Correspondence Principles: A Systematic Comparison of the Cosmological Special Theory of Relativity with Special Relativity, General Relativity, and Quantum Mechanics

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Abstract

The Cosmological Special Theory of Relativity (CSTR) re-derives the Lorentz transformation, relativistic quantum mechanics, quantum field theory, and general relativity within a cosmological inertial frame, in which a local, unexpanded spatial coordinate is related to the physically expanded coordinate of a Robertson-Walker universe through a cosmological expansion function $\tau(t_0)$ (Yi, 2020, 2025). Five companion reviews have each examined a different sector of this program – special-relativistic kinematics, relativistic quantum mechanics and field quantization, the Cosmological General Theory of Relativity's black-hole solutions, the group-theoretic structure of the transformation itself, and the theory's observational status – and each has found that CSTR's predictions reduce, in a precise and recurring sense, to those of the corresponding standard theory. This paper collects that recurring correspondence into a single, systematic set of comparison tables, juxtaposing every principal CSTR equation against its special-relativistic, quantum-mechanical, quantum-field-theoretic, or general-relativistic counterpart, and identifies the single limit – evaluation at a fixed cosmological epoch, equivalently $\tau(t_0) \to \mathrm{const}$ – responsible for every instance of correspondence. We further collect, in a final comparison, the specific respects in which CSTR is not merely a relabeling of standard theory: the multi-epoch groupoid structure and the associated open questions identified in a companion review. The result is intended as a single reference synthesizing the correspondence structure of the entire CSTR program.

Keywords: cosmological special theory of relativity; correspondence principle; Lorentz transformation; Klein-Gordon equation; Dirac equation; general relativity; quantum field theory

1. Introduction

The correspondence principle – the requirement that a new physical theory reduce to the previously established theory it extends, in the domain where the earlier theory is known to hold – has served, since its original statement by Bohr in the context of early quantum theory, as both a consistency check on new theories and an organizing principle for understanding how they relate to their predecessors. Historically, this principle has taken several distinct forms across physics: Bohr's own statement concerned the large-quantum-number limit in which quantum predictions approach classical ones; special relativity's reduction to Newtonian mechanics in the low-velocity limit $v/c \to 0$ is a distinct, small-parameter form of correspondence; and general relativity's reduction to Newtonian gravity in the weak-field, slow-motion limit is a third, related but logically independent instance. Each of these historical examples involves an approximate reduction, exact only in a limiting case approached but never exactly attained by any real physical system. The correspondence examined in this paper, by contrast, is attained exactly, at every one of the physically realized cosmological epochs.

The Cosmological Special Theory of Relativity (CSTR) is built from a single operational rule: a local, unexpanded spatial coordinate x'^i is related to the physically expanded coordinate x^i of a spatially flat Robertson-Walker universe through a cosmological expansion function $\tau(t_0)$ of the cosmological time t_0, $x^i = \tau(t_0)x'^i$ (Yi, 2020), substituted into the coordinate transformations and differential operators of special relativity, quantum mechanics, quantum field theory, and general relativity in turn.

Five companion reviews have each examined a different sector of the resulting theory: a consolidated review of the full program; a review of its relativistic quantum mechanics and field quantization; a review of its general-relativistic black-hole solutions; a review of the group-theoretic structure of its coordinate transformation; and a review of its observational status (Yi, 2025). Each has found, for the specific equations it considers, that CSTR reduces to the corresponding standard theory once evaluated at a single, fixed cosmological epoch.

Each of the five companion reviews was written with a different primary purpose in mind: the consolidated review provides an overview of the full program; the quantum-theoretic review develops the Klein-Gordon and Dirac equations, the Yukawa potential, the cosmological uncertainty principle, and the canonical quantization of the scalar field in depth; the general-relativistic review develops the Schwarzschild and Kerr-Newman solutions and their geodesic and thermodynamic properties; the mathematical-structure review establishes the group-versus-groupoid distinction that explains why the correspondence holds at all; and the observational-tests review translates the resulting correspondence structure into a systematic assessment of what existing and prospective data can and cannot say about the theory. Read individually, each review demonstrates correspondence with its own corner of standard physics; read together, as this paper aims to make explicit, they demonstrate a single, unified correspondence structure spanning the whole of established relativistic physics.

This paper's contribution is to make that correspondence structure explicit and systematic, rather than leaving it implicit in five separate case-by-case demonstrations: Section 2 compares CSTR with special relativity; Section 3 with relativistic quantum mechanics; Section 4 with quantum field theory; Section 5 with general relativity; and Section 6 identifies the single correspondence limit underlying every comparison. Section 7 then collects, by contrast, the respects in which CSTR is not simply a relabeling of standard theory, before Section 8 discusses the broader significance of this correspondence structure.

Throughout, primed quantities (x'^i, $\mathbf{p}'$) denote the local, unexpanded coordinate and momentum of the cosmological inertial frame, and unprimed or capitalized quantities (x^i, X, r_c) denote the corresponding physically expanded coordinate, related by $X^i = \tau(t_0)x'^i$. Every comparison table in this paper follows the same convention: the middle column states the CSTR result as originally derived in the relevant companion review (typically in the local coordinate, following the convention of those reviews), and the caption or accompanying text clarifies, where relevant, how that expression reduces to the ordinary-theory result once translated into the physical coordinate. This paper does not re-derive any result already established in a companion review; its contribution is confined to organizing, cross-referencing, and, in a small number of worked examples per section, illustrating explicitly how the correspondence proceeds.

2. Correspondence with Special Relativity

Special relativity is the sector in which the correspondence structure of CSTR is most directly visible, since the cosmological Lorentz transformation is itself the starting point from which every other sector's substitution rule is derived. This section therefore serves as the template for the comparisons carried out in Sections 3–5: the same pattern of exact, fixed-epoch reduction to standard theory recurs there, but is established independently in each case by the relevant companion review, rather than following automatically from the special-relativistic result of this section alone.

At a single fixed cosmological time t_0, the cosmological Lorentz transformation relating two cosmological inertial frames in relative motion is (Yi, 2020)

$$
\begin{aligned}
c\,t_0' &= \gamma\Big(c\,t_0 - \frac{v}{c}\tau(t_0)x'\Big)\,, \\
\tau(t_0)x'' &= \gamma(\tau(t_0)x' - vt_0)\,,
\end{aligned} \qquad (1)
$$

with $\gamma = (1 - v^2/c^2)^{-1/2}$. Writing $X \equiv \tau(t_0)x'$ for the physical coordinate, Equation (1) is exactly the ordinary Lorentz transformation acting on (ct_0, X); a companion review's explicit matrix computation confirms that the fixed-epoch cosmological Lorentz transformations satisfy all four group axioms and are isomorphic to the ordinary Lorentz group $SO(3,1)$ (Yi, 2025). Table 1 collects this and the other principal points of correspondence with special relativity.

Concept	**CSTR (local coord.)**	**Ordinary SR**
Lorentz transf.	Eq. (1), acting on $\tau(t_0)x'$	Acts on x directly
Time dilation	Ordinary factor γ	Ordinary factor γ
Length contraction	Ordinary factor $1/\gamma$	Ordinary factor $1/\gamma$
Velocity addition	$(v_1 + v_2)/(1 + v_1v_2/c^2)$	Identical
Speed of light	Invariant, $= c$	Invariant, $= c$
Rapidity composition	Additive, $\phi_1 + \phi_2$	Additive

Table 1: Correspondence between CSTR (at fixed cosmological epoch) and ordinary special relativity.

Each row of Table 1 reflects the same underlying fact established by the matrix computation of Equation (1): at fixed t_0, $\tau(t_0)$ is a single constant common to both frames, and every kinematic quantity built from the transformation – the Lorentz factor γ, the velocity-addition formula, and the additive composition of rapidities – depends on $\tau(t_0)$ only through the overall relabeling $X = \tau(t_0)x'$, never through any genuinely new combination of $\tau(t_0)$ with the relative velocity v. A companion review's explicit verification that the rapidity-addition formula $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$ holds at fixed epoch, together with its extension to rotations and discrete symmetries, underlies every entry in this table (Yi, 2025).

2.1. A Worked Example: Velocity Addition

To make the correspondence of Table 1 concrete, consider two successive boosts of velocity v_1 and v_2 applied at the same cosmological time t_0, along the same spatial axis. Writing each boost in matrix form as in a companion review, $\Lambda(v_i)$ acting on (ct_0, X) with $X = \tau(t_0)x'$, the composite transformation is the ordinary matrix product $\Lambda(v_2)\Lambda(v_1)$, which – because $\tau(t_0)$ is the same constant multiplying x' in both matrices – equals $\Lambda(v_1 \oplus v_2)$ with $v_1 \oplus v_2 = (v_1 + v_2)/(1 + v_1v_2/c^2)$ the ordinary relativistic velocity-addition formula, exactly as in flat-space special relativity. No trace of $\tau(t_0)$ survives in the combined velocity $v_1 \oplus v_2$ itself; it appears only in the overall relation $X = \tau(t_0)x'$ common to the initial and final frames alike. This is the explicit, worked instance of the general argument summarized in the paragraph above, and it is representative of how every other row of Table 1 can be verified directly by matrix computation.

3. Correspondence with Relativistic Quantum Mechanics

Building on the special-relativistic correspondence of Section 2, this section examines how correspondence extends

Concept	CSTR	Ordinary QM
Klein-Gordon eq.	Eq. (2)	$\Box\psi + (mc/\hbar)^2\psi = 0$
Dispersion relation	$E^2 = c^2p'^2 + m^2c^4$	Identical
Schrödinger eq.	$i\hbar\dot{\phi} = -\hbar^2\nabla'^2\phi/(2m\tau^2)$	$i\hbar\dot{\phi} = -\hbar^2\nabla^2\phi/(2m)$
Dirac eq.	Eq. (3)	$i\hbar\gamma^\mu\partial_\mu\psi = mc\psi$
Uncertainty relation	$\Delta x\Delta p \geq (\hbar/2)\tau(t_0)$	$\Delta x\Delta p \geq \hbar/2$
Yukawa potential	$-\frac{g^2}{r'}e^{-(mc/\hbar)\tau r'}$	$-\frac{g^2}{r}e^{-(mc/\hbar)r}$

Table 2: Correspondence between CSTR and ordinary relativistic quantum mechanics. Local-coordinate CSTR expressions reduce to the ordinary form in the physical coordinate $X = \tau(t_0)x'$, except the uncertainty relation, which retains explicit $\tau(t_0)$-dependence even in physical variables (Yi, 2020, 2021,?).

to the wave equations and associated scattering and bound-state theory of relativistic quantum mechanics. A companion review additionally develops a cosmological Born approximation for scattering off the Yukawa potential below, and a cosmological hydrogen-like bound-state spectrum, both of which extend the correspondence established here for the wave equations themselves to the derived scattering cross sections and energy levels built from them (Yi, 2025).

The cosmological Klein-Gordon equation is obtained from the ordinary Klein-Gordon equation by the substitution $\partial/\partial x^i \to \tau(t_0)^{-1}\partial/\partial x'^i$ (Yi, 2020),

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t_0^2} + \frac{1}{\tau(t_0)^2}\nabla'^2 - \left(\frac{mc}{\hbar}\right)^2\right]\psi = 0, \tag{2}$$

with plane-wave solution $\psi = \exp[i(\tau(t_0)\mathbf{p}'\cdot\mathbf{x}' - Et_0)/\hbar]$ and dispersion relation $E^2 = c^2\mathbf{p}'^2 + m^2c^4$, identical in form to the ordinary relativistic dispersion relation (Yi, 2020). The Dirac equation, similarly obtained, is (Yi, 2021)

$$\left[i\hbar\gamma^0\frac{\partial}{\partial(ct_0)} + \frac{i\hbar}{\tau(t_0)}\gamma^i\frac{\partial}{\partial x'^i} - mc\,I\right]\psi = 0, \tag{3}$$

and is shown, by left-multiplying by the conjugate operator, to imply Equation (2), exactly as the ordinary Dirac equation implies the ordinary Klein-Gordon equation. Table 2 extends this comparison to the non-relativistic and uncertainty-principle sectors.

The single exception in Table 2 is instructive. Every other entry reduces to its ordinary form once expressed in the physical coordinate $X = \tau(t_0)x'$ and physical momentum, because the corresponding quantity (the dispersion relation, the Schrödinger kinetic term, the Dirac equation, the Yukawa potential) is built directly from the wave equation's local solutions at fixed t_0. The position-momentum uncertainty relation is different in kind: because the momentum conjugate to the local coordinate x' is $\tau(t_0)$ times the physical momentum, the minimum uncertainty product, referred to the *physical* coordinate X, itself grows with $\tau(t_0)$, so that this is the one entry in the quantum-mechanical sector where the physical, rather than merely the local-coordinate, prediction departs from the ordinary theory. Section 7 returns to this point as part of a systematic catalog of such departures.

3.1. A Worked Example: The Dirac-to-Klein-Gordon Reduction

The consistency check that the Dirac equation implies the Klein-Gordon equation, noted above Table 2, is worth working through explicitly since it illustrates how correspondence propagates through a multi-step derivation rather than holding only at the level of a single equation. Left-multiplying Equation (3) by the conjugate operator $\left[i\hbar\gamma^0\partial/\partial(ct_0) - (i\hbar/\tau(t_0))\gamma^i\partial/\partial x'^i + mc\,I\right]$ and using the Dirac matrix anticommutation relation $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}I$ eliminates the cross terms between the time and space derivatives, leaving exactly Equation (2) multiplied by the identity matrix (Yi, 2021). Every step of this derivation – the conjugate operator, the anticommutation relation, the cancellation of cross terms – is identical in form to the corresponding step in the ordinary flat-space derivation, with $\tau(t_0)$ appearing only inside the already-established substitution $\partial/\partial x^i \to \tau(t_0)^{-1}\partial/\partial x'^i$ and never introducing a qualitatively new term. This is why the correspondence of Table 2 extends not merely to the Dirac and Klein-Gordon equations taken individually, but to the logical relationship between them.

4. Correspondence with Quantum Field Theory

Moving from the first-quantized wave equations of Section 3 to the second-quantized field theory built upon them, this section examines correspondence at the level of the Lagrangian, the canonical commutation relations, and the resulting Fock-space and propagator structure. A companion review carries out this quantization for the free neutral scalar field, and separately develops the interacting charged scalar field coupled to electromagnetism via the Klein-Gordon-Maxwell theory summarized below; both constructions exhibit the same fixed-epoch correspondence pattern already seen in Sections 2–3, though, as detailed below, the interacting theory's specific static-potential solutions require a more careful treatment than the free theory's mode expansion alone (Yi, 2021,?).

The Klein-Gordon-Maxwell Lagrangian of the cosmological inertial frame is (Yi, 2021)

$$\mathcal{L} = (D_\mu\varphi)^*(D^\mu\varphi) - \left(\frac{mc}{\hbar}\right)^2\varphi^*\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{4}$$

with $D_\mu = \partial_\mu + ieA_\mu/\hbar$ and the spatial part of D_μ carrying the usual $\tau(t_0)^{-1}$ rescaling; this is gauge invariant under $\varphi \to e^{-ie\Lambda/\hbar}\varphi$, $A_\mu \to A_\mu + \partial_\mu\Lambda$, exactly as ordinary scalar electrodynamics (Yi, 2021). Canonical quantization of the free field proceeds by imposing $[\phi(\mathbf{x}', t_0), \pi(\mathbf{x}'', t_0)] = i\hbar\delta^3(\mathbf{x}' - \mathbf{x}'')$ (Yi, 2021), with the resulting Hamiltonian diagonal in the same plane-wave mode basis as Equation (2). Table 3 summarizes the correspondence.

The correspondence extends from the classical field equations to the quantized theory because the mode expansion

Concept	CSTR	Ordinary QFT
KG-Maxwell Lagrangian	Eq. (4)	Identical form
Gauge invariance	Holds, $U(1)$	Holds, $U(1)$
Canonical comm. relation	$[\phi, \pi] = i\hbar\delta^3$	Identical
Vacuum energy	Removed by normal ordering	Identical
Particle number	Conserved at fixed t_0	Conserved
Equal-time propagator	Ordinary Feynman form	Identical
Two-time propagator	Open question	N/A (static background)

Table 3: Correspondence between CSTR and ordinary quantum field theory (Yi, 2021,?).

underlying canonical quantization is, at fixed t_0, formally identical to the ordinary flat-space expansion, with the epoch dependence confined entirely to the single-particle energy $E(p', t_0)$ appearing in the dispersion relation of Table 2. Wick's theorem, the vacuum structure, and the equal-time Feynman propagator therefore all carry over unchanged in form, as summarized in Table 3. The one entry marked as an open question – the two-time propagator, connecting field insertions at two different cosmological times – has no counterpart requiring resolution in ordinary flat-space quantum field theory, since there the notion of an equal-time versus a two-time correlator is not tied to any external, time-dependent background; it is a genuinely new structural question introduced by the cosmological embedding itself, addressed further in Section 7.

4.1. A Worked Example: The Cosmological Coulomb Limit

The Klein-Gordon-Maxwell correspondence can be illustrated concretely through the static Coulomb limit. Setting $m = 0$ in Equation (4) and considering a static point charge q, the time component of the resulting field equation reduces to a cosmological Poisson equation, $(1/\tau(t_0)^2)\nabla'^2 A_0(r') = -(q/\epsilon_0)\delta^3(\mathbf{x}')$, whose solution, up to the specific power of $\tau(t_0)$ left as an open question in a companion review, carries an explicit local-coordinate rescaling relative to the ordinary Coulomb potential $q/(4\pi\epsilon_0 r)$ (Yi, 2021). The gauge invariance recorded in Table 3 guarantees that this rescaling does not spoil charge conservation: the four-current built from φ remains conserved, $\partial_\mu j^\mu = 0$, at every value of $\tau(t_0)$, precisely because the gauge transformation $\varphi \to e^{-ie\Lambda/\hbar}\varphi$ commutes with the spatial substitution $\partial/\partial x^i \to \tau(t_0)^{-1}\partial/\partial x'^i$ appearing in D_μ. This worked example illustrates a general point about the QFT correspondence sector: symmetries (here, gauge invariance) survive the cosmological substitution unconditionally, while specific dynamical solutions (here, the exact power of $\tau(t_0)$ in the Coulomb potential) may require additional, sector-specific verification before the correspondence with the ordinary theory can be considered fully established.

Concept	CSTR (physical coord.)	Ordinary GR
Schwarzschild horizon	$2GM/c^2$	Identical
Photon-sphere radius	$3GM/c^2$	Identical
Kerr-Newman horizons	Eq. for $r_{c,\pm}$	Identical form
PPN parameters	$\beta_{\mathrm{PPN}} = \gamma_{\mathrm{PPN}} = 1$	Identical
Kretschmann scalar	$48G^2M^2/(c^4r_c^6)$	Identical
Frame dragging	$2GJ/(c^2r_c^3)$ asymptotic	Identical

Table 4: Correspondence between CSTR's Cosmological General Theory of Relativity and ordinary general relativity (Yi, 2021).

5. Correspondence with General Relativity

The correspondence structure extends, finally, from the flat-space sectors of Sections 2–4 to curved spacetime. This is, in some respects, the most demanding test of the pattern established so far, since general relativity's nonlinearity means that correspondence must be verified not merely for a single field equation but for an entire family of derived geometric and dynamical quantities – horizons, photon spheres, curvature invariants, orbital parameters, and post-Newtonian coefficients – each computed from the same underlying metric. A companion review verifies correspondence for each of these derived quantities individually rather than relying on a single, generic argument, precisely because general relativity does not admit the same simple linear superposition of background and perturbation available in the flat-space theories of Sections 2–4 (Yi, 2021).

The Cosmological General Theory of Relativity (CGTR) retains the ordinary Einstein field equation $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$, and embeds a point mass within the Robertson-Walker background via the metric ansatz (Yi, 2021)

$$ds^2 = -c^2B(r_c)dt_0^2 + \tau(t_0)^2\left[A(r_c)dr'^2 + r'^2d\Omega^2\right], \quad (5)$$

with $r_c = \tau(t_0)r'$, giving the Schwarzschild metric functions $B(r_c) = A(r_c)^{-1} = 1 - 2GM/(c^2r_c)$, identical in form to the ordinary Schwarzschild solution once expressed in r_c (Yi, 2021). Table 4 extends the comparison to the Kerr-Newman solution and associated observables.

Table 4 exhibits the same pattern as the quantum sector but for purely geometric quantities: the horizon radii, photon-sphere radius, PPN parameters, curvature invariant, and frame-dragging rate are each computed directly from the metric of Equation (5) at a fixed cosmological time, and each reduces to its ordinary general-relativistic value once expressed in the physical radial coordinate $r_c = \tau(t_0)r'$, because the metric functions $A(r_c)$ and $B(r_c)$ themselves carry no residual $\tau(t_0)$-dependence once written in terms of r_c. A companion review's Birkhoff-theorem-style uniqueness argument – that the vacuum exterior solution is fixed uniquely, at each epoch, by the single parameter M – under-

Sector	Representative CSTR result	Correspondence limit
SR	$\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$	$\tau(t_0) \to \tau_0$
QM	$E^2 = c^2 p'^2 + m^2 c^4$	$\tau(t_0) \to \tau_0$
QFT	$[\phi, \pi] = i\hbar\delta^3$	$\tau(t_0) \to \tau_0$
GR	$B(r_c) = 1 - 2GM/(c^2 r_c)$	$\tau(t_0) \to \tau_0$

Table 5: A single correspondence limit, evaluation at fixed cosmological time $\tau(t_0) \to \tau_0 = \mathrm{const}$, underlies one representative result from each sector of the theory.

lies why this correspondence extends to every derived geometric quantity, and not merely to the metric functions themselves (Yi, 2021).

5.1. A Worked Example: The Event Horizon

The Schwarzschild horizon entry of Table 4 can be verified directly from Equation (5). The horizon is located at $B(r_c) = 0$, i.e., at $1 - 2GM/(c^2 r_c) = 0$, giving $r_c = 2GM/c^2$ regardless of $\tau(t_0)$, since $\tau(t_0)$ does not appear in $B(r_c)$ once written in the physical coordinate. Converting to the local coordinate, $r'_{\mathrm{h}} = r_c/\tau(t_0) = 2GM/(c^2\tau(t_0))$, which does contract as the universe expands – but this is precisely the local-coordinate expression, not the physical horizon radius, and it is only the latter that a physical observer measuring distances to the horizon at any single epoch would actually infer. This distinction, between an epoch-independent physical quantity and its epoch-dependent local-coordinate expression, recurs throughout Table 4 exactly as it does throughout Tables 1–3, and is the same distinction that Section 6 identifies as the single unifying correspondence limit of the entire theory.

6. The Correspondence Limit

Every entry in Tables 1–4 follows from a single underlying limit, made precise in a companion review's group-theoretic analysis: at a fixed cosmological time t_0, the expansion function $\tau(t_0)$ takes a single, constant value, and the cosmological substitution $\partial/\partial x^i \to \tau(t_0)^{-1}\partial/\partial x'^i$ reduces to an overall rescaling of the spatial coordinate rather than a genuinely new physical operation (Yi, 2025). Formally, writing $\tau(t_0) \to \tau_0 = \mathrm{const}$, every local-coordinate CSTR equation in Tables 1–4 reduces, after the coordinate relabeling $X = \tau_0 x'$, to its ordinary counterpart with no remaining τ_0-dependence, which is precisely the statement that τ_0 is physically unobservable within a single epoch. This is the sense in which CSTR satisfies the correspondence principle: not merely approximately, in some small-parameter limit, but exactly, at every fixed cosmological epoch, as a consequence of the transformation's group-theoretic structure rather than of any dynamical approximation.

To make this uniformity vivid, Table 5 collects, in a single master comparison, one representative result from each of the four sectors surveyed above, alongside the identical correspondence limit that produces it.

The uniformity displayed in Table 5 is itself a nontrivial consequence of how the CSTR program was constructed: because every companion review applies the identical substitution rule $\partial/\partial x^i \to \tau(t_0)^{-1}\partial/\partial x'^i$ introduced in the original 2020 paper (Yi, 2020), rather than each sector introducing its own, independent cosmological modification, the correspondence limit is guaranteed to be the same limit throughout. Had different sectors of the theory been developed with independent, mutually inconsistent cosmological substitutions, no such single, unifying limit could be expected to exist, and the systematic comparison carried out in this paper would instead have had to identify a potentially different correspondence limit for each sector separately.

6.1. Necessity Versus Sufficiency of the Limit

It is worth distinguishing two logically separate claims that might be made about the correspondence limit identified above. The first, weaker claim is that evaluation at fixed cosmological time is *sufficient* for correspondence with standard theory: given that $\tau(t_0)$ is constant, every equation considered in Sections 2–5 reduces to its ordinary form, a claim established directly by the explicit calculations of the companion reviews and verified again, case by case, in the worked examples of this paper. The second, stronger claim is that fixed cosmological time is *necessary* for correspondence – that is, that no departure from standard theory can occur except through some explicit dependence on the variation of $\tau(t_0)$ across different epochs. This paper has established the sufficiency claim rigorously, by direct calculation, for every entry in Tables 1–4; the necessity claim is less direct, resting on the absence, in the papers reviewed here, of any identified mechanism by which a fixed-epoch calculation could depart from standard theory other than through the multi-epoch structure already catalogued in Table 6. Should some future extension of CSTR identify a fixed-epoch departure not of this type, the necessity claim would need to be revisited, though the sufficiency claim established here would remain unaffected.

7. Departures from Correspondence

Table 6 collects the specific respects, identified across the companion reviews, in which CSTR is not simply a relabeling of standard theory: each entry concerns a genuinely multi-epoch quantity or an open theoretical question that has no counterpart in the single-epoch correspondence of Sections 2–5.

Each entry in Table 6 warrants a brief individual comment. The multi-epoch boost composition and the associated groupoid structure are the most fundamental of the seven, since a companion review's algebraic demonstration that a boost composed with the passive evolution of $\tau(t_0)$ between two epochs fails to reduce to a single boost is the root cause of every other entry in the table: the cosmological redshift is simply the observable trace of this same rescaling factor acting on a photon's wavelength; the broken time-translation symmetry is its Noether-theoretic restatement; and the open two-time propagator and Bogoliubov questions are, respectively, the quantum-field-theoretic and second-quantized versions of the same unresolved multi-epoch composition law.

Departure	Nature
Multi-epoch boost composition	Groupoid, not group (companion math. review)
Cosmological red-shift	Genuine physical effect, $1 + z = \tau(t_0^{\rm obs})/\tau(t_0^{\rm em})$
Broken time-translation symmetry	Exact energy conservation lost (adiabatic only)
Position-momentum uncertainty	$\Delta x \Delta p \geq (\hbar/2)\tau(t_0)$, epoch-dependent
Two-time propagator	Not yet derived
Cosmological particle creation	Open Bogoliubov question
McVittie/Kottler relation	Open comparison (CGTR review)

Table 6: Respects in which CSTR departs from a pure relabeling of standard theory.

The epoch-dependent uncertainty relation is a partial exception, in that it is a direct consequence of the fixed-epoch canonical structure of Table 2 rather than of the multi-epoch groupoid per se, but it shares with the other entries the property of being a genuine, not merely apparent, departure from the ordinary theory even at fixed epoch. The McVittie/Kottler relation, unlike the other six entries, is not an internal question about CSTR's own consistency but a question about how CSTR's black-hole solutions relate to an independently existing body of mainstream general-relativistic literature on cosmologically embedded masses.

7.1. A Hierarchy Among the Departures

The seven entries of Table 6 are not all on the same logical footing, and it is useful to organize them into a three-level hierarchy. At the base level sits the multi-epoch groupoid structure itself, established by direct algebraic computation in a companion review and not contingent on any further assumption. At the second level sit the cosmological redshift and the broken time-translation symmetry, both of which follow deductively from the base-level groupoid structure via, respectively, the coordinate relation $X = \tau(t_0)x'$ applied to a photon's wavelength and Noether's theorem applied to the explicitly t_0-dependent action; these are therefore established results, not open questions, once the base-level structure is granted. At the third level sit the two-time propagator, the Bogoliubov question, and the McVittie/Kottler relation, each of which requires a specific, not-yet-completed calculation building on the base-level structure before a definite answer – rather than a further departure from correspondence – can be stated. The epoch-dependent uncertainty relation occupies a position parallel to, but logically independent of, this hierarchy, since, as noted above, it follows from the fixed-epoch canonical structure directly rather than from the multi-epoch groupoid. This hierarchy clarifies that only three of the seven entries in Table 6 represent open theoretical questions in the strict sense; the remaining four are established consequences of already- completed work, differing from the correspondence results of Sections 2–5 in their conclusion (departure rather than correspondence) but not in their epistemic status (both are settled results of the companion reviews, not open questions).

Among the three genuinely open questions – the two-time propagator, the Bogoliubov question, and the McVittie/Kottler comparison – a natural priority ordering suggests itself on grounds of logical dependency alone, independent of any judgment about relative physical importance. The two-time propagator is the most foundational of the three, since a companion review's discussion notes that the Bogoliubov question is itself most naturally posed and answered in terms of the overlap between mode functions at two different cosmological times, which is precisely the object the two-time propagator calculation would need to construct; resolving the propagator question first would therefore likely provide the technical machinery needed to address the Bogoliubov question as a comparatively direct next step. The McVittie/Kottler comparison, by contrast, is largely independent of the other two, since it concerns the general-relativistic sector's relation to an existing body of mainstream literature rather than any internal quantum-field-theoretic construction, and could in principle be pursued in parallel with, rather than only after, progress on the other two questions.

8. Discussion

Having established the correspondence structure systematically (Sections 2–6) and catalogued its departures into a three-level hierarchy (Section 7), this section turns to the broader significance of that structure: how it relates to historical statements of the correspondence principle, to the methodology of effective field theory, to the mainstream comoving-versus- physical coordinate distinction already used in standard cosmology, and to the observational assessment carried out in a companion review. Each of the following subsections addresses one such point of contact.

8.1. Relation to Mainstream Comoving-Coordinate Cosmology

Before turning to these historical and structural comparisons, it is worth relating the local-versus-physical coordinate distinction that underlies every table in this paper to its counterpart in mainstream cosmology. Standard Robertson-Walker cosmology already distinguishes a comoving coordinate, fixed for an observer at rest with respect to the cosmic fluid, from the physical or proper distance, which grows with the scale factor; this distinction is used routinely, for instance, in computing luminosity and angular-diameter distances for supernova and baryon-acoustic-oscillation cosmology. What is specific to CSTR, and absent from this mainstream usage, is the further step, examined systematically in this paper, of substituting the comoving-versus- physical relation directly into the Lorentz transformation and every subsequent wave equation and field theory, rather than treating the expanding background as a fixed geometric arena within which ordinary special-relativistic physics occurs locally, as the mainstream treatment implicitly does. The correspondence tables of Sections 2–5 show that, at fixed cosmological time, this further step reproduces exactly what the mainstream implicit treatment already assumes; the multi-epoch departures of Table 6 are where the further step's genuinely additional content, if any, must be sought.

8.2. Relation to Bohr's Correspondence Principle

Bohr's original correspondence principle required that quantum predictions approach classical predictions in the limit of large quantum numbers, a limit in which the two theories' predictions converge asymptotically rather than coinciding exactly at any finite quantum number. The correspondence identified in this paper is of a different and, in one respect, stronger character: CSTR's single-epoch predictions do not merely approach those of ordinary relativity in some limit, but coincide with them exactly at every fixed epoch, with no residual $\tau(t_0)$-dependence whatsoever once expressed in the physical coordinate. The companion mathematical-structure review's group-theoretic argument is what accounts for this stronger form of correspondence: because the fixed-epoch transformations are isomorphic, not merely approximately similar, to the ordinary Lorentz group, exact correspondence follows as a mathematical consequence rather than as an approximation valid only in some regime.

A further historical comparison can be drawn with Ehrenfest's theorem, which shows that expectation values of quantum operators obey the classical equations of motion under quite general conditions, again a form of correspondence that holds as an identity of the formalism rather than as an approximation requiring a specific limit to be taken. The correspondence identified in this paper shares this exact, formalism-level character with Ehrenfest's theorem more than with Bohr's original, asymptotic large-quantum-number statement: just as the Ehrenfest theorem follows directly from the structure of the quantum-mechanical equations of motion for any state, the fixed-epoch correspondence of Tables 1–4 follows directly from the group-theoretic structure of the cosmological Lorentz transformation for any physical process confined to a single epoch, independent of any additional assumption about the state, process, or regime under consideration.

8.3. Relation to Effective Field Theory

A related comparison can be drawn with the general framework of effective field theory, in which a more fundamental theory reduces to a simpler, established theory upon integrating out or neglecting physics above some energy or length scale. CSTR's correspondence structure differs from this pattern in an important respect: the reduction to standard theory is not obtained by neglecting some small effect (as, for instance, higher-dimension operators are neglected in an effective field theory expansion), but by restricting attention to a single cosmological epoch, a restriction that is exact rather than approximate. The genuinely approximate step in the CSTR program, identified in a companion review, is instead the adiabatic treatment of quantities that vary slowly across different epochs (Table 6), which is a small-parameter expansion in $\hbar H(t_0)/(mc^2)$ in the ordinary effective-field-theory sense; the exact fixed-epoch correspondence and the approximate multi-epoch adiabaticity are thus two logically distinct forms of approximation at play in different parts of the theory, and this paper's tables are organized so as to keep the two separate.

This distinction can be sharpened with a brief analogy to the Wilson- coefficient language of effective field theory. In an ordinary effective field theory, higher-dimension operators are suppressed by inverse powers of a large mass or energy scale, and the correspondence with the low-energy theory improves smoothly as that scale is taken to infinity. In CSTR, by contrast, there is no analogous operator expansion in $\tau(t_0)$ that becomes exact only in some limit: the fixed-epoch theory is exactly the ordinary theory at every value of $\tau(t_0)$, not merely for $\tau(t_0)$ near some reference value. If CSTR is to be compared with the effective-field-theory paradigm at all, the more apt analogy is to a theory formulated with a spurion field frozen to a fixed, externally prescribed value within any single calculation: $\tau(t_0)$ plays a role structurally similar to such a spurion, exact and calculable at fixed t_0, with all of the theory's genuinely dynamical content residing in how the spurion's value changes from one calculation (epoch) to the next – precisely the multi-epoch sector catalogued in Table 6.

8.4. The Value of a Systematic Comparison

Presenting the correspondence structure systematically, as in Tables 1–6, offers two practical benefits beyond what is available from the five companion reviews considered individually. First, it makes visible that the fixed-epoch correspondence is not particular to any one sector of the theory (special relativity, quantum mechanics, quantum field theory, or general relativity) but is a single, unified consequence of the group-theoretic result established in the companion mathematical-structure review, applicable uniformly across every sector. Second, collecting the departures from correspondence into a single table (Table 6) makes clear that every open question raised across the five companion reviews is, in fact, a manifestation of the same underlying gap: the absence, at present, of a fully worked-out multi-epoch extension of the fixed-epoch theory. Closing that gap, rather than further elaborating the already-established fixed-epoch correspondence, is therefore the single most valuable direction for the continued development of CSTR identified by this synthesis.

Third, a systematic comparison of this kind provides a template against which any future extension of CSTR – to a new physical sector not yet considered in the five companion reviews – can be checked for consistency before detailed calculation. Given a proposed new application of the cosmological substitution rule, Table 5 suggests an immediate consistency requirement: the result should reduce, at fixed cosmological epoch, to the corresponding standard-theory result via the identical limit $\tau(t_0) \to \tau_0$ that governs every entry already tabulated. A proposed extension that failed this requirement – for instance, one in which some new fixed-epoch quantity retained explicit $\tau(t_0)$-dependence for reasons not analogous to the uncertainty-relation exception noted in Section 3 – would warrant particular scrutiny, since it would break the otherwise uniform pattern established across every sector considered in this paper.

8.5. Limitations of This Synthesis

Three limitations of this synthesis should be stated explicitly. First, this paper has not re-derived any result from first principles; every entry in Tables 1–7 is drawn from, and should be verified against, the specific companion review cited in its row. Second, the correspondence established here is, by construction, only as reliable as the underlying companion-review derivations themselves; if a future reanalysis of any companion review's specific calculation were to identify an error, the corresponding entry in this paper's tables would need to be revised accordingly, and this paper provides no independent verification beyond the cross-checking already performed, case by case, in Section 2–5's worked examples. Third, the three-level hierarchy of Section 7 organizes the currently known departures from correspondence; it does not, and cannot, guarantee that no further, currently unidentified departure exists in some sector of the theory not yet examined by any companion review. With these limitations noted, the synthesis presented here is intended as a faithful and complete organization of the correspondence structure established to date, rather than as an independent source of new physical results.

8.6. Relation to the Observational-Tests Review

The correspondence structure catalogued in this paper has a direct observational counterpart, developed in a companion review devoted to CSTR's observational status. Because every single-epoch prediction of the theory coincides exactly with the corresponding ordinary-relativistic prediction, as Sections 2–5 establish systematically, that companion review concludes that no single-epoch observation – however precise – can distinguish CSTR from ordinary relativity, and identifies genuinely multi-epoch observables, most concretely the redshift-drift or Sandage-Loeb test, as the only avenue toward a genuine observational test. The present paper's Table 6 and the observational review's catalog of open multi-epoch questions are, in effect, the same list viewed from two different angles: this paper organizes it as a matter of theoretical completeness, while the observational review organizes the identical content as a matter of empirical testability. Reading the two reviews together makes clear that theoretical and observational progress on CSTR's multi-epoch sector are not independent tasks but two aspects of the same outstanding problem.

8.7. A Consolidated Reference Table

For ease of reference, Table 7 combines the principal entries of Tables 1–4 into a single, comprehensive listing, organized by sector, with each row citing the specific companion review from which it is drawn.

Table 7 is deliberately organized so that the handful of rows marked "departs" or "open" stand out visually against the majority of rows exhibiting ordinary correspondence, reinforcing the paper's central finding: the overwhelming majority of CSTR's fixed-epoch content is standard physics in a relabeled coordinate, and the theory's entire distinctive content is concentrated in a small, clearly identifiable set of multi-epoch questions.

Sec.	Result (CSTR → standard theory)	Source
SR	Lorentz transf. → $SO(3,1)$ at fixed t_0	Yi (2020)
SR	Velocity addition → ordinary formula	Yi (2025)
SR	Rapidity composition → additive	Yi (2025)
QM	Klein-Gordon eq. → ordinary KG eq.	Yi (2020)
QM	Dirac eq. → ordinary Dirac eq.	Yi (2021)
QM	Yukawa potential → ordinary Yukawa	Yi (2021)
QM	Uncertainty relation: *departs*	Yi (2020)
QFT	KG-Maxwell Lagrangian → ordinary form	Yi (2021)
QFT	Canonical quantization → ordinary Fock space	Yi (2021)
QFT	Two-time propagator: *open*	Yi (2021)
GR	Schwarzschild horizon → $2GM/c^2$	Yi (2021)
GR	Kerr-Newman horizons → ordinary form	Yi (2021)
GR	PPN parameters → $\beta = \gamma = 1$	Yi (2021)
Math	Multi-epoch composition: groupoid	Yi (2025)
Obs	Single-epoch tests: uninformative	Yi (2025)

Table 7: Consolidated reference table combining the principal correspondence and departure results of Sections 2–7.

It is worth counting explicitly: of the fifteen rows in Table 7, twelve exhibit ordinary fixed-epoch correspondence with standard special relativity, quantum mechanics, quantum field theory, or general relativity, one records the epoch-dependent uncertainty relation as an explicit, established departure, and two record genuinely open questions (the two-time propagator and the multi-epoch groupoid structure, the latter of which is in turn the shared root cause of every other departure catalogued in Table 6, as discussed in Section 7's hierarchy). This twelve-to-three ratio, while not a precise measure of anything beyond the specific set of results selected for this particular consolidated table, is nonetheless a reasonably representative reflection of the balance found throughout the five companion reviews as a whole: the large majority of specific calculations carried out to date confirm correspondence, and a small, specifically identified minority establish or leave open a genuine departure from it. Any future companion review extending CSTR to a new physical sector should expect, on the basis of this pattern, a broadly similar ratio, with the bulk of new results confirming correspondence and only a small number, if any, contributing new entries to Table 6 itself.

9. Conclusion

This paper has collected the correspondence structure of the Cosmological Special Theory of Relativity, previously established piecemeal across five companion reviews, into a single, systematic set of comparison tables spanning special relativity (Section 2), relativistic quantum mechanics (Section 3), quantum field theory (Section 4), and general relativity (Section 5). We have identified the single limit responsible for every instance of this correspondence – evaluation at a fixed cosmological epoch, under which the cosmological substitution rule reduces to a physically unobservable coordinate relabeling – and shown that this correspondence is exact, as a consequence of the underlying group-theoretic structure of the transformation, rather than approximate in the manner of Bohr's original correspondence principle or of an effective-field-theory expansion. We have further collected, in a single table, the specific respects in which CSTR departs from this correspondence: the multi-epoch groupoid structure, the cosmological redshift and broken time-translation symmetry, the epoch-dependent uncertainty relation, and the open questions concerning the two-time propagator, cosmological particle creation, and the relation to the McVittie and Kottler solutions. Taken together, this synthesis is intended to serve as a single reference against which the correspondence structure – and the genuinely open content – of the entire CSTR program can be assessed. Organizing those departures further into the three-level hierarchy of Section 7 shows that only three specific, well-posed calculations – the two-time propagator, the Bogoliubov question, and the McVittie/Kottler comparison – remain as open theoretical questions in the strict sense, with the remaining departures from correspondence following as already-established deductive consequences of those three calculations' shared root cause, the multi-epoch groupoid structure itself.

The broader lesson of this synthesis is that the five companion reviews, though addressing distinct physical sectors and written for somewhat different purposes, are not five independent bodies of work but five facets of a single underlying mathematical structure: the fixed-epoch triviality of the cosmological Lorentz transformation established in the mathematical-structure review. Every instance of correspondence identified in Sections 2–5, and every departure from it catalogued in Section 7, can be traced back to that single structural fact and its multi-epoch complement. Future work extending CSTR to additional sectors of physics – whether further quantum-field-theoretic constructions, additional exact solutions of the Cosmological General Theory of Relativity, or entirely new applications not yet considered – should accordingly expect, on the basis of the pattern established here, to find the identical fixed-epoch correspondence with standard theory, with any genuine new content appearing only in the corresponding multi-epoch sector. Confirming or refuting this expectation in a new context would itself be a useful test of whether the correspondence structure identified in this paper is a truly general feature of the CSTR program or a pattern specific to the particular sectors examined so far.

In closing, it is worth restating plainly what has, and has not, been accomplished. This paper has not introduced any new physical result: every equation appearing in Tables 1–7 was already established in one of the five companion reviews cited throughout. What this paper has contributed is the observation, made explicit through systematic tabulation and a handful of worked examples, that these previously separate results share a single, common explanation – the group-theoretic triviality of the fixed-epoch cosmological Lorentz transformation – and a single, common exception – the multi-epoch groupoid structure and its three open calculational consequences. Whether this unifying observation proves useful going forward will depend on whether it succeeds in redirecting future theoretical effort on CSTR toward the three specific open calculations identified here (Section 7), rather than toward further, individually instructive but structurally repetitive demonstrations that some new sector of physics also happens to reduce to its standard form at fixed cosmological time.

Acknowledgments

The author thanks the editors and readers of the International Journal of Advanced Research in Physical Science, in which the component results reviewed here were originally published.

A. Glossary of Key Symbols

Symbol	Meaning
t_0	Cosmological time
$\tau(t_0)$	Cosmological expansion function
x^i, X, r_c	Physically expanded coordinate
x'^i, r'	Local (unexpanded) coordinate
ψ, ϕ, φ	Klein-Gordon wave function / field
γ^μ	Dirac matrices
A_μ, $F_{\mu\nu}$	Electromagnetic potential, field tensor
M, a, Q	Mass, spin, charge (CGTR sources)
$\beta_{\rm PPN}$, $\gamma_{\rm PPN}$	Parametrized post-Newtonian parameters
SR, GR, QM, QFT	Special/general relativity, quantum mechanics/field theory
CSTR, CGTR	Cosmological Special/General Theory of Relativity

Table 8: Glossary of symbols used in this paper.

B. Index of Companion Reviews

Table 9 indexes the companion review from which each principal comparison in this paper is drawn.

Companion review	Comparisons drawn from it
Consolidated review	Overall framework (Sections 2–5)
Quantum-theoretic review	Table 2, Table 3
CGTR review	Table 4
Mathematical-structure review	Section 6, Table 6 (groupoid)
Observational-tests review	Table 6 (open-question status)

Table 9: Index of companion reviews underlying each comparison table.

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