

A Relativistic Vector Field Theory of Gravitation
and a First Principles Approximation of
the Graviton

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This paper presents a Vector Gravitational Field Theory asymptotically equivalent to standard Minkowski spacetime in the limit of a classical vacuum. From such a theory we then derive a new definition of rest mass energy as well as an approximation of the quantum of gravitational action via a graviton particle of energy hf proportional to the gravitational charge quantified by matter-energy density and compute solar system graviton energy ranges.

We investigate the question of the quantum of gravitational action from relativity, classical mechanics, and quantum mechanics. We shall rely upon the Newtonian description of gravity and the relativistic vector framework henceforth developed to describe gravitating systems. The mathematical reasoning in this paper leads to a set of dynamical quantities relating the gravitational field quantified by geometric variables and matter-energy distribution. Finally we shall introduce the Hamiltonian of a graviton particle via a wave analogy and first principles deduction from the aforementioned dynamical theory.

1. Dynamical Variables of the Relativistic Gravitational Field:

We consider a system of 2 massive bodies of gravitational charges m . and M . respectively, where gravitational charge is measured in kilograms. Per the Galilean principle of equivalence stating that gravitation=inertia per all observations, and in agreement with our choice of units, the more general definition of the gravitational charge should be E/c^2 where E quantifies both rest mass, momentum, jerk, and accounts for all possible purely “inertial” or kinematic contributions to the gravitational charge in arbitrarily moving frames. Additionally, this convention allows for radiation to contribute to the field, since radiation and other forms of energy gravitate in General Relativity, which was confirmed by modern observations. Energy is such should be defined generally as:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} \quad (1)$$

Per the original derivation of Paul Dirac in 1928. Therefore the gravitational charge E/c^2 can be written more generally as:

$$E/c^2 = \frac{\sqrt{m_0^2 c^4 + p^2 c^2}}{c^2} \quad (2)$$

In deriving the relativistic field Hamiltonian, we shall first begin with the quantity m and M by the Newtonian convention, before generalizing it to the more empirically accurate and general notion of a E/c^2 gravitational charge, in agreement with relativity. We shall now consider Max Planck's 1906 definition of relativistic force $\bar{F}$ written as:

$$\bar{F} = \gamma \cdot \bar{F}_c \quad (3)$$

Where $\bar{F}_c$ is the classical force. Therefore a relativistic universal force law of gravitation can be written as:

$$\bar{F} = \gamma \cdot \frac{m_0 M_0 G}{R^2} \quad (4)$$

For kinetic energy we simply use the standard relativistic kinetic energy definition written as follows:

$$E_K = \gamma(m_0 c^2 - 1) \quad (5)$$

For potential energy we rely upon the Coriolis's Work-Energy Theorem (1829) written canonically in integral form as follows:

$$\int_{r=0}^R \bar{F} \cdot dR = W = dE \quad (6)$$

For an arbitrary choice of stationary linear coordinates in the rest frame of an object of mass m_0 moving along R . In agreement with this theorem, we integrate the relativistic gravitational force $\bar{F}$ over infinity with respect to r continuously. We obtain:

$$\int_{r=0}^R \bar{F} dr = \gamma \frac{m_0 M_0 G}{R} + C \quad (7)$$

for by equation (4), the factor $\gamma m_0 M_0 G$ does not vary differentially in this integration. We write this result by the convention of an arbitrary convenient choice of coordinates where $C = 0$ as follows:

$$E_p = \gamma \frac{m_0 M_0 G}{R} \quad (8)$$

Which is the Relativistic Potential Energy in the gravitational field written in the massive gravitational charge form. In the mass-energy gravitational charge form, the problem of the

two-body bias in the Newtonian Classical Mechanical bias arises and shall be discussed and resolved in the subsequent sections.

2. Time-Independent Hamiltonian of the Field:

Let there be an isolated physical system composed of two distinct physical sub-systems S_1 and S_2 containing arbitrary non-zero amounts of mass-energy-momenta quantified symbolically for convenience in the massive gravitational charge form m_0 and M_0 . We further qualify that, as earlier mentioned in section (1), the gravitational charge obeys the mass-energy equivalence relation $m = E/c^2$ where m is an equivalent mass quantity symbolizing the observed mass-energy-momentum distribution, which cannot be purely mass in any empirically observable situation in the first place. We substantiate this claim using the following argument: for instance, a zero Kelvin temperature in any physical body is impossible per the Heisenberg uncertainty principle, whereby even a stationary massive body will necessarily have non-mass energy contributions in addition to rest mass-energy, and therefore gravitational charge contributions from temperature alone in this instance, ad-absurdum suggesting the m term in both relativity and classical newtonian mechanics is only symbolic at best, and at any rate is insufficient in accurately representing the gravitational charge. Therefore in principle we should rather default to E/c^2 to represent the gravitational charge, but we are starting with m for compatibility with earlier theories from which we inherit those ideas, and from which we ought to derive the more general physical principles relevant to describing the gravitational field. Additionally for the Newtonian paradigm is built upon the study of 2 body systems, one ought to be careful in the generalization of the quantifier of a system's mass-energy-momentum distribution upon which the intensity of the gravitational field depends, for a great many scenarios involve a larger number of bodies, warranting a more general formalism.

Having clarified this choice, we begin with the classical definition of the time-independent Hamiltonian for isolated systems in agreement with the law of conservation of mechanical energy written as follows:

$$H(R, \bar{v}) = E_p + E_k \quad (9)$$

$$H(R, \bar{v}) = \gamma \frac{m_0 M_0 G}{R} + \gamma (m_0 c^2 - 1) \quad (10)$$

Where $\bar{v}$ is the linear velocity of a body of rest mass m_0 traveling inertially in a Minkowski coordinate spacetime quantified by R . In the rest frame of our system, this space is of spatial dimension one, where the time dimension is not specified explicitly for the Hamiltonian is time-independent. We now consider the case of a null Hamiltonian and obtain the following equation:

$$- \gamma (m_0 c^2 - 1) = \gamma \frac{m_0 M_0 G}{R} \quad (11)$$

Where the sign convention designates opposite directions.

Expanding this equation we obtain the following equation:

$$1 = m_0 c^2 + \frac{m_0 M_0 G}{R} \quad (12)$$

Suggesting that in general, for time-independent Hamiltonian, the difference between rest energy and potential energy is equal to a constant energy value, here symbolized by 1 in reference to relativistic kinetic energy. We replace 1 by E_0 the total rest energy of the physical sub-system in a gravitational field and thusly write:

$$E_0 = m_0 c^2 + \frac{m_0 M_0 G}{R} \quad (13)$$

Which we interpret as the rest frame Hamiltonian of a body of mass m_0 under the influence of a gravitational field quantified by a potential $\frac{m_0 M_0 G}{R}$. Asymptotically, as $R \rightarrow 0$, the total rest energy approaches infinity, implying that the gravitational field contributes to the rest energy of physical systems proportionally to the magnitude of the gravitational field strength. As such, there is $R \neq 0$, therefore there must exist a minimum distance value R_{min} we posit to be Planck's length defined as $\sqrt{\frac{\hbar G}{2\pi c^3}}$ in agreement with Heisenberg's uncertainty principle. Therefore the Planck length bounds the range of maximum mass-energy density for gravitating bodies. In the opposite asymptotic direction, that is as $R \rightarrow \infty$, the total rest energy of the system approaches $m_0 c^2$, corresponding to the limit of an empty Minkowski spacetime, suggesting close agreement with ordinary special relativity in non-gravitational cases. The equation can be written as a factor of rest mass in the following manner:

$$E_0 = m_0 \left(c^2 + \frac{M_0 G}{R} \right) \quad (14)$$

Whereby rest mass may be written as $\frac{E}{(c^2 + \frac{M_0 G}{R})}$, directly implying that gravitational field strength increases the rest mass of free falling physical systems. This is in fact to be expected from the Galilean equivalence principle, for in linear acceleration so called effective mass increases linearly according to classically inertial observers per the total relativistic energy equation, which does apply to a free fall scenario. In any case, by symmetry with special relativity, one simply represents the total rest energy of a body of rest mass m_0 in a given gravitational field characterized by a total classical gravitational charge $m_0 + M_0$ by E_0 above defined function of gravitational product $m_0 M_0$.

We notice that the total gravitational charge depends on the total rest mass m_0 of a gravitational sub-system undergoing free fall. In light of this fact and the Galilean equivalence principle, as well as the unit kg^2 of the gravitational product, we consider by convention that the gravitational charge corresponds on average to the total mass of the gravitational system. As such, we write the gravitational charge simply as M_{t0} defined as $\frac{\sqrt{s}}{c^2}$ including the Mandelstam variable s . For any matter distribution of N particles in an inertial system of coordinates with their respective individual linear 4-momenta $\bar{P}_n$, let the Mandelstam variable s be the total Minkowski inner product, that is to say the Minkowski square of the total 4-momentum of the matter distribution. Thus, we may write $s = P^2$. Furthermore, we define P as follows:

$$P = \left\| \left(\sum_{n=1}^N (\bar{P}_{1\mu} + \bar{P}_{2\mu} + \dots + \bar{P}_{(N-1)\mu} + \bar{P}_{N\mu}) \right) \right\|^2 \quad (15)$$

Therefore we may write:

$$E_0 = m_0 c^2 + \frac{M_{t0}^2 G}{R} \quad (16)$$

But we may also write:

$$E_0 = m_0 c^2 + \frac{\sqrt{s} G}{c^4 R} \quad (17)$$

Which is the more general formulation of rest energy, for it includes all inertial frames of motion per the momenta-distribution. We may also write on account of our definition of the gravitational charge M_{t0} the following expression:

$$E_0 = m_0 c^2 + \frac{E_{total}^2 G}{c^4 R} \quad (18)$$

Where E_{total} represents the total matter-momentum-energy distribution of the entire gravitational system at a time t in the rest frame of a body of classical vacuum mass m_0 and total rest energy E_0 . From such an expression, we must conclude that the rest energy of a particle is also determined by the local gravitational field.

3. Derivation of the Energy of the Graviton:

We attempt to derive the quantum of gravitational action via an analogy between gravitational phenomena and classical wave mechanics, on account of the fact that per General Relativity, gravitational energy propagates in the form of a wave. Therefore we postulate the following assertions:

- (i) All wave mechanical phenomena apply to gravitation.
- (ii) The Planck-Einstein photonic energy definition hf applies to the graviton.

Now let us consider the following situation: let there be a physical matter-energy system K of total gravitational charge $\frac{E_{total}}{c^2}$ as defined in section (2). The implicit consequence of our postulation (ii) is that the system, by analogy with classical wave mechanics, does have a resonant frequency determining the gravitational energy exchanges in the field. In classical wave mechanics the resonant frequency of any arbitrary physical system is defined as $\sqrt{\frac{k}{m}}$ where m is the gravitational charge. Therefore for our system K we may define the following frequency:

$$f = c \cdot \sqrt{\frac{k}{E_{total}}} \quad (19)$$

Where the classical mass term m has been replaced by $\frac{E_{total}}{c^2}$. This frequency determines in principle the maximum gravitational energy transfer of our system K. For the graviton should travel at the speed of light and therefore be massless, we shall quantize it by Planck's constant directly in agreement with the Planck-Einstein relation for photonic energy. Therefore the average energy E_g of a graviton emitted by system K is found via the following derivation:

$$c \cdot \sqrt{\frac{k}{E_{total}}} = \frac{E_g}{h} \quad , \quad (20)$$

$$E_g = hc \cdot \sqrt{\frac{k}{E_{total}}} \quad (21)$$

Which relates directly to the vacuum rest mass m_0 of a macroscopic gravitating body emitting gravitons of energy:

$$E_g = h \cdot \sqrt{\frac{k}{m_0}} \quad (22)$$

Where k is the force parameter in the parabolic potential energy equation. We write such an equation as:

$$E_p = \gamma \frac{1}{2} k R^2 \quad (21)$$

Per the Planck-derived energy correspondence relations found in section (1). To establish a complete wave-mechanical analogy with gravitation, we simply set the parabolic potential energy equation to the gravitational potential energy relation of section (1) and obtain:

$$\gamma \frac{m_0 M_0 G}{R} = \gamma \frac{1}{2} k R^2 \quad (22)$$

Solving for Hooke's constant we obtain:

$$k = \frac{2m_0 M_0 G}{R^3} \quad (23)$$

4. Numerical Predictions:

We are now in a position to calculate average graviton energies for all gravitational scenarios. We will in this section calculate using equation (21) an approximation of the average energy of a graviton emitted due the gravitation of Planet Earth and an approximation of the average energy of gravitons emitted by the Sun.

- a) In the case of Planet Earth, of rest mass M_0 being 5.9722×10^{24} kg, the average graviton will be found by this relation to have an energy of $3.36061296 \times 10^{-60}$ J. In this computation, we are assuming a gravitational constant of $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, a Planck's constant of $6.62607015 \times 10^{-34}$ J·s and Hooke's constant of $1.53624075 \times 10^{-28} \text{ kg s}^{-2}$ for an observer of mass m_0 on the order of 50kg sitting near the equator at the surface namely at radius $R = 6,378,137 \text{ m}$.
- b) In the case of the Sun, of rest mass M_0 being 1.989×10^{30} kg, the average graviton will be found by this relation to have an energy of $2.95002629 \times 10^{-51}$ J. In this computation, we are assuming a gravitational constant of $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, a Planck's constant of $6.62607015 \times 10^{-34}$ J·s and Hooke's constant of $3.94252537 \times 10^{-5} \text{ kg s}^{-2}$ for an observer of mass m_0 on the order of 50kg orbiting the earth, assuming energy was unchanged from the Sun's emission at surface radius $R = 6.957 \times 10^8 \text{ m}$.

Concluding Remarks:

We have laid out relativistic dynamical principles for gravitation and computed an energy for theorized graviton particles. Such energy ranges for hypothetical gravitons are orders of magnitude below the lowest energy photon ever measured in 2019 by Gely, M-F, et al [6], signifying the detection of solar system gravitons may pose a significant technical challenge to experimenters. However, gravitons emitted by violent cosmic events may show more promise in terms of detectability. Further studies in the direction of such notable cosmic events are required to improve odds of experimental detection. Alternatively, one may hypothesize that the graviton may simply be an extremely low energy photon obeying reacting proportionally weakly to electrical charge itself, explaining why such a particle was not yet observed. a hypothesis which might form the basis of subsequent papers in this direction.

Citations:

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 - a. Note: This work by Coriolis is where he introduced the concept of mechanical work and kinetic energy in a way that led to the work-energy theorem.
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