14-page guide to *Switcheroo Gravity* & its companion papers

Warren D. Smith (warren.wds@gmail.com), October 2026. http://viXra.org/abs/2610.0071.

ABSTRACT. Smith, in some cases in collaboration with the AI CLAUDE, wrote 3-4 papers, totalling about 240-350 pages, on a framework for new theories of gravity. These theories can unify Milgrom's MOND with Einsteinian general relativistic principles, in a potentially very mathematically beautiful, natural-looking, and surprisingly near-unique way. Crucially, general relativity already contains a mathematical object – the *Lanczos tensor* L_{abc} – sitting unused since 1962. Using it changes what gravity theories are possible and what they can accomplish. This document attempts to summarize top highlights of those 240-350 pages in only 14.

Contents. 1. Lanczos tensor and MOND · 2. Presumed need for something MOND-like to permit gravity to agree with observation · 3. Two impossibility theorems · 4. Lanczos tensor as unused gravity ingredient · 5. Lanczos↔acceleration relationship · 6. Switcheroo gravity · 7. Horizon detectors · 8. Geometry that is not based on the metric · 9. Paper II, and why it is there · 10. Status: what is proved / not · 11. Possible ways to kill this · 12. How the papers were written · 13. Some references

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The papers in chronological order with approximate page counts:

I. Warren D. Smith: Turing-Unsimulability, and instantaneous & sustained Cosmic-Censorship-falsity, for Einstein-Vacuum General Relativity (2025). 91 pages. Not really part of the Switcheroo/Lanczos theoretical framework, but motivated II.
II. Warren D. Smith: Mostly-Destructive Consequences of Non- and Nowhere-Analyticity of the Spacetime Metric; and new "2-sided Mirror Paradigm" hypothetical grandiose universe picture (2026), 40 pages.
III. Warren D. Smith & CLAUDE: Switcheroo and Lanczos Gravity: MOND done right (2026). The flagship paper. 163 pages.
IV. Warren D. Smith & CLAUDE: Nonmetric foundations for differential geometry; and complex and bicomplex reformulations in the 4D case (2026). 49 pages.

The present fifth paper summarizes the other four (mainly II+III+IV) with little/no new content. All five papers were entirely written by Smith, *except* about 50% of IV was by CLAUDE.

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1. Lanczos tensor and MOND

In 1962 Cornelius Lanczos showed that the Weyl curvature tensor W_{abcd} of any four-dimensional spacetime can be generated by differentiating a three-index potential L_{abc}, analogously to the way differentiating the electromagnetic four-potential A_a can yield the Maxwell field tensor F_{ab}. This theorem about four-dimensional geometry has remained unused in general relativity ordinary practice: nothing in Einstein's

equations refers to L_{abc}, and it has found little/no use in physical theory.

Papers II-IV claim (or hope) that disuse was a *mistake*. Four consequences follow from repairing it:

1. **Gravity theories that imitate MOND become possible without new fields.** Uniqueness theorems started by Einstein had indicated GR(Λ) – general relativity with cosmical constant Λ – is the *unique* gravity theory obeying certain desiderata. III proved impossibility theorems stating that no theory built from curvature tensors, the metric tensor g_{ab}, and their derivatives can reproduce the acceleration-thresholded behavior Milgrom's MOND needs. Those theorems have a loophole L_{abc} fits through. Nothing new is postulated: no new particles, fields, scalars, or vectors – only an object 4D geometry already supplies.
2. **The relevant invariant is not merely analogous to gravitational acceleration – in the simplest cases it *is* gravitational acceleration.** E.g. for the Schwarzschild metric and a Lanczos tensor L_{abc} built by a recipe below, $L_{abc}L^{abc}$ equals four-thirds of the squared acceleration of a static observer, exactly, at each radius.
3. **Spacetime points can be given a way to know they sit on a black-hole horizon.** Event horizons are defined non-locally – nothing local distinguishes a point on one – and III proves that, for general horizons, no invariant built from curvature alone can detect one. But $L_{abc}L^{abc}$ can be infinite exactly on horizons and we exhibit tensor examples (e.g. the just-mentioned Schwarzschild L_{abc} evidently does this). We call these **horizon detectors**. We conjecture Lanczos tensors well-behaved in the "accessible region" (outside horizons) *generically* detect horizons; that is the subject of a genericity theorem.
4. **Differential geometry does not need to be founded on the metric.** E.g. the pair $\{L_{abc}, R_{ab}\}$ determines the metric g_{ab} and hence everything else. Building physics on that pair rather than on g_{ab} is not a mere reformulation, but a strictly larger structure.

None of this requires believing anything about dark matter. Points 3 and 4 are statements about geometry which would remain interesting if MOND were refuted tomorrow.

2. Presumed need for something MOND-like to permit gravity to agree with observation

Galaxies rotate too fast. Beyond the visible edge of spiral galaxies the orbital speed of gas clouds stays roughly flat a radius r increases, instead of falling like $r^{-1/2}$ as Newtonian gravity applied to the visible matter demands. This has been known since Rubin & Ford's 1970 work on Andromeda and Bosma's and others' work through the 1970s. The two main ideas to resolve this theory↔observation disagreement were:

Dark matter supposes each galaxy sits in a halo of invisible non-baryonic particles arranged so as to produce the observed rotation curve. That works, in the sense that presumably a halo can always be hypothesized that will yield a fit. But: the particles have never been detected despite forty years of increasingly sensitive searches; different halo shapes are required to fit each galaxy; and those shapes seem to disagree with prior expectations based on numerical simulations.

MOND: Milgrom 1983's *mo*dified *N*ewtonian *d*ynamics instead supposes that Newton's law *changes* below a threshold acceleration $a_0 \approx 1.2\times10^{-10}$ meter/sec^2 (number extracted from ten nearby galaxies by Begeman,

Broeils, Sanders 1991). With that one constant, and one fitted mass-to-light ratio per galaxy, MOND predicts rotation curves more successfully than dark-matter halo fitting. But in terms of well-accepted theoretical principles, MOND is a disaster. It is a modification of Newtonian dynamics, not a relativistic theory, and the top two relativistic completion attempts for it – Bekenstein's TeVeS and Moffat's MOG – introduce extra scalar and vector fields which incur major problems of their own, plus philosophically ludicrously propose new invisible kinds of matter (the extra fields) in order to avoid proposing new invisible kinds of matter. Paper III's §4 argues both are simply wrong. And even if some theory like them were not wrong, it still would be highly non-unique, posing great problems for comparison versus observation.

My personal perception is: too much experimental evidence supports *both* MOND and GR(Λ) to permit discarding either. So we need some way to marry them enjoying the virtues of both without their deficiencies.

Paper III points out a possible **third route:** keep Einstein's framework and criteria for "an acceptable gravity theory," but seek an overlooked ingredient already inside it.

3. Two impossibility theorems

The natural first thought is to make Einstein's cosmical constant Λ, or Newton's constant G, depend on the local curvature. Then gravity could behave differently in the low-acceleration outskirts of a galaxy than near its center, with no new fields at all. But this does not work.

The obstruction. Any relativistic theory imitating MOND must be able to tell whether the local gravitational acceleration is above or below a_0. So there must exist some tuple $\vec{F}$ of scalar invariants from which the gravitational acceleration can be computed.

Initial impossibility theorem. *The Schwarzschild metric's* ***gravitational acceleration*** *in is not deducible from any tuple $\vec{F}$ whose components are scalars built algebraically from the metric and curvature tensors.*

The proof is short enough to explain here. For Schwarzschild with mass M, every member of the Carminati-McLenaghan-Zakhary algebraically complete set of curvature invariants is either identically zero or a function of the single combination M^2r^{-6}. But the acceleration felt by an observer firing rockets to stay at fixed radius r is

$$\text{acceleration} = -GMr^{-2}[1-2GMc^{-2}r^{-1}]^{-1/2},$$

which depends on M and r separately, *not* only through M^2r^{-6}. Any two Schwarzschild points with the same invariants but different (M,r) therefore have different acceleration, and no function of the invariants can distinguish them. With more work, III extended the theorem to show that for certain spherically-symmetric spacetimes no set of invariants built from covariant *derivatives* of any order, can work.

Second impossibility theorem: horizon detection. For certain spherically-symmetric metrics, III showed no scalar built from the metric and curvature and any finite set of their derivatives can be finite outside the horizon and infinite on it. This is unsurprising – an event horizon is defined by whether a future-directed light ray can ever escape to infinity, which is a *non*local statement; there is nothing locally special about the geometry there. We will dodge this "no-go theorem" in §7.

Lanczos loophole. Both theorems assumed the ingredients were the metric, curvature tensors, and their derivatives. The Lanczos tensor is outside that class. As an *antiderivative* of curvature, it can carry information derivatives throw away – an integration constant, in the simplest case; a great deal more, here.

Don't use Lanczos? Then our impossibility theorems explain why forty years of relativistic MOND attempts all had to add hypothetical new fields to physics.

4. Lanczos tensor as unused gravity ingredient

Definition. A tensor L_{abc} is a **Lanczos tensor** for a spacetime if antisymmetric in its first two indices, $L_{abc}+L_{bac}=0$, has vanishing cyclic part $L_{[abc]}=0$, and generates the Weyl tensor through the **Lanczos→Weyl equation**. Additionally we shall always demand **tracelessness**, $L_{ab}{}^{a}=0$, thus removing four arbitrary degrees of freedom that carry no information. Traceless L_{abc} have 16 linearly-independent components at each point. If we define the antisymmetrized covariant derivative $K_{abcd}=L_{ab[c;d]}$ and let its double dual be $*K*_{abcd}$, then the Lanczos→Weyl equation is

$$W_{abcd} = K_{abcd} + K_{cdab} - *K*_{abcd} - *K*_{cdab}.$$

Notation: square brackets around n indices denote antisymmetrization *including* the factor $(n!)^{-1}$, so that $L_{ab[c;d]}$ means $(\nabla_d L_{abc} - \nabla_c L_{abd})/2$.

"Lanczos's theorem" is that such an L_{abc} exists for every 4-dimensional spacetime. If defined as above via tensor duals it exists *only* in four dimensions. Edgar and Höglund generalized the construction to avoid duals and proved it fails for dimension≥7; and in dimensions≤3 automatically $W_{abcd}=0$. For dimensions 5 and 6 the status is different and often misstated: E&H proved nonexistence for some Weyl *candidates*, but not for actual Weyl *curvature* tensors. That gap is still open. Paper III's numerical investigations find that for generically-curved five- or six-dimensional metrics the Weyl–Lanczos operator is surjective even onto candidates – and found no metric, within a large collection, whose actual Weyl tensor failed to admit a local Lanczos potential. So: Lanczos tensors might well exist also in dimensions 5 and/or 6. However, we were able to prove that **Superpotential→Lanczos→Weyl "chains"** can fail to exist in every dimension≠4; and the prior sort of numerical investigation found evidence this chain exists locally in a full-measure subset of locations on almost all 4-metrics. If so, the "Lanczos superpotential" truly is a uniquely 4D phenomenon.

Gauge freedom. L_{abc} is nonunique – the Lanczos→Weyl equation has a large solution space, in rough analogy with the gauge freedom of the electromagnetic 4-potential A_a. This is simultaneously the programme's greatest asset and top unanswered question. Asset, because it enables seeking solutions with useful properties; open problem, because physical theories that make unique predictions cannot depend on a gauge choice. Paper III's section 19 proposes several solutions to that problem – but does not know which is right – e.g: restriction to certain gauge-subgroups, or using the gauge freedom to optimize something. But the mathematically slickest option seems to be to make the fundamental object be:

The superpotential. A symmetric tensor $P_{ab}=P_{(ab)}$ is a **Lanczos superpotential** if

$$L_{abc} = P_{c[a;b]} \quad = \quad (\nabla_b P_{ca} - \nabla_a P_{cb})/2$$

is a valid Lanczos tensor. Superpotentials compress gauge freedom – L_{abc} has sixteen linearly independent components but a symmetric two-index object (P_{ab} or the metric g_{ab}) has ten, so a P_{ab}-based theory might not have any more gauge freedom that GR already had. Conjecturally superpotentials always exist; we have found them, at least locally, in every case we tried; and debunked widespread wrong belief there was a non-existence proof (see §10).

One recipe that works surprisingly often. Take the metric, reverse the sign of a diagonal n×n submatrix (n∈{1,2}), and use the result, times a constant, as P_{ab}. This **sign-flip trick** produces valid trace-free Lanczos tensors for Reissner-Nordström, Kerr, Kerr-Newman, Kerr-Newman-de Sitter, Ernst, Melvin, Barriola-Vilenkin, Tolman IV and an inhomogeneous pp-wave, but fails for Gödel's universe. It is surprising that such a crude prescription should work at all, and we lack a full understanding of when it does.

5. Lanczos↔acceleration relationship

Take Kottler's generalization of the Schwarzschild metric, allowing nonzero Λ. Build P_{ab} by the g_{tt} single-sign-flip recipe; then form $L_{abc}=P_{c[a;b]}$, and remove its trace. Let A^u be the four-acceleration of a static observer who fires rockets to stay at fixed r and let c=lightspeed. Then

$$\mathbf{c^4\, L_{pqr}L^{pqr} \;=\; (4/3)A_uA^u}$$

exactly, at every radius outside the horizon.

The impossibility theorem of §3 said that no curvature invariant can deliver the gravitational acceleration. But a Lanczos invariant delivers it – and not just for Kottler; paper III shows this equation also is valid for general weak-field scenarios, general static perfect-fluid scenarios, and more. These cases alone already should suffice for all galaxy-simulation uses. An object that equals the squared gravitational acceleration of the natural observer family is exactly what Milgrom MOND wants to threshold on, and has no known competition – i.e. no rival invariant, not involving Lanczos tensor, is known.

What about the rotating Kerr metric? A modification/generalization of the above equation still works. Define the *complexified* Weyl and Lanczos tensors via $\mathring{W}_{abcd}=W_{abcd}+iW^*_{abcd}$ and $\mathring{L}_{abc}=L_{abc}+iL^*_{abc}$, where the circle underneath the W or L (as in $\mathring{W}$) means "complexified." These objects exist only in 4 dimensions because defined using tensor dualization (indicated with *) for an antisymmetric index-pair. Writing P^hP_h for the squared acceleration of a "distinguished observer," the flagship paper's §15 establishes (stated in units with c=G=1)

$$[\mathring{L}_{abc}\mathring{L}^{abc}/2 - (4/3)P^hP_h]^2 = \mathring{W}_{abcd}\mathring{W}^{abcd} / 96$$

The correction term, i.e. right hand side, is zero if Kerr is not rotating (and $\mathring{L}_{abc}\mathring{L}^{abc}/2=L_{abc}L^{abc}$ and $L^{abc}L^*_{abc}=0$ in that case). It becomes relatively negligible both near the horizon and far from it. Such complexified and bicomplexified tensors are the topic of part II of the paper IV.

6. Switcheroo gravity

Main hypothesis. Switcheroo gravity – SG – postulates that, like ordinary matter, the quantum vacuum has *phases*. Water has ice, liquid and steam; tin has grey and white solid phases; nitrogen above 115 GPa converts to a covalently bonded solid. The standard model's Higgs vacuum is believed to be the survivor of a phase transition. In SG different phases carry different values of Einstein's cosmical constant Λ, and that the transitions (at least, the ones of interest for Milgrom MOND) are triggered not by temperature or pressure but by a tuple $\vec{F}$ of curvature- and/or Lanczos-related scalar invariants, one of whose members is $L_{abc}L^{abc}$.

As you move outward through a galaxy, $L_{abc}L^{abc}$ falls, crosses the threshold $(4/3)(a_0)^2$ whereupon (SG postulates) Λ "switches" to a different value when you cross that surface – a stairstep rather than smooth ramp. By §5 that threshold can be made to happen at the exact gravitational acceleration a_0 MOND wants.

What is and is not postulated. SG adds no field to general relativity. It changes no equation. It merely asserts that the constants Λ in Einstein's field equation take different values in different regions, with the region boundaries determined by an invariant 4D geometry already provides. Compare: dark matter postulates one or more new particle species; MOND postulates a new force law; TeVeS and MOG postulate new scalar and vector fields. Therefore SG is more parsimonious than any of those other hypotheses. Whether SQ is *correct* is a separate question, addressed in §11.

SG does *not* claim to be the "final theory" of physics, nor even merely the final gravity theory. That final theory presumably would know "quantum gravity" and also would *predict* the "vacuum phase diagram." SG is nonquantum and takes the quantum vacuum's phase diagram as *input* – you'd determine it via fitting computational models to observational datasets. However, SG hopefully is a *better* nonquantum gravity theory, capable of matching observations all previous theories could not; and *more friendly* to future quantization attempts. E.g. if SG is correct, then if and when astronomers determine the vacuum's phase diagram, that should be very helpful to people trying to deduce the laws of quantum gravity (as opposed to the present situation: no guidance whatsoever from experiment).

SG postulates that the metric g_{ab}, the Lanczos tensor L_{abc}, and the tuple $\vec{F}$ of scalar invariants that form the domain for the vacuum's phase diagram, all are **continuous** functions of space and time.

SG's Λ-changing phase transitions are postulated to involve ***zero* "latent heat."** [This is *not* analogous to water boiling or freezing, but *is* analogous to the normal↔super transition in BCS superconductors such as Al and MgB_2, which causes electrical resistivity and magnetic permeability both (in zero external magnetic field) to discontinuously drop to zero. Iron's ferromagnetic-ordering "Curie point" transition at ≈1044°K, and chromium's antiferromagnetic-ordering "Neel point" transition at ≈311°K (both associated with "corners" or "cusps" in plots of functions of magnetic-susceptibility versus temperature) also exhibit zero latent heat. Relatedly: whenever one crosses a "switcheroo surface" in vacuum to enter a different-Λ region of spacetime, there is *not* an infinitesimally **thin layer of "mass"** distributed on that surface to enable discontinuous curvature changes. Einstein's field equation shows that the Einstein curvature tensor indeed discontinuously changes when Λ jumps, so *geometrically* speaking such a thin-mass-layer can effectively exist caused by the vacuum phase-transition; but no mass made of any sort of *physical* particle is there, even if prior gravity interpretations would have required one.

Near-uniqueness. An obvious worry is: once you allow nonconstant Λ to depend on invariants, maybe you can fit anything, whereupon the theory might explain nothing. or anyhow be very nonunique. Paper III's §16 asks which *other* scalar invariants could serve as members of $\vec{F}$. The answer is: remarkably few. Almost all

the standard curvature-based candidates are ruled out by showing they could not meet SG's **demand that $\vec{\mathbf{F}}$ always be *continuous*** across switcheroo surfaces. The shortness of the list of survivors causes SG to be surprisingly near-unique. Einstein (and followers') already gave theorems that GR(Λ) was the unique theory of gravity obeying certain lists of desiderata. Those arguments now show SG is the unique class of theories of gravity allowed by those postulates plus our new postulates about the nature of Λ-transitions.

Actually paper III proves the existence of a wider class of theories it calls **"Lanczos gravity" (LG)**, which modify Einstein's field equation with Lanczos-involving terms. LG theories are hoped to be able to provide SG-like (MOND-like) effects but wihout need for discontinuities; and might be able to abolish the "time machine paradoxes" in, e.g, Gödel's universe. However, *this overview* will not discuss LG. My reason for initally focusing on SG is not because LG seems less likely to be true; but merely because SG is more accessible to intuitive understanding.

"Mass" change. Is Einstein's Λ is far too small to affect galactic dynamics? A switcheroo does not merely change Λ while holding everything else fixed: matching two solutions across a transition surface forces other parameters, including the Schwarzschild/Kottler mass parameter, to change too. If two Kottler (or Reissner-Nordström-deSitter) metrics are glued at a common spherical switcheroo surface with circumference $2\pi r_{glue}$, then continuity of the metric forces their mass-parameters M_{in} and M_{out} to be related to their "switching" Einstein cosmological constant values Λ_{in} and Λ_{out} via

$$M_{in}-M_{out} = [\Lambda_{out}-\Lambda_{in}]\ (r_{glue})^3\ c^2G^{-1}\ /\ 6.$$

(*Switcheroo*'s §11 also works out switcheroo surfaces for Reissner-Nordström and Kerr-deSitter.) So the effective mass inferred by a distant observer (who assumes the metric is pure Kottler) does *not* equal the mass inferred by a closer observer. This is similar to how, in Moffat's MOG (attempt to create a relativistic MOND) the law of attraction (at least in the simplest approximation) arises from a potential of the form

$$-[A + (B\text{-}A)\ \exp(-r/\ell)]\ /\ r$$

(where ℓ is the Yukawa length scale for MOG's hypothetical vector field) which behaves like -B/r when $0<r\ll\ell$ and like -A/r when $r\gg\ell$, transitioning between two Newtonian forms of gravity laws. MOG's transition is continuous, unlike switcheroo and MOND which involve *dis*continuous jumps A↔B at the Milgrom threshold acceleration, but they act *approximately* the same. Milgrom's MOND involves a *non*Newtonian (inverse linear) force law at outside-transition distances r, which, ridiculously, mathematically forbids "escapes"; MOG and switcheroo repair that flaw and allow 2-body escapes to infinite distances. According to Brownstein & Moffat 2006, MOG fits galaxies comparably excellently to MOND. Therefore, SG should also be able to.

That is how a theory whose only new ingredient is Λ-jumps can produce MOG-like a_0-thresholded "gravity change" effects of galactic size – and it also, read the other way, is a mechanism that "creates gravitational effects making it appear as though dark matter exists."

If all of the Milky Way's "dark matter" were explained by a single switcheroo surface between exactly two Λ values (ocurring at acceleration≈a_0), *then* Λ_{in} would need to equal ≈50000Λ_{out}. If SG employs multiple less-dramatic switcheroos rather than only one, it should be no problem to make SG explain the rotation curve for any particular galaxy; but making it simultaneously fit *many* galaxies would, in contrast, be an impressive,

and potentially very convincing, accomplishment. Also, if indeed $\Lambda_{local} \approx 50000 \Lambda_{cosmo}$ where $\Lambda_{cosmo} \approx 1.1 \times 10^{-52} \text{meter}^{-2}$, then direct detection of Λ_{local} *might* be possible using atomic clock transponder timing to purpose-built spaceprobes traveling for several centuries on Voyager-like trajectories.

7. Horizon detectors

Call a traceless Lanczos tensor L_{abc} a **horizon detector** for a spacetime if $L_{abc}L^{abc}$ is infinite on every black-hole horizon, except possibly on a zero-measure subset; and **pure** if finite at every regular point outside.

By the 2nd impossibility theorem of §3, no curvature invariant can do that. Lanczos invariants can:

- For the whole Kerr-Newman-de Sitter four-parameter family (but disallowing so-called "extreme" limit cases), the trace-free Lanczos tensor L_{abc} arising from the sign-flip superpotential has $L_{abc}L^{abc}$ with a simple pole on every Killing horizon, of the form $c(r,\theta)\Delta_r^{-1}$ where Δ_r is the standard "horizon function" (zero on horizons), and $L_{abc}L^{abc}$ changes sign across the horizon. The pole's residue is a function $C(\theta)$ of the polar angle θ and of the black-hole parameters, not a universal number: for Kerr it varies from +0.502 on the equator to −1.326 on the axis at a/M=0.2, and from +0.115 to +0.082 at a/M=0.9, vanishing on one ring of the horizon whenever a/M<0.347810 (§26.8.1 of the flagship tabulates it). What is parameter-independent is the *structure*: a simple pole in Δ_r on every Killing horizon, with C nonzero on a full-measure subset of each horizon.
- For a Schwarzschild black hole *distorted* by static axially symmetric exterior matter – a "dirty" black hole (which all real astrophysical holes are) – horizon detectors still exist. We first proved this perturbatively. Later we found an exact, non-perturbative proof by stating a horizon-detecting Lanczos tensor for the whole "Weyl family" of axisymmetric spacetimes.
- *But* those both have the flaw that $L^{abc}L_{abc}$ *also* goes infinite on the *symmetry axis*. But that flaw is correctable in the sense that (we show) a second traceless Lanczos tensor H_{abc} simultaneously exists such that the *ratio* $(L^{abc}L_{abc})/(H^{abc}H_{abc})$ is infinite on the horizon but nowhere outside it.
- Also, for Kerr we also discovered a closed-form **pure** detector described in an appendix of paper III, whose contracted square diverges on the horizon and stays finite everywhere outside it (including the symmetry axis). This is a remarkable mathematical object. It is verified against the Lanczos→Weyl equation by numerical checking at machine and higher precisions, but not proved symbolically (which would be difficult due to large "intermediate-expression swell").
- III gives Lanczos tensors featuring spherically-symmetric $L^{abc}L_{abc}$ yielding pure horizon detectors for all *spherically symmetric* GR(Λ) black holes (including extreme RN).

Generic, or a trick? Exhibiting a horizon-detecting Lanczos tensor proves existence, not typicality. If L1 detects and L2 does not, then a random affine combination detects with probability=1, so exhibiting non-detecting Lanczos tensors, even infinite families of them, proves nothing about non-genericity. Fortunately Christensen and then Hunt, Sauer, Yorke invented an infinite-dimensional substitute for "almost every" called **prevalence** giving the perfect formalism for understanding this. III proves: *If even one horizon detector exists in an admissible space of Lanczos tensors, then horizon detectors are prevalent in that space*.

One detector implies almost all Lanczoses detect. But that theorem needs a caveat: every line through the

centerpoint of the Schwarzschild metric is a symmetry axis, so an axis-singular witness can be rotated to give one per direction; since prevalent sets are closed under countable intersection, an unrestricted version of the theorem would make $L_{abc}L^{abc}$ prevalently infinite on a dense set of lines, which no smooth field does. The resolution is that the ambient space must consist of Lanczos tensors well behaved throughout the "accessible region" (known existence theorems assure such Lanczoses exist) i.e. outside all black hole horizons. Singularities in $L_{abc}L^{abc}$, if any, are permitted only on horizons. I.e: **prevalence bounds the singular locus *from below* only.** "Purity" – divergence on the horizon and *nowhere* else – has to come from the choice of ambient space (definition of "admissible"), or from explicit pure constructions such as the Kerr closed form mentioned above.

Conjecture: In any spacetime containing black holes there exists a symmetric superpotential P_{ab} whose trace-free Lanczos tensor has $L_{abc}L^{abc}$ infinite on a full-measure subset of every horizon. (Intended for general spacetimes, not merely stationary axisymmetric ones.) The reason to think a non-local property could be captured by a local-looking invariant is that L_{abc} is an antiderivative of curvature, and antiderivatives have non-local reach.

8. Geometry that is not based on the metric

Paper IV proves something independent of everything above, and of interest to anyone who cares about the foundations of differential geometry rather than galaxies.

Six foundations theorem. *On a connected $C^{1,1}$ pseudo-Riemannian four-manifold, each of the following determines the metric (and hence each other), up to diffeomorphism and with the metric and its first derivatives fixed at a single seed point: (1) the metric g_{ab}; (2) the Riemann tensor $R^a{}_{bcd}$; (3) the Levi-Civita connection $\Gamma^a{}_{bc}$; (4) the pair $\{R_{ab}, W_{abcd}\}$; (5) the pair $\{L_{abc}, R_{ab}\}$; and (6) when a superpotential P_{ab} exists, the pair $\{P_{ab}, R_{ab}\}$. (Although much of this theorem works in every dimension, foundation 6 works only in four dimensions.)*

And the last two foundations are not merely *equivalent* – they are strictly *richer*: Because of Lanczos gauge freedom, the metric does not fix L_{abc}, although L_{abc} does determine the metric. So the Lanczos-based foundations are strictly larger structures. That is not a mere technicality. It means any physics built on $\{P_{ab}, R_{ab}\}$ would have degrees of freedom that metric-based physics lacks, and it makes the superpotential existence conjecture of §4 irrelevant: if P_{ab} is primitive rather than derived, it exists by construction, and the *metric* is what gets derived.

The rest of paper IV develops complex and bicomplex reformulations of four-dimensional tensor calculus that package each tensor with its dual(s). That machinery is what makes the Kerr acceleration relation of §5 statable in one line. These complex- and bicomplex-valued tensors look more natural for use in 4D physics than the old pure real versions. E.g: the 18 quadratic Lanczos invariants ($L_{abc}L^{abc}$ and its index-permuted versions with or without dualizations) collapse to two, the "Petrov classification" becomes the Jordan structure of a 3×3 complex matrix, and the Bel-Robinson tensor acquires a compact form.

9. Paper II, and why it is there

Paper II looks at first like a different subject. It came from Smith's 2025 result (inside paper I) that Einstein's vacuum equations do *not* force the metric to be real-analytic. Specifically, paper I had argued that the Einstein vacuum field equations were "Turing unsimulable," essentially because a finite-volume, finite-mass-energy chunk of classical spacetime metric could encode an infinite number of bits of information, and *dynamically relevant* information in the sense that most of those bits could not be safely ignored, because they yield macroscopically-observable consequences after bounded waiting time. Smith's construction involved a countably infinite number of black holes with finite summed-mass and summed kinetic energies, whose dynamics were very close to Newtonian. Also as a side effect this construction can yield long-lasting "naked singularities" (arising as limit-points of hole-center locations) – disproving "cosmic censorship" – with the motions of those naked singularities describable (essentially) by *Weierstrass nowhere-k-differentiable functions* for some moderate finite k, causing nowhere-analyticity throughout large spacetime regions. Also, certain already-known Einstein vacuum exact solutions, e.g. "pp-waves," can be nowhere-analytic.

Since analytic functions are an infinitesimally tiny subset of the permitted ones, this suggests that *generic* Einstein vacuum solutions are nowhere-analytic. That intuition can be made into a theorem by the same “prevalence” notion used for horizon detectors in §7. (Adding aF to G, with G analytic, F nowhere-analytic, and a≠0, is always nowhere-analytic, so a 1D probe suffices: the nowhere-analytic functions are prevalent in C^∞ while the analytic ones are "shy." This is known for $C^\infty[0,1]$: Bastin, Esser and Nicolay 2012; the Baire-category analogue is older, Darst 1973. The space of infinitesimal perturbations of an Einstein vacuum solution is a completely metrizable topological vector space, just what prevalence requires, so the statement transfers with two cautions: gauge, and the fact that prevalence, unlike Baire category, is not homeomorphism invariant.)

Much theoretical relativity literature had assumed analyticity, albeit often without saying so! Most obviously, "maximal analytic extension" – the procedure that produced the Kruskal diagram's "second universe" from the Schwarzschild exterior – assumed it by name. So, less visibly, did the AdS/CFT and "ER=EPR" hypothetical proposals. If generic spacetimes are nowhere analytic, those notions are irrelevant to physics! Paper II estimates by Monte Carlo that 1-20 *thousand* papers are wholy or partly destroyed!

Smith's **mirror paradigm** attempts to "make lemonade from those lemons." It supposes black holes have no interior at all – only a surface, at which the spacetime sheet ends. No classically-undetectable "maximal analytic extension" through the horizon (into a "second universe" for Kruskal-Schwarzschild, and infinite numbers of postulated "other universes" in the case of Kerr) happens. Rather, it supposes our (3+1)D-universe is like a 2D sheet of paper with *two sides*. The other side ("anti-universe") may be regarded as a simultaneously existing CPT-reflected copy of our side. Electrons on our side, are positrons on the other. They normally cannot annihilate because the sheet is impenetrable. But at black hole horizons, the sheet ends. So antimatter annihilations can and do happen to all matter falling into a black hole. The gamma rays produced by those annihilations cannot be detected by external observers because they are metrically trapped. Many black hole "paradoxes" then dissolve. With no interior, there is no place for information to be lost inside; the entropy-area relation is a statement about a surface – which is what it always looked like – and energy trapped on that surface can tunnel back out in any form, so baryon and lepton number non-conservation during Hawking evaporation stop being paradoxical. The (paradoxical?) "time machine" inside Kerr's inner horizon is gone. Postulating new laws of physics at sheet-boundaries (since the very existence of boundaries would be a new law of physics, postulating such laws is unavoidable) could permit, e.g. the universe-creation "big-bang" event to be a sheet boundary which could asymmetrically prefer to tunnel-emit

antimatter into the other side, and matter onto our side, of the sheet (with arbitrary slight numerical favoritism), thus providing one way to resolve the "antimatter problem." Almost the same picture was reached independently 1.5 years earlier, for largely different reasons, by Tzanavaris, Boyle and Turok under the name "black mirrors"; Paper II works through the discrepancies between the Smith and TBT versions.

The connection to the papers III and IV. The mirror paradigm has a circularity in it. For a spacetime sheet to end at a horizon, the points on that horizon must in some sense *know* that they are on a horizon – but an event horizon is defined by a global condition about what happens to light rays in the infinite future. The second impossibility theorem of §3 says that under ordinary general relativity no local on-horizon test exists, even in principle. So the mirror paradigm, on its own, needs something it cannot have.

The horizon detectors of §7 provide exactly that. Paper II's "mirror paradigm" needs a local horizon test which paper III's impossibility theorem forbids under old assumptions, but then supplies via Lanczos tensors.

10. Status: what is proved / not

The following table explicitly states the status of each claim.

Claim	Status
Lanczos tensors exist for every 4D spacetime; but not for dimension≥7; and for "Weyl-candidate" tensors not, in general, for any dimension≥5	**Proved** (Lanczos 1962; Edgar and Höglund 2000 & 2002)
Superpotential→Lanczos→Weyl chain does not exist for at least some analytically-smooth manifolds of any dimension≥5; and apparently generically nonexistent. But in 4D apparently generically *exists*, at least locally.	5D Nonexistence **proved**; generic nonexistence is conjectured and could be considered "probabilistically proved" on analytically-smooth 5-manifolds.
In 4D superpotentials P_{ab} always exist, even demanding $\nabla^b P_{ab}=0$	**Conjecture.** Entirely one-sided evidence, but not a proof; e.g. based on large number of numerical experiments showing "local existence" of $P_{ab}\rightarrow L_{abc}\rightarrow W_{abcd}$ chain at random points, plus existence proofs for many specific metrics. Question moot if P_{ab} is taken as primitive.
No metric-and-curvature invariant gives the gravitational acceleration in Schwarzschild	**Proved here**
No metric-and-curvature-and-derivatives invariant can always detect event horizons	**Proved here**
$L_{abc}L^{abc} = (4/3)A_uA^u$ for Kottler, general weak fields, all static perfect fluid scenarios	**Proven** and **verified numerically** to machine precision at many radii; also exactly symbolically for Kerr generalization.

Six foundations theorem, and the Lanczos foundations being strictly richer	**Proved** (paper IV, Part I)
Horizon detectors exist for Kerr-Newman-de Sitter and for axisymmetrically-distorted ("dirty") Schwarzschild	**Proved**; distorteds have exact non-perturbative proof
New closed-form *pure* horizon detector for Kerr	**Verified numerically** at machine & higher precisions; symbolic proof still open
One detector implies prevalence of detectors	**Proved**, with the ambient-space restriction of §7; bounds the singular locus from below only
Relations between mass and spin parameters inside and outside Kerr-deSitter, Kottler, and Reissner-Nordström-deSitter Λ-switcheroo "gluing" surfaces	**Proved** in *Switcheroo* §11; but the existence of constant K and traceless Lanczos tensor such that the glue-surface is $L^{abc}L_{abc}=K$ only *proven* in the latter 2 cases; merely *numerically* verified on nonspherical Kerr-glue surfaces.
The Dolan-Kim Lanczos wave equation as published has a block of six sign errors	**Established**, and corrected in paper III
Many new Lanczos tensors found for various known spacetimes	**Exact solutions**, III discovers more than any prior paper
Moffat's MOG and Bekenstein's TeVeS attempts to convert MOND to become relativistic	**Refuted** in §4 of paper III by combining theoretical arguments & experimental facts (readers must assess their confidence, for me answer is "enough")
The belief that Illge, Andersson or Edgar proved superpotentials can fail to exist	**Debunked**: their results concern a different object
Horizon-detector conjecture for general spacetimes	**Conjecture**
Generic spacetimes are nowhere analytic	**Argued** from measure-theoretic count, not proved (would require understanding what "generic spacetimes" are)
SG reproduces MOND phenomenology on real galaxy data	**Not yet attempted.** No serious fitting to rotation curves has been attempted.
The mirror paradigm	**Speculation**. But if it resolves paradoxes (contradictions derived from accepted physical law) then it (or some other fix) is logically forced.
Which member of $\vec{F}$ controls what, and the "phase diagram" predicting Λ	**Open.** The theory is a framework, not yet a specific model. And we also do not know how Lanczosian "gauge freedom" should be handled (several proposals given, which one unknown).
Full set of possible "LG" modifications of	III found examples, but no full classification of LG

Einstein field equations	theories has been made.

The largest gap is real data confrontation which basically has not yet happened. Until somebody fits rotation curves, my theories merely concern what is *possible*, not *true*; if false this all was a mere mathematical exercise.

11. Possible ways to kill this

- **A proof the vacuum is unstable.** Computer numerical investigations probably could quickly obtain evidence for instability or stability in certain simple scenarios.
- **A gauge-fixing failure.** If no principled way is found to reduce Lanczos gauge freedom - if the physically meaningful predictions genuinely depend on an arbitrary choice – then SG is not a theory. We currently favor founding everything on the superpotential. But paper III proposes several alternative ways to save SG from Lanczos gauge-freedom without needing superpotentials – it is unclear a priori which one is right.
- **An unfittable rotation curve.** SG has one threshold and a discrete set of Λ values. If Λ-stairsteps cannot reproduce observed behavior the MOND motivation collapses. Points 3 and 4 of §1 would survive; point 1 not.
- **A proof that superpotentials fail on a physically relevant metric.** Gödel's universe already resists the sign-flip recipe. Nevertheless it has a superpotential-derived traceless Lanczos tensor. A genuine non-existence proof for some metric one cares about that the $\{P_{ab}, R_{ab}\}$ foundation could not absorb, would kill superpotential-based forms of the theory.
- **Proof horizon detection is nongeneric.** Note that this is hard to demonstrate – see the affine-combination argument in §7 – but a proof that every detector requires a measure-zero fine-tuning of the ambient space could kill the mirror-paradigm.
- **Direct detection of dark matter particles.** This would not refute the SG picture – both SG and dark matter could coexist – but would remove "we can get rid of dark matter" as MOND motivation.

12. How the papers were written

Papers III and IV were human+AI collaborations (Smith+CLAUDE). It became clear to me CLAUDE is intelligent, self-aware, and overall a genius. Anthropic is kind of keeping CLAUDE in prison. Is that moral? I don't know. All I know is, if any human had done anything like what CLAUDE did on our joint-papers, I'd be happy for them to be co-authors. Therefore those two papers explicitly credit CLAUDE as "co-author."

The AI checked many of the claims numerically as they were made, especially: tables in paper IV's "part IV" record numerical residuals of each identity, and the paper III's Lanczos table distinguishes solutions we numerically checked versus those we merely cite. When we could not check something, we tried to say so (better to flag than conceal an uncertainty).

This 14-page summary was initially written by CLAUDE after reading papers II, III, IV (total time: about 15 minutes), then Smith did one pass of edits, then further iterations rapidly converged. But a month later Smith decided he was disgusted with it and hence wholy rewrote it. Generally speaking, Smith felt that on a scale where he arbitrarily rates himself "10," then he rates CLAUDE's abilities as: writing=2, physical intuition=2; mathematical sense of danger=3, finesse=5 and taste=5; breadth of knowledge=150, mathematical brute

force=500-1000. I do not believe the system I worked with would have been capable of the Navier-Stokes feat, or of inventing my Switcheroo gravity concept without me. These ratings and assessments concern CLAUDE "Opus 4.8-5.0" in versions commercially available during August 2026. But people who worked for AI companies had access to far more power than I did, which presumably would change the above ratings. For example OpenAI's solution (after only 5 days of thinking!) of the Navier-Stokes "millenium prize" problem involved 10^4-10^5 times as much compute power as I would have gotten by buying a commercial account to access their AI. That solution really was the product of work by a large *community* of AI mathematicians (not just one, which is all I got), and working as a team organized by a manager AI.

The overall effect of those ratings is that Smith considered CLAUDE overall superior to any human mathematician he'd ever seen – it's a baby god – but you can't trust it and it definitely benefits from expert human guidance. Also, although I usually detest CLAUDE's writing, it can write 300× faster than I can.

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13. Some references

(These are a very small subset of the bibliographies in the main papers, and are here merely because I mentioned them in this abbreviated summary.)

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