

Theoretical derivation of the spiral galaxy rotation curves obtained by Vera Rubin, without dark matter

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Abstract

In this work, it is demonstrated that dark matter does not exist. Through a strictly analytical derivation based on classical mechanics and the geometry of spiral galaxies, a formula is obtained that precisely reproduces the rotation curves observed by Vera Rubin. The results show that the flat behavior of stellar velocities in outer regions does not require invisible mass halos, but arises naturally from the proper application of fundamental gravitational laws to extended mass systems.

Keywords: Galactic rotation curves, Newtonian mechanics, stellar dynamics, Dark matter.

1. Introduction

Dark matter is one of the great enigmas of modern cosmology. From the time Fritz Zwicky predicted the existence of dark matter in 1933 to its apparent confirmation by Vera Rubin based on observed rotation curves in spiral galaxies the components of this dark matter have remained undiscovered. Postulating the existence of such matter to reconcile observations with the predictions of Newton's and Kepler's laws reveals a flaw in the theoretical methodology underlying Newton's laws of gravitation. Newton's law applies only to constant masses M , such as in planetary systems where the mass is quasi-constant (for instance, in the Solar System, the Sun accounts for approximately more than 99.60% of the total mass). In spiral galaxies, however, this mass increases with the galaxy's radius; consequently, the mass must be a function of r . The challenge lies in determining the correct formula for $M(r)$.

2. Mass of spiral the galaxies as a function of r

The gravitational field is spherically symmetric $v^2 = \frac{GM}{r}$; it does not depend on the shape of the mass M , but only on the radius r measured from the center of M . Therefore, the Lagrangian of that system is:

$$L = \frac{1}{2}\omega^2 r^2 m + \frac{GMm}{r} \quad (1)$$

$$\text{De (1)} \quad M_{(r,\omega)} = \frac{Lr}{Gm} - \frac{1}{2} \frac{\omega^2 r^3}{G} \quad (2)$$

Using equation (2), the mass M could be found, but there are two variables: ω and r .

$$\text{But} \quad \omega^2 = \frac{l^2}{r^4 m^2} \quad (3)$$

Substituting (3) into (2):

$$M_{(r)} = \frac{Lr}{m} - \frac{l^2}{2Grm^2} \quad (4)$$

Removing the dependence on m from (4)

$$l = vrm \quad \frac{l}{m} = vr = l_{(r)} \quad L_{(r)} = \frac{L}{m} \quad (5)$$

Substituting (5) into (4):

$$M_{(r)} = L_{(r)} \frac{r}{G} - \frac{l_{(r)}^2}{2Gr} \quad (6)$$

3. Rotation speed of spiral galaxies

$$v_r = \sqrt{\frac{GM_{(r)}}{r}} \quad [2] \quad (7)$$

Substituting (6) into (7):

$$v_r = \sqrt{L_{(r)} - \frac{1}{2} \frac{l_{(r)}^2}{r^2}} \quad (8)$$

Formula (8) yields the graph in Figure (1)

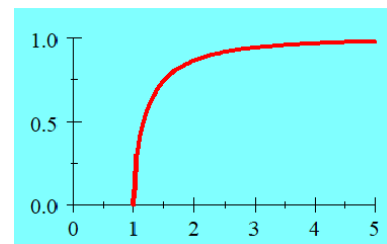


Fig. (1)

The graph in Figure (1) represents a case where the mass outside the galaxy's radius of rotation is constant, as shown in Figure (2).

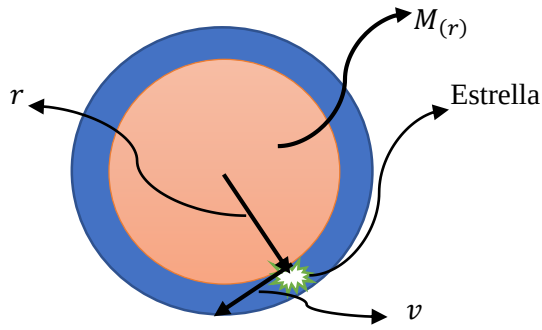


Fig. (2)

But the actual curve has bumps, as shown in Figure (3)

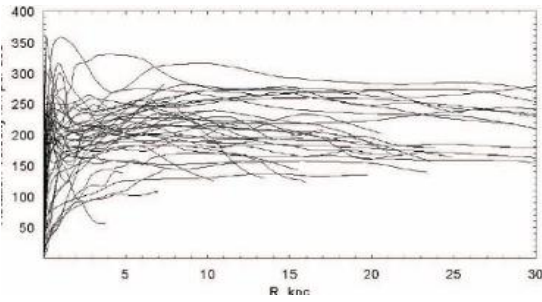


Fig. (3) [1]

The bumps are due to the mass outside the white circle not being constant.

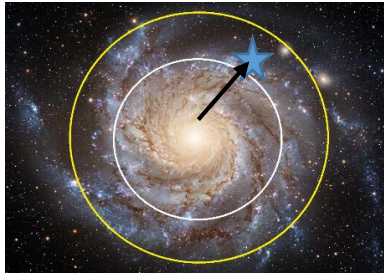


Fig. (4)

Using formula (7), one can determine the mass extending from the center of the galaxy to the star whose velocity has been measured.

$$M(r) = \frac{v^2 r}{G} \quad (9)$$

The influence of the mass located outside the radius r (blue) cancels out, as explained by Newton's shell theorem.

5. Initial conditions of the galaxy.

The problem with formula (6) is that we do not know L_r or l_r however, we can determine L_i and l_i if we know M_i and r_i , where M_i is the initial mass of the galaxy which, in the case of the Milky Way, is Sagittarius A*. Yet, we do not know the value of r_i the radius of the Sagittarius A* black hole, although we do know the periastron distance of the star closest to Sagittarius A*, which is S4714.

So:

$$M_i = 8.54 * 10^{36} \text{ Kg} \quad r_i = 1.88 * 10^{12} \text{ m}$$

$a = 1.26 * 10^{14} \text{ m}$ Where a is the semieje major axis?

With this data, one obtains v_i, l_i y L_i

$$v_i = \sqrt{G = M_i \left(\frac{2}{r_i} - \frac{1}{a} \right)} = 2.45 * 10^7 \frac{\text{m}}{\text{s}}$$

$$l_i = v_i r_i = 4.61 * 10^{19} \frac{\text{m}^2}{\text{s}}$$

$$L_i = \frac{1}{2} \frac{l_i^2}{r_i^2} + \frac{GM_i}{r_i} = 6.041 * 10^{14} \frac{\text{m}^2}{\text{s}^2}$$

$$M_i = \frac{L_i}{G} r_i - \frac{1}{2} \frac{l_i^2}{r_i}$$

$$M_r = \frac{L_i}{G} r - \frac{1}{2} \frac{l_i^2}{r} \quad (10)$$

Formula (10) is the same as (6), but expressed in terms of the initial conditions.

Substituting (10) into (7):

$$v_r = \sqrt{L_i - \frac{1}{2} \frac{l_i^2}{r^2}} \quad (11)$$

6. Speed of the Sun

$$r = 2.523 * 10^{20} \text{ m} \quad l_i = 4.61 * 10^{19} \frac{\text{m}^2}{\text{s}}$$

$$L_i = 6.041 * 10^{14} \frac{\text{m}^2}{\text{s}^2}$$

Using these data in (11):

$$v_r = 2.45 * 10^7 \frac{\text{m}}{\text{s}} \quad (12)$$

The most widely accepted value for the Sun's tangential velocity is $2.30 * 10^5 \frac{m}{s}$, which differs from (12). The difference arises because the mass distribution extending from the center of the Milky Way is not homogeneous; there are regions with lower mass density, such as the spaces between the spiral arms.

4. Conclusions

This model demonstrates that there is no breakdown in the laws of physics when transitioning from the planetary to the galactic scale. The apparent anomaly in the rotation curves observed by Vera Rubin is the direct result of treating the galaxy as a point-mass system (a Keplerian model) rather than a distributed-mass system. By mathematically integrating the mass-radius dependence, $M(r)$, within the framework of Newton's law of gravitation, a constant velocity profile is obtained; this explains galactic dynamics without the need for dark matter, thereby validating the universality and sufficiency of classical mechanics.

If mass is not constant, Keplerian behavior is not required.

The rotation curves of planetary systems represent a special case such as that of the Solar System where the mass M is practically constant.

5. References

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