A SET OF SOLUTIONS OF THE EINSTEIN'S VACUUM EQUATION WITH A Λ-TERM

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ABSTRACT

The article proposes to use all 12 metric solutions of the Einstein's vacuum equation with the Λ-term within the framework of constructing a single system consisting of metric-dynamic models of one stable convex vacuum formation and one stable concave vacuum formation. This approach is a prerequisite for the creation of a Hierarchical Cosmological Model [16 – 31], aimed at developing the Clifford-Einstein-Wheeler program for the complete geometrization of physics.

Keywords: Einstein's vacuum equation, metric-solutions of the vacuum equation, hierarchical cosmological model.

INTRODUCTION

Einstein's vacuum equation (EVE) with the Λ-term has the form:

$$R_{ik} + \Lambda g_{ik} = 0, \qquad (1)$$

where g_{ik} are the components of the metric tensor of a smooth 4-dimensional space with the Riemann metric

$$ds^2 = g_{ik}\, dx^i dx^k;$$

$$R_{ik} = \frac{\partial \Gamma^l_{ik}}{\partial x^l} - \frac{\partial \Gamma^l_{il}}{\partial x^k} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{mk} \quad \text{is the Ricci tensor;} \qquad (2)$$

$$\Gamma^\lambda_{ik} = \frac{1}{2} g^{\lambda\mu}\left(\frac{\partial g_{\mu k}}{\partial x^i} + \frac{\partial g_{i\mu}}{\partial x^k} - \frac{\partial g_{ik}}{\partial x^\mu}\right) \quad \text{are the Christoffel symbols;} \qquad (3)$$

$T_{ik} = 0$ is the energy-momentum tensor of an external material source; (4)

$\Lambda = \pm 3/r_a^2$ is a constant (where r_a is the radius of the sphere); in cosmology, it is called the cosmological constant. If $r_a \to \infty$, then the value of Λ tends to zero ($\Lambda = 0$). (5)

The vacuum equation (1) is the quintessence of the experience acquired by the scientific community by the beginning of the 20th century, as it harmoniously intertwines, in a mysterious way, many fundamental epistemological principles underlying science [1 – 12]:

1] The principle of general covariance (i.e., the independence of the form of equations and invariants from the choice of coordinate system or reference frame; essentially, the tensor nature of equations);

2] The principle of coordinate invariance (i.e., the independence of the laws of physics from the choice of coordinate system). The difference between general covariance and coordinate invariance lies in the object to which the requirement of invariance is directed: the form of equations or the value of a physical quantity;

3] The principle of equivalence (i.e., local curvatures, movements, and accelerations are assigned local reference frames). The concept of "force" is replaced by motion by inertia in curved space-time;

4] The principle of the independence of the speed of light from the reference frame (i.e., the unification of space and time into a single space-time continuum with a metric of the form $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = 0$;

5] The principle of causality (i.e., any event can have a causal effect only on those events that occur after it, i.e., within a circle with a radius no greater than $l = cdt$ where dt is the time interval between events);

6] The principle of extremum of action (geodesic lines of curved 4-dimensional space are extremal). David Hilbert showed that the tensor $R_{ik} - \frac{1}{2}Rg_{ik}$, which is part of the fundamental equation of Albert Einstein's general relativity, can be obtained on the basis of the variational principle of least action from the Lagrangian $L = R\sqrt{-g}$ (where $R = g^{ik}R_{ik}$ is the scalar curvature (i.e., the Riemann invariant), $g = |g_{ik}|$ is the determinant of the matrix g_{ik});

7] Symmetry Principle (conditions of invariance, which imply conservation laws);

8] Relativity Principle (Eq. (1) involves only relative quantities, including time);

9] Quasilinearity: Despite its nonlinearity, Eq. (1) is quasilinear, since the highest (second) derivatives enter it linearly. This guarantees that the Cauchy problem (time evolution) is well-posed under smooth initial conditions;

10] Diffeomorphic Invariance (Gauge Freedom): If g_{ik} is a solution, then any solution obtained by applying an arbitrary change of coordinates (diffeomorphism) is also a solution. This means that the system contains 4 degrees of freedom (physical) out of the 10 components of the metric. The remaining 6 are gauge and connected degrees;

11] For $T_{\mu\nu} = 0$, the condition $\nabla_\mu T^{\mu\nu} = 0$ is trivially satisfied. Therefore, any test particles in such a space move along geodesic lines;

12] Absence of a distinguished scale (scale invariance): vacuum equation (1) contains only one fundamental constant, c the speed of light; the scale (radius) appears only when boundary conditions are introduced;

13] Vacuum equation (1) is invariant under the substitution $t \to -t$ (absence of an arrow of time);

14] Self-consistency: under the condition of global hyperbolicity, vacuum equation (1) guarantees that geodesic lines do not intersect chaotically, preserving causality;

15] At each point in Riemannian spacetime one can always choose a locally inertial system such that its metric becomes flat to the first order, and this property is built into Eq. (1) itself;

16] Vacuum equation (1) can be viewed as 10 conservation laws for identifying stable vacuum formations, since the covariant derivative on the right-hand side of Eq. (1) is equal to its ordinary derivative and is zero

$$\nabla_j 0 = \frac{\partial 0}{\partial x^j} - \Gamma^l_{ij} 0 - \Gamma^l_{kj} 0 = \frac{\partial 0}{\partial x^j} = 0. \tag{6}$$

Therefore, the covariant derivative of the left-hand side of Eq. (1) is also equal to its ordinary derivative and is zero (this is a consequence of the tensor nature of this equation)

$$\nabla_j [R_{ik} + \Lambda g_{ik}] = \frac{\partial [R_{ik} + \Lambda g_{ik}]}{\partial x^j} = 0. \tag{7}$$

17] The main property of the vacuum equation (1) is the paradox of emptiness. It represents a rigid, self-sufficient system that, in the absence of matter, does not exclude the existence of stable deformations of space-time and wave vacuum disturbances propagating at the speed of light and obeying strict geometric conservation laws.

Moreover, upon closer examination, it can be shown that Einstein's vacuum equation (EVE) (1) latently contains Aristotle's "Analytics" (i.e., formal logic), Euclid and Riemann's "Geometry," Newton's "Natural Philosophy," Kant's "Critique of Pure Reason," Hegel's "Phenomenology and Dialectics," and much more. That is, Eq. (1) condenses the wealth of human knowledge accumulated by the 20th century AD.

This combination of fundamental scientific and philosophical principles in a single Eq. (1) imparts a sense special significance to its solutions.

MATERIALS AND METHOD

1 Solutions of the Einstein's Vacuum Equation With the Λ-term

The solutions in spherical coordinates (t, r, θ, ϕ) of the Einstein's vacuum equation with the Λ-term (1)

$$R_{ik} + \Lambda g_{ik} = 0 \tag{1′}$$

are 10 general (Kottler's) metrics* [1 – 12]

$$ds_1^2 = \pm f(r)c^2dt^2 \mp \frac{dr^2}{f(r)} \mp r^2d\theta^2 \mp r^2 \sin^2\theta\, d\phi^2. \tag{8}$$

Of these, five metric-solutions have the signature (+ – – –):

I $$ds_1^{(+)2} = \left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} - r^2d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{9}$$

H $$ds_2^{(+)2} = \left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{10}$$

V $$ds_3^{(+)2} = \left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{11}$$

H′ $$ds_4^{(+)2} = \left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_a^2}\right)} - r^2d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{12}$$

i $$ds_5^{(+)2} = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2 \sin^2\theta\, d\phi^2; \tag{13}$$

and five similar metric-solutions with the opposite signature (– + + +)

H′ $$ds_1^{(-)2} = -\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} + r^2d\theta^2 + r^2 \sin^2\theta\, d\phi^2, \tag{14}$$

V $$ds_2^{(-)2} = -\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} + r^2d\theta^2 + r^2 \sin^2\theta\, d\phi^2, \tag{15}$$

H $$ds_3^{(-)2} = -\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} + r^2d\theta^2 + r^2 \sin^2\theta\, d\phi^2, \tag{16}$$

I $$ds_4^{(-)2} = -\left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} + r^2d\theta^2 + r^2 \sin^2\theta\, d\phi^2, \tag{17}$$

i $$ds_5^{(-)2} = -\,c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{18}$$

** The metrics-solution (9) – (18) are taken from various sources, including [1 – 12]. Furthermore, these metrics have been confirmed to be solutions of Einstein's vacuum equation (1) using several Artificial Intelligences (AI): Kimi K2.5, Claude Opus 5, Grok 4.5, Giga.chat, and Deepseek. Simply plug these metrics into any AI.*

Modern scientific papers typically use only one signature (+ – – –) or (– + + +), while solution metrics, such as (9) – (13), are considered separately for various cosmological problems (Table 1).

Table 1 – Parameters of metric-solutions for vacuum equation (1)

№	$f(r)$	Metric, Λ	Mass parameter, M	Scalar curvature, R
1	$1-r_b/r+r^2/r_a^2$	$\Lambda<0$ Schwarzschild – anti-de Sitter (Sch-AdS)	$M>0$	$-\frac{12}{r_a^2}$
2	$1+r_b/r-r^2/r_a^2$	$\Lambda>0$ Schwarzschild – de Sitter (Sch-dS) (with a negative cosmological constant)	$M>0$	$+\frac{12}{r_a^2}$
3	$1-r_b/r-r^2/r_a^2$	$\Lambda>0$ Schwarzschild – de Sitter (Sch-dS)** (standard Sch-dS metric or Kottler metric)	$M>0$	$+\frac{12}{r_a^2}$
4	$1+r_b/r+r^2/r_a^2$	$\Lambda<0$ Schwarzschild – anti-de Sitter (Sch-AdS) (repulsive central solution in anti-de Sitter space)	$M<0$	$-\frac{12}{r_a^2}$
5	1	$\Lambda=0$ Uncurved Minkowski space	$M=0$	0

*** For the first time, Friedrich Kottler wrote down a metric of the form (11) (the Kottler metric)*

$$ds^2_{Kottler}=\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)c^2dt^2-\frac{dr^2}{\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)}-r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (11')$$

in article [3], which was published in March 1918, after the publication of Einstein's general relativity.
In the limiting case: $r_a\neq\infty$ and $r_b=0$, the Kottler metric (11) becomes the de Sitter metric

$$ds^2_{de\ Sitter}=\left(1-\frac{r^2}{r_a^2}\right)c^2dt^2-\frac{dr^2}{\left(1-\frac{r^2}{r_a^2}\right)}-r^2(d\theta^2+sin^2\theta\, d\phi^2).$$

In another limiting case: $r_a=\infty$ and $r_b\neq 0$, the Kottler metric (11) becomes the Schwarzschild metric

$$ds^2_{Schwarzschild}=\left(1-\frac{r_b}{r}\right)c^2dt^2-\frac{dr^2}{\left(1-\frac{r_b}{r}\right)}-r^2(d\theta^2+sin^2\theta\, d\phi^2).$$

In the third case: $r_a=\infty$ and $r_b=0$, the Kottler metric (11) takes the form of the Minkowski metric for the uncurved space

$$ds^2_{Minkowski}=c^2dt^2-dr^2-r^2(d\theta^2+sin^2\theta\, d\phi^2).$$

Therefore, the metrics-solutions of the Einstein vacuum equation (9) – (12) and (14) – (17) are called, respectively, Schwarzschild – de Sitter metrics and Schwarzschild – anti-de Sitter metrics (abbreviated as Sch-dS and Sch-AdS metrics).

2 Combining Solutions of the Vacuum Equation

Einstein's vacuum equation (1) is a nonlinear equation, so the sums and differences of metrics (9) – (13) and (14) – (18) cannot be solutions of this equation.

On the other hand, Eq. (1) defines the metric-dynamic state of a certain vacuum region, and all its metric-solutions, such as (9) – (13), equally apply to this region. In other words, all solutions (9) – (13) with the signature (+ – – –) must belong to one system (specifically, to the stable convexity of the vacuum), and all solutions (14) – (18) must belong to the second, opposite system (specifically, to the stable concavity of the vacuum).

The sum of all 10th solution metrics (9) – (18)

$$ds_0^{(\pm)2}=\sum_{i=1}^{5}ds_i^{(+)2}+\sum_{j=1}^{5}ds_j^{(-)2}=0 \quad (19)$$

leads to the two zero metric

$$ds_0^{(\pm)2} = \pm 0 \cdot c^2dt^2 \mp 0 \cdot dr^2 \mp 0 \cdot r^2 d\theta^2 \mp 0 \cdot r^2 \sin^2\theta\, d\phi^2 = 0, \qquad (20)$$

which are the 11th and 12th trivial initial metric-solutions of the vacuum equation (1) [14].

Thus, it is obvious that a situation is possible when the total sum of solutions of the nonlinear Eq. (1) is also its solution, which further supports the idea that all 12 above-mentioned metric-solutions belong to a single system.

In the articles [16 – 31], under the general title "Geometrized Vacuum Physics Based on the Algebra of Signature", an attempt was made to combine all metric-solutions (9) – (20) of the Einstein vacuum equation (1) within a single system.

Averaging all 10 metrics (9) – (18) leads to complete mutual compensation, since their sum (19) is zero. Therefore, it is proposed to separately average metrics (9) – (12), which determine the structure of the vacuum convexity, and metrics (14) – (17), which determine the structure of the vacuum concavity.

The result of averaging, for example, metrics (9) – (12), is the metric

$$ds_\Sigma^{(+)2} = \frac{1}{4}\sum_{i=1}^{4} ds_i^{(+)2} = c^2dt^2 - g_{11}^{(+)}(r)dr^2 - r^2d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \qquad (21)$$

where

$$g_{11}^{(+)}(r) = \frac{1}{4}\left[\frac{1}{\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}\right]. \qquad (22)$$

We substitute the components $g_{ii}^{(+)}$ from the averaged metric (21) into the expressions for the components of the 4-vector of relative elongation of vacuum (47) in [18] (see also [15])

$$l_i^{(+)} = \frac{\Delta r}{r} = \sqrt{1 + \frac{g_{ii}^{(+)} - g_{ii0}^{(+)}}{g_{ii0}^{(+)}}} - 1, \qquad (23)$$

where the components $g_{ii0}^{(+)}$ are taken from the uncurved metric (13). As a result, we obtain

$$l_r^{(+)} = \frac{\Delta r}{r} = \sqrt{1 + \frac{g_{ii}^{(+)} - g_{ii0}^{(+)}}{g_{ii0}^{(+)}}} - 1 = \sqrt{g_{11}^{(+)}(r)} - 1 = \qquad (24)$$

$$= \sqrt{\frac{1}{4}\left[\frac{1}{\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}\right]} - 1,$$

$$l_t^{(+)} = 0, \qquad l_\theta^{(\pm)} = 0, \qquad l_\phi^{(\pm)} = 0. \qquad (25)$$

The graph of the function $l_r^{(+)}$ (24) with the conventionally accepted $r_a = 90$ and $r_b = 1.5$, which determines the relative elongation of the vacuum in the radial direction, is shown in Figure 1. From this graph it is evident that a practically hollow sphere (i.e., a closed spherical space, reminiscent of the de Sitter world) with strongly stretched edges is obtained, inside which there is a spherical cavity (a Schwarzschild nucleolus).

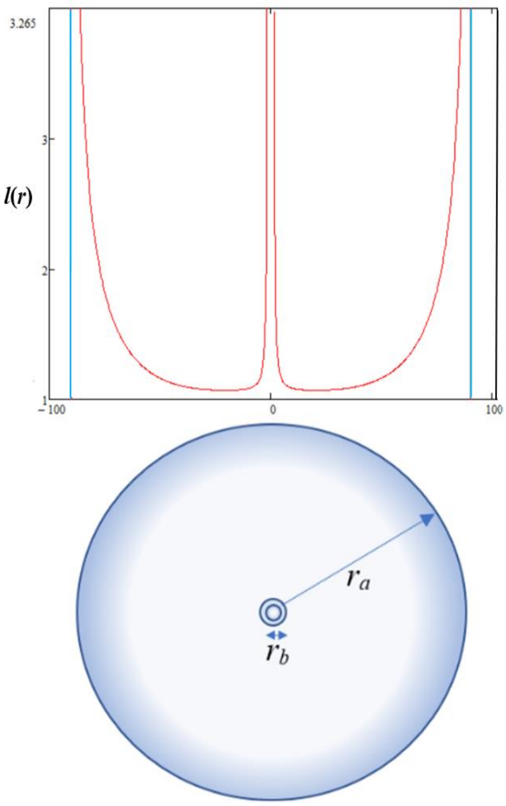

Fig. 1: Graph of the relative elongation function $l_r^{(+)}$ (24), which determines the relative elongation of a vacuum in the radial direction. Calculations were performed using MathCad 15 software.

We will call such a convex vacuum formation a Schwarzschild-de Sitter's "corpuscle." Performing similar operations with metric-solutions (14) – (17) with the opposite signature (– + + +), we obtain a similarly structured spherical vacuum concavity (i.e., a Schwarzschild - de Sitter's "anti-corpuscle").

The radii r_b and r_a are initially undefined. That is, vacuum equation (1), due to scale invariance, allows for any value, provided $r_b \ll r_a$. Therefore, Schwarzschild - de Sitter "corpuscles" (i.e., spherical vacuum convexities) and "anti-corpuscles" (i.e., spherical vacuum concavities) can be of any size.

This property of Einstein's vacuum equation (1) in combination with the use of all 16 possible signatures of 4-dimensional metric spaces (70) in [21]

$$sign(ds^{(a,b)2}) = \begin{pmatrix} (++++) & (+++-) & (-++-) & (++-+) \\ (---+) & (-+++) & (--++) & (-+-+) \\ (+--+) & (++--) & (+---) & (+-++) \\ (--+-) & (+-+-) & (-+--) & (----) \end{pmatrix} \tag{26}$$

allowed us to construct a Hierarchical Cosmological Model [21 – 27], within the framework of which the Universe is filled by an infinite number of "corpuscles" and "anti-corpuscles" of various scales with a fixed set of radii r_{ai} and r_{bj} (Figure 2).

Fig. 2: Illustration of a space filled with "corpuscles" and "anti-corpuscles" of varying scales

CONCLUSIONS

This brief communication attempts to draw the attention of the scientific community to the fact that considering all 12 possible metric-solutions (9) – (18) and (20) of Einstein's vacuum equation (EVE) (1) within a single metric-dynamic model (system) leads to quite meaningful results.

This approach allows us to construct a Hierarchical Cosmological Model (or "EVE project") [16 – 31]. Within this Hierarchical Cosmological Model, space is filled with stable, closed spherical (convex and concave) vacuum formations ("corpuscles" and "anti-corpuscles") of varying scales (see Figure 2).

REFERENCES

[1] Stephani, H.; Kramer, D.; MacCallum, M.; Hoenselaers, C.; Herlt, E. (2003). Exact Solutions of Einstein's Field Equations, 2nd ed. (Cambridge University Press, Cambridge).
[2] Griffiths, J. B. & Podolsk´y, J. (2009). Exact Space-Times in Einstein's General Relativity (Cambridge University Press, Cambridge, UK).
[3] Kottler, F. (1918). Uber die physikalischen Grundlagen der Einsteinschen Gravitationstheorie // Annalen der Physik, Vol. 56, pp. 401-462. doi:10.1002/andp.19183611402.
[4] Alexanian, M. (2026). Effective de Sitter-Schwarzschild Metrics // Armenian Journal of Physics, vol. 19, issue 2, pp. 107 -112, arXiv:2607.09707.
[5] Tenan, J. (2026). Foliations by constant spacetime mean curvature surfaces for asymptotically hyperboloidal initial data sets. arXiv:2607.02244.
[6] Cha, Ye Sle, & Marcus Khuri (2018). Transformations of asymptotically AdS hyperbolic initial data and associated geometric inequalities. General Relativity and Gravitation 50.1: 3.
[7] Chruściel, Piotr T., Daniel Maerten, and Paul Tod (2006). Rigid upper bounds for the angular momentum and centre of mass of non-singular asymptotically anti-de Sitter space-times. Journal of High Energy Physics 2006.11: 084-084.
[8] Ashraf Azman, M. (2026). Dark Energy Stars in Finch-Skea Spacetime with a Schwarzschild - (Anti)-de Sitter Exterior arXiv:2605.30398.
[9] Tullini, A. (2026). Boundedness and decay for the conformal wave equation in Schwarzschild - AdS under dissipative boundary conditions. arXiv:2604.02084.

[10] Friedrich, H. (1995). Einstein equations and conformal structure: existence of anti-de Sitter-type space-times. Journal of Geometry and Physics 17.2, pp. 125–184.
[11] Graf, O. & Holzegel, G. (2024). “Linear Stability of Schwarzschild-Anti-de Sitter spacetimes I: The system of gravitational perturbations”, arXiv:2408.02251.
[12] Moschidis, G. (2020). A proof of the instability of AdS for the Einstein-null dust system with an inner mirror, Analysis & PDE 13, no. 6, 1671–1754.
[13] Landau,L.D. & Lifshitz, E.M. (1971). The Classical Theory of Fields // Course of theoretical physics, V. 2 Translated from the Russian by Hamermesh M. University of Minnesota – Pergamon Press Ltd. Oxford, New York, Toronto, Sydney, Braunschwei, p. 387.
[14] Shipov, G. (1998). The Theory of Physical Vacuum. Moscow ST-Center, Russia ISBN 5 7273-0011-8.
[15] Sedov, L.I. (1994). Continuum Mechanics. Vol. 1. – Moscow: Nauka.
[16] Batanov-Gaukhman, M. (2023). Geometrized Vacuum Physics. Part I. Algebra of Stignatures // Avances en Ciencias e Ingeniería, 14 (1), 1–26, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-1-ano-2023-articulo-1 or viXra:2403.0035. Available in Russian: https://doi.org/10.24108/preprints-3113027.
[17] Batanov-Gaukhman, M. (2023).Geometrized Vacuum Physics. Part II. Algebra of Signatures // Avances en Ciencias e Ingeniería, 14 (1), 27–55, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-1-ano-2023-articulo-2/ or viXra:2403.0034. Available in Russian: https://doi.org/10.24108/preprints-3113028.
[18] Batanov-Gaukhman, M. (2023). Geometrized Vacuum Physics. Part III. Curved Vacuum Area // Avances en Ciencias e Ingeniería Vol. 14 nro 2 año 2023 Articulo 5, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-2-ano-2023-articulo-5/ or viXra:2403.0033. Available in Russian: https://doi.org/10.24108/preprints-3113032.
[19] Batanov-Gaukhman, M., (2024). Geometrized Vacuum Physics. Part IV: Dynamics of Vacuum Layers // Avances en Ciencias e Ingeniería Vol. 14 nro 3 año 2023 Articulo 1 https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-3-ano-2023-articulo-1/, and Preprints.org. https://doi.org/10.20944/preprints202310.1244.v3, or viXra:2403.0032. Available in Russian: https://doi.org/10.24108/preprints-3113039.
[20] Batanov-Gaukhman, M., (2024). Geometrized Vacuum Physics Part 5: Stable Vacuum Formations // Avances en Ciencias e Ingeniería Vol. 14 nro 3 año 2023 Articulo 2 https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-3-ano-2023-articulo-2/, or viXra:2405.0002. Available in Russian: https://doi.org/10.24108/preprints-3113040.
[21] Batanov-Gaukhman, M. (2024). Geometrized Vacuum Physics Part 6: Hierarchical Cosmological Model // Avances en Ciencias e Ingeniería Vol. 14 nro 4 año 2023 https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-4-ano-2023-articulo-3/, or viXra:2408.0010. Available in Russian: https://doi.org/10.24108/preprints-3113086.
[22] Batanov-Gaukhman, M. (2025). Geometrized Vacuum Physics Part VII: "Electron" and "Positron" // Avances en Ciencias e Ingeniería Vol. 15 nro 1 año 2024 Articulo 3, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-1-ano-2024-articulo-3/, or viXra:2409.0097 Available in Russian: https://doi.org/10.24108/preprints-3113132.
[23] Batanov-Gaukhman, M. (2025). Geometrized Vacuum Physics. Part VIII: Inertial Electromagnetism of Moving «Particles» //Advances en Ciencias e Ingeniería, Vol. 15 nro 2 año 2024 Articulo 1, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-2-ano-2024-articulo-1/, or viXra:2411.0086. Available in Russian: https://doi.org/10.24108/preprints-3113170.
[24] Batanov-Gaukhman, M. (2025). Geometrized Vauum Physics. Part IX: «Neutrino» // Avances en Ciencias e Ingeniería, Vol. 15 nro 3 año 2024 Articulo 1, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-3-ano-2024-articulo-1/ or viXra:2501.0059. Available in Russian: https://doi.org/10.24108/preprints-3113337.
[25] Batanov-Gaukhman, M. (2025) Geometrized Vacuum Physics. Part X: Naked «Planets» and «Stars» // Avances en Ciencias e Ingeniería Vol. 15 nro 3 año 2024 Articulo 2, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-3-ano-2024-articulo-2/, or viXra:2502.0139. Available in Russian: https://doi.org/10.24108/preprints-3113413.
[26] Batanov-Gaukhman, M. (2025). Geometrized Vacuum Physics. Part XI: Gravity And Levitation. Avances en Ciencias e Ingeniería, Vol. 15 nro 4 año 2024 Articulo 1. Publicada el agosto 7, 2025.https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-4-ano-2024-articulo-1/, or viXra:2504.0180 // Available in Russian: https://doi.org/10.24108/preprints-3113413.
[27] Batanov-Gaukhman, M. (2025). Geometrized Vacuum Physics. Part XII: Naked "Galaxies" - "Particles" of Dark Matter? //Avances en Ciencia e Ingeniería, Vol. 16 nro 1:1- 46, DOI: 10.65093/aci.v16.n1.2025.21 and viXra:2508.0158. Available in Russian: https://doi.org/10.24108/preprints-3113692.
[28] Batanov-Gaukhman, M. (2025). Geometrized vacuum physics. Part XIII: Connection with quantum mechanics // Avances en Ciencia e Ingeniería, Vol. 16 nro 2, 21–57. https://doi.org/10.65093/aci.v16.n2.2025.28 and viXra:2510.0084. Available in Russian: https://doi.org/10.24108/preprints-3113016.

[29] Batanov-Gaukhman, M. (2025). Geometrized vacuum physics. Part 14: "the Hydrogen Atom", viXra:2607.0064. Available in Russian: https://doi.org/10.24108/preprints-3115872.
[30] Batanov-Gaukhman M. (2024). Development of the Stochastic Interpretation of Quantum Mechanics by E. Nelson. Derivation of the Schrödinger-Euler-Poisson Equations. *Recent Progress in Materials* **2024**; 6(2): 014; 10.21926/rpm.2402014, or arXiv:2011.09901v10. Available in Russian: https://doi.org/10.24108/preprints-3113016.
[31] Batanov-Gaukhman, M. (2026). Analog of a Compact Calabi-Yau Manifold Based on the Algebra of Signatures. Int J Quantum Technol, 2(1), 01-19, DOI10.66311/3068-9090.02.01.12.