Integral Reduction Formulas Enhanced With a TI84

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Abstract

The TI84 has both a definite integral function and a recursive list generator. We explore whether the combination can be used to solve single and double integral problems that reference recursive formulas for integral evaluations.

Introduction

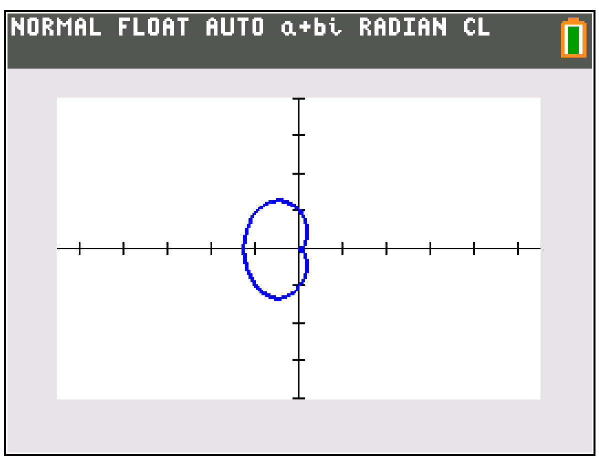

Figure 1: The cardioid $r = 1 - \cos(\theta)$.

Thomas's Calculus text has an example problem that references a reduction formula for $\cos^n$ integrands: Find the moment of inertia, about the y-axis, about the area enclosed by the cardioid $r = a(1 - \cos(\theta)$ [1]. The TI84-CE can graph functions expressed in polar coordinates: Figure 1.

With a few manipulations and references to formulas, the solution distills to solving the integral

$$\int_0^{2\pi} \frac{a^4}{4} \cos^2(1 - \cos(\theta))^4 \, d\theta. \tag{1}$$

There are several instances of powers of $\cos$ in the integrand: $\cos^2$, $\cos^3$, $\cos^4$, $\cos^5$, and $\cos^6$. Each has a coefficient. These coefficients can be stored in a TI84 list.

The evaluation of the indefinite integral of $\cos^n$ can proceed with an integration by parts. A reduction formula pops out; in simplified form it is

$$\int \cos^n = \frac{n-1}{n} \cos^{n-1} \sin + \int \cos^{n-2}.$$

If the definite integrals have upper and lower bounds that make the first term on the right 0 (typical as in this problem), this further reduces to

$$\int \cos^n = \frac{n-1}{n} \int \cos^{n-2}.$$

Although this is termed a reduction formula, it really works through recursion. If the limits of integration make either $\sin$ or $\cos$ zero, we can proceed; a simple test on a TI84 using an IF with intFn – a definite integral evaluation. Let $F(n)$ give the n case in the formula. Then once

$$F(0) = \int_a^b \cos^0 = \int_a^b 1 = b - a \text{ and } F(1) = \int_a^b \cos = |_a^b \sin = \sin(b) - \sin(a)$$

are determined we can crunch others: for example,

$$F(2,3,4,5,6) = \frac{1}{2}F(0), \frac{2}{3}F(1), \frac{3}{4}F(2), \frac{4}{5}F(3), \frac{5}{6}F(4),$$

all the integrals in (1).

We will show that the TI84 recursive sequence feature can generate a list with all these integral evaluations. When this list is multiplied by a list of coefficients, $\{0, 1, -4, 6, -4, 1\}$ a summed dot product gives the value of the integral. With a little trick, π can be incorporated into the evaluation for an exact answer. Without the $a^4/4$ factor in the integrand of (1), the exact answer is $49\pi/8$.

Details

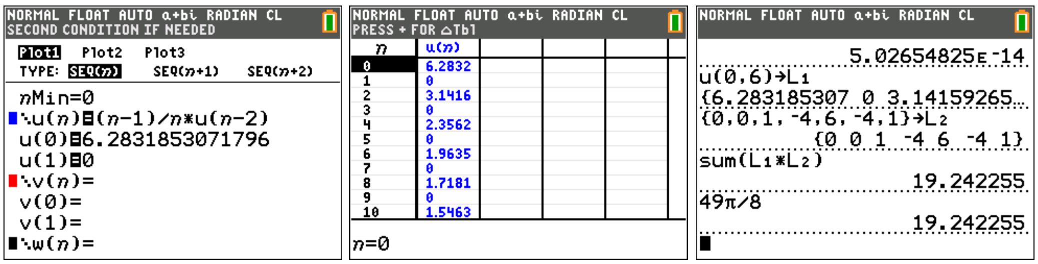

Figure 2: a) Set mode to SEQ and hit the Y1 key; b) Table shows integral values; c) Make lists using u and coefficients then take the sum of the dot product.

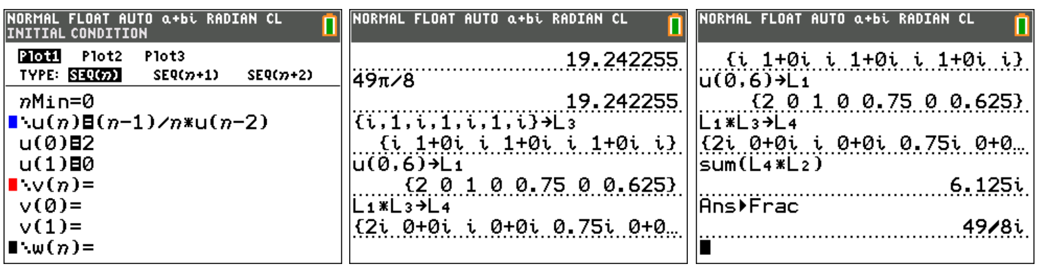

Figure 3: a) Reinitialize the u variable; b) Make a mask for π using i; c) Sum the dot product.

References

[1] G. B. Thomas, *Calculus and Analytic Geometry*, 4th ed., Addison-Wesley (1968).