Blankets for the Worm

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Abstract

We introduce some constants. The first is well-known under the name 'Mother Worm's Blanket Problem.' From all curves of a length 1 we look for the smallest area of a 'blanket' which can cover all. The area may be $\frac{\pi}{12}$. Also we look for the maximum area of the convex hull of the curves. We suppose that it is $\frac{1}{8}$. The two areas may be equal. We will not calculate them.
We generalize the construction if we are not allowed to rotate the curves. Also, we generalize the problem by the restriction of the shape of the blankets.
There is another generalization, if we do not consider curves, but areas, which we call 'Sofa'. As above, we can define two constants. In the first, it is allowed to shift and rotate the sofas. In the second, rotation is forbidden. We can restrict both the shape of the blankets and the sofas.
In some cases, the area of the blankets tends to be infinite. Also, in some cases, the maximum area of the convex hull of a sofa of area 1 tends to be infinite.

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1 Introduction

There is a well-known problem in elementary geometry which is called 'Mother worm's blanket problem' or 'Moser's worm problem'. See, for instance, [1], [2], [3], or [4]. We seek for the smallest area of a 'blanket'which can cover all curves with a length 1, if we can shift and rotate the curve arbitrarily.

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2 Blankets for the Worm

Definition 1. A *Sofa* is defined as a subset of $\mathbb{R}^2$ such that it is homeomorphic to the unit circle area $\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$.

We say that the real number Rot is defined as the smallest area of a sofa which can cover all curves of length 1, where we can shift and rotate the curve arbitrarily. Perhaps the real number Rot is $\frac{\pi}{12}$.

We say that the real number NRot is defined as the smallest area of a sofa which covers all curves of length 1, where we can shift the curve arbitrarily. It is not allowed to rotate the curve.

We say that the real number Hull is defined as the maximum area of the convex hull of a curve of length 1.

Proposition 1. $\mathsf{NRot} = \frac{\pi}{4}$

Proof. Easy. A circle with diameter 1 can cover all curves. It must have a diameter 1 at least since a line segment with length 1 has to be covered. Note that it is not allowed to rotate the curve. □

3 Generalizations

Let n be a natural number larger than 3.

A *polygon* is homeomorphic to a circle, and it has a finite number of vertices.. The boundary consists of a finite number of line segments. This number is also the number of vertices.
On the straight line which connects two vertices is not the following vertex, i.e. three consecutive vertices are not collinear.
Also, the first vertex and the last two vertices are not collinear. Also, the first two vertices and the last vertex are not collinear.

An *n-gon* is a polygon that has precisely n vertices.
A *chordal n-gon* is an n-gon such that all vertices are on a circle.
A *chordal* 3*-gon* is a a triangle such that the vertices are on a circle.
We define an *orthogonal n-gon* as an n-gon such that the inner angles either are 90 °or 270 °.
We define an *n-equalside* as an n-gon, where all sides have the same length.

We define a set of shapes.
$\mathsf{Shapes} := \{circles, ellipses, triangles, isosceles\ triangles, equilateral\ triangles, squares, rec-tangles, isosceles\ trapezoids, trapezoids, chordal\ quadrilaterals, tangential\ quadrilaterals, orthodiagonal\ quadrilaterals, rhombs, convex\ kites, kites, parallelograms, Cassini\ curves, convex\ Cassini\ curves, regular\ n-gons, orthogonal\ n-gons, convex\ n-equalsides, n-equalsides, chordal\ 3-gons, chordal\ n-gons, convex\ n-gons, n-gons, convex\ Sofas, Sofas\}$

We define the real number Rot_{xxx} as the smallest area of a sofa of shape xxx, where xxx is an element from the set Shapes which can cover all curves of length 1, and we can shift and rotate the curve.
We define the real number NRot_{xxx} as the smallest area of a sofa of shape xxx, where xxx is an element from Shapes, which can cover all curves of length 1, and we can shift the curve arbitrarily, but rotation is forbidden.
We define the real number $\mathsf{Rot}_{xxx}(yyy)$ as the smallest area of a 'blanket' called xxx which can cover all sofas of shape yyy. They are called yyy, and we can shift and rotate yyy, and both xxx and yyy are sofas, and they have shapes from Shapes, and yyy has the area 1.
We define the real number $\mathsf{NRot}_{xxx}(yyy)$ as the smallest area of a blanket which is called xxx which can cover all sofas of shape yyy. They are again called yyy, and we can shift the sofa arbitrarily, but rotation is forbidden, and both xxx and yyy have shapes from Shapes, and yyy has the area 1.

We say that the real number $\mathsf{Hull}(yyy)$ is defined as the maximum area of the convex hull of a sofa with shape yyy of area 1, where yyy is element from Shapes.

Remark. The constants may be infinite, for instance, $\mathsf{Rot}_{\mathsf{squares}}(\mathsf{triangles})$.

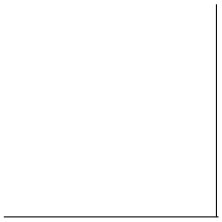
Proposition 2. The constant Hull is at least $\frac{1}{8}$.

Proof. See the following figure.

The line has length 1.

It has in the middle

a rectangular kink.



□

References

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