Exact Expression for Relativistic Kinetic Energy

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Abstract. In this paper, we will demonstrate that relativistic kinetic energy follows an expression that is just as precise as the non-relativistic one, that one is related to the other, and that a corpuscular topology can be derived from it.

1. Introduction

Relativistic kinetic energy can be expressed in a single term, and just as accurately, as non-relativistic kinetic energy. The analysis I am going to present for this purpose is based on the study of the relativistic dispersion relation (RDR) that I conducted as part of the article I am writing on kinetic energy and momentum in the context of the symmetrized wave packet (SWP) [1]. From this context emerges a toroidal topology and certain implications regarding particle speed, studied in relation to the Zitterbewegung (zitter) effect [2], which obviously applies to all the physical variables that comprise it. A topological configuration that, as we shall see, is supported by the result obtained, not so much because the particles are composed of two semitoroids attached to the two WPs of the SWP (as we postulate), as because they are composed of two halves, regardless of their nature or shape. Since the discussion is much more extensive and detailed, it seems appropriate to highlight this result here without accompanying it with issues that might obscure it or lead to unnecessary questions, while the rest of the ongoing work—which addresses all the structural and kinetic implications—can be better developed based on this progress.

2 Relativistic Kinetic Energy

Using the relativistic dispersion relation

$$E_T^2 = E_0^2 + (cp)^2, \tag{1}$$

and the usual energy balance,

$$E_T^2 = (E_k + E_0)^2 = E_k^2 + 2E_k E_0 + E_0^2 \tag{2}$$

we obtain

$$(cp)^2 = E_T{}^2 - E_0^2 = E_k^2 + 2E_k E_0 \rightarrow cp = \sqrt{E_k^2 + 2E_k E_0}. \tag{3}$$

The energy E_0 can also be written as

$$E_k = E_T - E_0 = E_0\left(\gamma - 1\right) \quad \rightarrow \quad E_0 = \frac{E_k}{\left(\gamma - 1\right)}, \tag{4}$$

which allows us to write Eq. (3) as

$$\begin{aligned} cp = \sqrt{E_k^2 + 2E_k E_0} &= E_k \sqrt{1 + \frac{2}{\left(\gamma - 1\right)}} \\ &= E_k \sqrt{\frac{\left(\gamma - 1\right)}{\left(\gamma - 1\right)} + \frac{2}{\left(\gamma - 1\right)}} = E_k \sqrt{\frac{\left(\gamma + 1\right)}{\left(\gamma - 1\right)}} \equiv \mho E_k, \end{aligned} \tag{5}$$

and we can set

$$\mho^2=\frac{(\gamma+1)}{(\gamma-1)}=\frac{\dfrac{1+\gamma^{-1}}{\gamma^{-1}}}{\dfrac{1-\gamma^{-1}}{\gamma^{-1}}}=\frac{1+\gamma^{-1}}{1-\gamma^{-1}}=\frac{(1+\gamma^{-1})(1+\gamma^{-1})}{(1-\gamma^{-1})(1+\gamma^{-1})}$$
$$=\frac{(1+\gamma^{-1})^2}{1-\gamma^{-2}}=\frac{(1+\gamma^{-1})^2}{(\upsilon/c)^2}=\frac{(1+\gamma^{-1})^2}{\beta^2}\quad\Rightarrow\quad\mho=\frac{(1+\gamma^{-1})}{\beta} \tag{6}$$

On the other hand,

$$E_k=E_T-E_0=E_0(\gamma-1)=m_0c^2\left(\frac{1}{\left(1-\upsilon^2/c^2\right)^{1/2}}-1\right)=\frac{1}{2}m_0\upsilon^2+\frac{3}{8}\frac{m_0\upsilon^4}{c^2}+\frac{5}{16}\frac{m_0\upsilon^6}{c^4}+\ldots, \tag{7}$$

which we can express as

$$E_k=\left(\frac{\upsilon}{2}+\frac{3\upsilon^3}{8c^2}+\ldots\right)p_0=\frac{1}{\gamma}\left(\frac{\upsilon}{2}+\frac{3\upsilon^3}{8c^2}+\ldots\right)p\equiv\hat{\upsilon}p, \tag{8}$$

This result would allow us, using Eq. (5), to write

$$cp=\mho E_k=\mho\hat{\upsilon}p\Rightarrow c=\mho\hat{\upsilon} \tag{9}$$

Using this last equation and Eq. (6), we obtain

$$\hat{\upsilon}=\frac{c}{\mho}=\frac{c\beta}{(1+\gamma^{-1})}=\frac{\upsilon}{(1+\gamma^{-1})}=\frac{\upsilon}{1+(1-\beta^2)^{1/2}} \tag{10}$$

from which it follows that,

$$E_k=\hat{\upsilon}p=\frac{c}{\mho}p=\frac{\upsilon}{1+(1-\beta^2)^{1/2}}p=\frac{p\upsilon}{1+(1-\beta^2)^{1/2}}=\frac{1}{1+(1-\beta^2)^{1/2}}(m\upsilon^2), \tag{11}$$

that is, the relativistic kinetic energy E_k (its characteristic power series) becomes an exact expression in terms of $\hat{\upsilon}$, which, as we can verify, turns out to be $E_k=½m\upsilon^2$ for $\beta=0$.

3 Corpuscular Topology

Equation (11) for E_k is relevant in that, as we can see, it yields an exact result (of the sum of the series itself), given that it connects this relativistic result to the form of the classical result, and given that the result obtained has implications regarding the topology of the particle. A result and topological implications that, let us not forget, stem from the RDR (supporting what we propose regarding the SWP [1] and zitter [2]), that is, it is the RDR alone that leads us to the topological conception through the analytical result, which cannot be understood except through the interaction of two elements, one of which (modulated by υ) has a modulating effect on the set. A set that, in a strict sense, as we can see in the following equation,

$$E_k=\frac{1}{1+(1-\beta^2)^{1/2}}(m\upsilon^2)=\frac{1}{(1+\gamma^{-1})}(m\upsilon^2)=\frac{1}{\left(\dfrac{1}{1}+\dfrac{1}{\gamma}\right)}(m\upsilon^2), \tag{12}$$

would be nothing more than the harmonic mean of those two elements, that is, the fixed value $\gamma_0\equiv\gamma(\upsilon_0)=1$ of one branch and $\gamma(\upsilon)$ of the other, on the total reference energy. Here we see that, in particular, for $\upsilon=\upsilon_0$, and $\gamma(\upsilon_0)=\gamma_0$ (in both branches), we would obtain the classical result, valid for non-relativistic speeds υ, as we mentioned earlier,

$$E_k=\frac{1}{\left(\dfrac{1}{\gamma_0}+\dfrac{1}{\gamma}\right)}(m\upsilon^2)=\frac{\gamma}{(\gamma_0+\gamma)}(m\upsilon^2)\quad\rightarrow\quad\frac{\gamma_0}{(\gamma_0+\gamma_0)}(m\upsilon^2)=½m\upsilon^2, \tag{13}$$

in which the harmonic mean coincides with the arithmetic mean.

The physical interpretation derived from the harmonic mean, in contrast to other statistical solutions, is that we are dealing with two different speeds over two equal segments. Obviously, when we speak of two equal segments associated with the particle, we are referring to two internal segments (this does not exist in the trajectory itself). Of which we can be certain is that this speed in the two branches has a topological character, that is, it is not that the particle moves at a certain speed—which it also does (as a result of the three-dimensional space created and as a way of interacting with others within it)—but rather that the particle has that speed "written" into it because that speed is the result of what the particle expresses regarding itself in each of its two branches (according to the zitter [2]).

4. Summary and Discussion

The purpose here has been to demonstrate that relativistic kinetic energy can be expressed with a single, exact term, just like non-relativistic kinetic energy, as well as to establish that the form of the term is due to corpuscular topology. Why this topology gives rise to that form in the term, how it operates internally within matter, and the rest of the evidently understudied issues of the RDR (such as the one discussed here) are precisely what justify the study of momentum and energy that we are developing within the aforementioned theoretical framework. Meanwhile, we see that the result achieved is beyond dispute and that it is significant both in and of itself and for what it implies. Although it is important, its true significance—both in and of itself and for what it implies—stems from what we have already presented in previous works, even though those works may be considered, under certain circumstances, to be of little importance, to have little impact, or to be out of step with the current state of physics. It could happen with this one, too, even though it seems crucial.

References:

[1] Cañete Mesa, R. (2023) Phasic Structure of the Standard Model. In *Cosmology Research - Addressing Current Problems with Astrophysics*, IntechOpen. DOI: 10.5772/intechopen.109384

[2] Cañete Mesa, R. (2026) The Zitterbewegung, Such as It Is, and Etiology of the Corpuscular Speed. *Journal of High Energy Physics, Gravitation and Cosmology*, **12**, 945-983. DOI: 10.4236/jhepgc.2026.122052