}{2m_e}\frac{r}{r_s} = \mu_B \frac{r}{r_s} \tag{21}$$

but this result must agree with the previous calculation, thus $r = r_s$ and again the result is that the magnetic moment of an electron is the Bohr magneton.

The Dirac equation also predicts this result. I see problems in the Dirac equation, see [6][7][8], and I preferred to make this calculation based on the old quantum theory, but the result is the same: the magnetic momentu of an electron should be the Bohr magneton.

However, measurements of the magnetic moment of an electron show that the magnetic moment of an electron is a bit larger. The formula is

$$\mu_e = g_e \mu_B \frac{S}{\hbar} = \frac{g}{2}\mu_B \tag{22}$$

where $S = \hbar/2$ is the spin angular momentum of an electron, g is called the coupling constant and it is used in QED as a coupling constant of the electromagnetic field (the Lagrangean of the Maxwell's equations in the Lorentz covariant form) to the Dirac spinor field. The finding from the measurements is

$$\frac{g}{2} = 1 + a_e \tag{23}$$

and here we have the anomalous magnetic moment of an electron. Let us see what is wrong in this measurement in the next section.

3. Why does the measurement give $a_e \neq 0$?

We only need to look at the latest measurement in [1] to find the error. The authors trap a single electron in a magnetic field and then they apply magnetic and electric fields in order to get two frequencies: the cyclotron frequency v_c and the spin-precession frequency ν_a. By dividing these frequencies they get the coupling constant and the anomalous magnetic moment of an electron

$$1 + a_e = \frac{g}{2} = \frac{\nu_a}{\nu_c}. \tag{24}$$

From this method we can see that they use the following formulas: The cyclotron frequency is

$$\nu_c = \frac{eB}{m} \tag{25}$$

and the spin-precession frequency is

$$\nu_a = \frac{geB}{2m_e}. \tag{26}$$

Then

$$\frac{\nu_1}{\nu_c} = \frac{geB}{2m_e}\frac{m}{eB} = \frac{g}{2}\frac{m}{m_e} \tag{27}$$

and assuming that $m = m_e$, the formula does give $g/2$. We see that the measured electron must have moved slowly because otherwise the authors would have used the Thomson precession formula for the spin-precession:

$$\nu_a = \frac{eB}{2m_e c}\left(g - 2 + \frac{2}{\gamma}\right) \tag{28}$$

where $\gamma = \left(1 - (v_p/c)^2\right)^{-1/2}$ with v_p being the speed of the electron, i.e., the speed by which the electron moves in the space. If the speed of the electron had been large, they also would have used $m = \gamma m_e$ in the cyclotron frequency formula where $\gamma = \left(1 - (v_p/c)^2\right)^{-1/2}$. The authors did not use these formulas for high speed electrons, therefore the electron was slow. But here is the caveat. The speed of the electron was small, but the spinning speed v_s of the electron is not small.

It was noticed by early researches, like Kaufmann [4] and Lorentz [5], that an electron moving in a constant magnetic field appears to have more mass. This was named apparent transverse mass. Einstein incorrectly understood that a moving mass grows and defined the relativistic mass, today it is presented as relativistic kinetic energy, but earlier the formula γm_0 represented apparent transverse mass. It is transverse because the magnetic force vector is transverse to the velocity vector and it is apparent because the mass does not increase, the force in a fixed frame of reference cannot influence a fast moving mass with full force, the force is weakened which can also be expressed as the mass appearing as larger. In those times experimenters, like Kaufmann, could reach speeds $v = 0.7c$ in a cathode ray tube. At those speeds they could see clearly the effect of the apparent transverse mass in the trajectory that the electron took. They could not have seen the effect of the spinning speed of the electron provided that the spinning speed was much slower than $0.7c$, for instance if $v_s \approx 0.048c$, as it will turn out, these early researchers could not notice this effect, but this effect would show up in the very precise measurement of a_e in .

It is clear that the spinning speed v_s must cause the same apparent transverse mass effect when the magnetic field of the cyclotron is the fixed frame and the rest frame of the spinning electron is the moving frame. When measuring the torque of the electron in the cyclotron frequency calculation the apparent transverse mass of the electron is $m = \gamma_s m_e = m_e \left(1 - (v_s/c)^2\right)^{-1/2}$.

Consequently, what the authors of [1] took as $1 + a_e$ is in fact

$$1 + a_e = \frac{\nu_1}{\nu_c} = \frac{geB}{2m_e}\frac{\gamma_s m_e}{eB} = \frac{g}{2}\gamma_s. \tag{29}$$

The magnetic moment of an electron is indeed the Bohr magneton and $g = 2$. Solving v_s from the remaining equation

$$1 + a_e = \gamma_s \tag{30}$$

gives

$$v_s = \frac{\sqrt{2a_e + a_e^2}}{1 + a_e} c \tag{31}$$

and the radius r_s comes from the quantization rule for spin angular momentum as

$$r_s = C \frac{\hbar}{m_e c} \qquad C = \frac{1 + a_e}{2\sqrt{2a_e + a_e^2}}. \tag{32}$$

Calculating C from a_e in [1] gives $(2C)^{-1} = 0.048117317159$, so $C \approx 10.39$

$$v_s = C_1 c \approx 0.048c \qquad r_s = C \frac{\hbar}{m_e c} = 10.39 \frac{\hbar}{m_e c}. \tag{33}$$

These values do not violate (16).

The value of C does not relate to the fine-structure constant in any obvious way:

$$C_1 = (2C)^{-1} = \frac{\sqrt{2a_e + a_e^2}}{1 + a_e} = 6.593804636\alpha. \tag{34}$$

Probably there is no simple connection between the two constants. From Bohr's atomic model we get Sommerfeld's original definition of α. Setting the centrifugal force to equal the Coulomb force

$$F_{c.f} = \frac{m_e v_o^2}{a_0} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{a_o^2} \tag{35}$$

and using the quantization rule for L gives

$$\alpha = \frac{v_o}{c} \tag{36}$$

while the apparent anomalous moment of an electron a_e is related to the spinning speed as

$$C(a_e) = \frac{v_s}{c}. \tag{37}$$

The quantization rules for L and S demand that

$$v_0 a_o = 2 v_s r_s. \tag{38}$$

The radius r_s is smaller than the Bohr radius a_0, thus v_s is larger than v_o, but v_s is clearly smaller than c. Indeed, $v_s \approx 0.048c$.

4. References

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