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First-principles derivation of lepton masses, neutrino mixing angles, and CP-violating phase

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Abstract

We present a first-principles derivation of lepton masses, neutrino mixing angles, and the CP-violating phase solely from theoretical principles without relying on experimental data. Each particle is modeled as a point particle regarding its charges and wave function (or field), consistent with quantum field theory calculations. However, each particle also possesses a radius determined by geometric constraints within our framework. This radius, alongside other properties like spin and charge, is a defined property that enables us to resolve the issue of divergent loop integrals in quantum field theory and determine particle masses without relying on experimental input. We find that the smaller neutrino masses are primarily related to the relatively small radius associated with the tau lepton in weak interaction. Furthermore, our theory predicts three generations of fundamental particles and derives the neutrino mixing angles and CP-violating phase within the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix from first principles. The resulting theoretical predictions for masses, mixing angles, and CP-violating phase exhibit excellent agreement with experimental data. A long-standing limitation of the Standard Model of particle physics has been its reliance on numerous experimentally determined input values. This work significantly enhances the Standard Model by deriving these fundamental quantities directly from first principles.

Keywords: fundamental particle masses, self-energy, lepton masses, neutrino masses, neutrino mixing angles, cp-violating phase

1. Introduction

The Standard Model of particle physics has been remarkably successful in describing fundamental particles and their interactions [1]. Its validity is finally confirmed by the experimental discoveries of the Higgs boson [2,3]. However, the Standard Model relies on a substantial number of experimentally determined input parameters to make accurate predictions [1]. Predicting these parameters from first principles remains a significant challenge in theoretical physics.

Accurate prediction of fundamental particle masses requires careful treatment of self-energy contributions. Experimental results consistently demonstrate that electrons are point-like, lacking substructure [4–8]. Similarly, a variety of particle interaction experiments show that leptons and quarks behave as point particles without substructure [9–15]. These findings support the Standard Model's treatment of fundamental particles as point particles with charges localized at their positions. However, classical electrodynamics predicts extremely high energy densities in the strong electric field near a point charge, leading to infinite self-energy. Similarly, the point particle description of fundamental particles in quantum field theory results in divergent self-energy due to loop integrals. These divergences suggest potential incompleteness in our current understanding of fundamental particles and highlight the need for a more comprehensive description capable of theoretically determining their masses, mixing angles, and CP-violating phase.

For nearly a century, observations have confirmed three generations of fundamental particles, each containing one pair of leptons and one pair of quarks [1]. The masses of particles in the first generation are significantly lighter than those in the third generation. The reason behind the existence of exactly three generations of fundamental particles and why the masses of particles in the three generations are arranged in such a manner remain unexplained by the Standard Model.

Leptons, being fundamental particles that do not experience the strong force, represent a crucial area for theoretical advancement. Although numerous models have been proposed to predict or correlate the lepton masses [16–26], the ultimate origin of these masses remains a subject of ongoing debate. Deriving their masses and other Standard Model parameters solely from theoretical principles—without relying on experimental data—is a key objective in making the Standard Model more complete. Success in this endeavor would provide profound insights into the nature of the universe and fundamental physics and may potentially help address unresolved questions such as the nature of dark matter [27–30] and the hierarchy problem [31].

In this paper, we derive the masses of charged leptons and neutrinos, as well as neutrino mixing angles and CP-violating phase within the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix from theoretical principles without relying on experimental data. The paper is organized as follows. Section 2 details the derivation of the masses of charged leptons from first principles. Section 3 presents the derivation of neutrino masses. Section 4 focuses on determining the neutrino mixing angles and CP-violating phase. Section 5 presents the theoretical determination that there are three generations of fundamental particles. Finally, Section 6 offers a discussion of these findings, followed by conclusions in Section 7.

2. The masses of charged leptons

In calculating the charged lepton masses (this section), neutrino masses (Section 3), and mixing angles and CP-violating phase (Section 4), we utilize the following coupling constants:

$$g_w^2 = \frac{4}{\pi^2}, \tag{1}$$

$$g_z^2 = \frac{4(\pi-2)^2}{\pi^2}. \tag{2}$$

These coupling constants were derived from first principles without experimental input [32,33], based on our fundamental particle spin theory. The Weinberg angle is determined by the following equation, also derived from first principles [32,33]:

$$s_w^2 = sin^2\theta_w = \frac{(\pi-2)^2-1}{(\pi-2)^2}. \tag{3}$$

Our fundamental particle spin theory predicts that the fine structure constant at an energy scale of $m_\tau c^2$ is [32]:

$$\alpha(m_\tau c^2) = \frac{g_e^2}{4\pi} = \frac{(\pi-2)^2-1}{\pi^3(\pi-2)^2}. \tag{4}$$

Eq. (4) yields a theoretical value of 0.0943 for g_e^2 at the energy scale of $m_\tau c^2$, which closely matches the reported value of 0.0942 ($\alpha(m_\tau)$=1/133.45) as presented in [1,34]. The fine-structure constant is energy-dependent. The value changes to 0.0917 ($\alpha(m_e)$ = 1/137.04) at the energy scale of $m_e c^2$.

Charged leptons participate in electromagnetic, weak, and gravitational interactions. While the gravitational interaction between two charged fundamental particles is negligible compared to the electromagnetic interaction, the weak interaction is stronger at short distances but becomes negligible at large distances. To derive their masses, we treat the charges of all leptons and W bosons as point charges located at their respective positions, consistent with the Standard Model of particle physics. However, each particle also possesses a radius determined by geometric constraints within our framework. The energy of a charged lepton is field energy beyond its radius. The electron's mass and radius were determined in [35]. The rest energy (the mass) of the tau lepton is determined as follows:

$$m_\tau c^2 = \frac{g_\tau^2 \hbar c}{8\pi r_\tau}. \tag{5}$$

The rest energy of the tau lepton arises from the weak field energy. According to our fundamental particle spin theory [32], the radius of the weak interaction is the radius at which the circular motion of the W boson mass, traveling at the speed of light, generates a spin angular momentum of $\hbar$. Consequently, the radius of the tau lepton is determined by the W boson mass as follows:

$$r_\tau m_w c = \hbar\,, \tag{6}$$

$$r_\tau = \frac{\hbar}{m_w c} = \frac{\hbar c}{m_w c^2}. \tag{7}$$

The coupling constant, g_τ, is determined by the weak interaction in which the tau lepton participates. The Lagrangian for the weak interaction [36], used to determine the coupling constant of the tau lepton, is:

$$-\mathcal{L}_{weak} = \frac{g_w}{\sqrt{2}}\overline{\psi}_L\gamma^\mu(W_\mu^+\tau_+ + W_\mu^-\tau_-)\psi_L + g_z\overline{\psi}\gamma^\mu(I_{3L} - Qs_w^2)\psi Z_\mu\,. \tag{8}$$

Based on this Lagrangian, the square of the coupling constant of the tau lepton is:

$$g_\tau^2 = (\frac{g_w}{\sqrt{2}})^2 + (\frac{g_w}{\sqrt{2}})^2 + (g_z I_{3L})^2 + (g_z Q s_w^2)^2$$

$$= g_w^2 + g_z^2(I_{3L}^2 + s_w^4)$$

$$= \frac{1}{\pi^2}(\pi^2 - 4\pi + 8 + 4(\frac{\pi^2 - 4\pi + 3}{\pi - 2})^2) . \quad (9)$$

At radial distance r_τ, the tau lepton is surrounded by the W^+, W^-, and Z^0 fields. The total energy of the surrounding fields of the tau lepton is the sum of the energies of these fields. Consequently, the square of the coupling constant of the tau lepton is the sum of all these terms in Eq. (8). For the tau lepton, the charge Q is -1 and I_{3L} is -0.5. Thus, the rest energy of the tau lepton is:

$$m_\tau c^2 = \frac{g_\tau^2 \hbar c}{8\pi r_\tau} = \frac{g_\tau^2 m_w c^2}{8\pi} = \frac{g_w^2 + g_z^2(1/4 + s_w^4)}{8\pi} m_w c^2 . \quad (10)$$

Using the first-principles theoretical W boson mass of 79.9 GeV [35], Eq. (10) yields a tau lepton mass of 1.799 GeV/c^2, which is 1.2% higher than the experimental value [1]. This discrepancy is reasonable, given that our calculation is at the tree level and the result is directly proportional to the square of the coupling constant. Eq. (10) demonstrates a clear relationship between the tau lepton mass and the W boson mass.

Analogous to the rest energies of the electron and tau lepton, the rest energy of the muon is:

$$m_\mu c^2 = \frac{g_\mu^2 \hbar c}{8\pi r_\mu} . \quad (11)$$

Because the $W^\pm$ boson is massive and the radius of the tau lepton, r_τ, is equal to $\hbar/m_w c$, the strength of the weak coupling decreases exponentially with radial distance as $\exp(-r/r_\tau)$. Consequently, the square of the effective weak coupling constant is:

$$g^2 = g_0^2 e^{-\frac{r}{r_\tau}} , \quad (12)$$

$$g_\tau^2 = g_0^2 e^{-1}, \quad (13)$$

$$g^2 = g_\tau^2 e^{1-\frac{r}{r_\tau}} . \quad (14)$$

The radius of the muon r_μ is the radial distance at which the effective weak coupling strength is equal to the electromagnetic coupling strength:

$$g_e^2 = g_\tau^2 e^{1-\frac{r_\mu}{r_\tau}} . \quad (15)$$

Solving Eq. (15) for r_μ, we obtain:

$$r_\mu = r_\tau(1 + ln\frac{g_\tau^2}{g_e^2}) = \frac{\hbar c}{m_w c^2}(1 + ln\frac{g_\tau^2}{g_e^2}) . \quad (16)$$

Substituting Eq. (16) into Eq. (11), we obtain the rest energy of the muon:

$$m_\mu c^2 = \frac{g_\mu^2 \hbar c}{8\pi r_\mu} = \frac{g_e^2 m_w c^2}{8\pi\left(1 + ln(\frac{g_\tau^2}{g_e^2})\right)} . \quad (17)$$

The mass of the muon calculated with Eq. (17) depends on the value of $g_e^2 = 4\pi\alpha$ used in the calculation. For $\alpha(m_\tau)$ = 1/133.3, the resulting muon mass is 103.4 MeV, whereas for $\alpha(m_e)$=1/137.03, the muon mass is 107.4 MeV. The value of α that yields the observed muon mass of 105.7 MeV is 1/134.83 = 0.007417. Based on these values of α at energies of $m_\tau c^2$ and $m_e c^2$, the value of α at energy of $m_\mu c^2$ should be very close to 1/134.83, ensuring that the theoretical muon mass calculated with Eq. (17) closely matches the observed muon mass of 105.7 MeV [1].

In summary, the mass of a charged lepton or W boson is determined by the following equation:

$$m_p c^2 = \frac{g_p^2 \hbar c}{8\pi r_p} , \quad (18)$$

where g_p is its coupling constant and r_p is its radius. The coupling constants of charged leptons are determined as follows:

- The coupling constant of the electron is the electromagnetic coupling constant $g_e = (4\pi\alpha)^{1/2}$.
- The coupling constant of the tau lepton is determined by the total weak coupling to W^+, W^-, and Z^0 bosons.
- The coupling constant of the muon is equal to the electromagnetic coupling constant, which is equivalent to the effective weak coupling strength at the muon's radius.

The radii of charged leptons are determined by geometric constraints as follows:

- The radius of the electron (r_e) is determined in [35].
- The radius of the tau lepton (r_τ) corresponds to the radius of the weak interaction, specifically the radius at which the circular motion of the W boson mass, traveling at the speed of light, generates a spin angular momentum of $\hbar$.
- The radius of the muon (r_μ) corresponds to the radial distance at which the effective weak coupling strength equals the electromagnetic coupling strength.

In determining the masses, charged leptons are treated as spheres. While this may seem inconsistent with the standard quantum mechanical or quantum field theory description of fundamental particles, it is actually consistent. In quantum theory, a fundamental particle is typically treated as a point particle and described by a wavefunction that determines the probability amplitude of finding the particle at a given position. However, treating a particle as a sphere allows for a different description: the particle can be characterized by the same wavefunction but with the spherical center defining the probability distribution. The radius of the sphere then becomes a fundamental property, alongside other known properties such as charge and spin.

The spherical representation of fundamental particles is also consistent with their point-like behavior observed in scattering experiments. Each fundamental particle possesses spherical symmetry, and the interaction between two particles depends solely on the distance between their centers. This effectively models the particle's behavior as a point particle. Another key benefit of this spherical representation is that it causes the integrands in loop integrals to fall off more rapidly at high momenta, preventing the integrals from diverging [37]—a crucial requirement for consistent quantum field theory.

Charged leptons also interact with the Higgs boson. However, based on the Higgs interaction term in the Lagrangian [36], the strength of this coupling, characterized by the square of the coupling constant, $(2mc^2/v)^2$, is significantly weaker than the couplings to the photon and W and Z bosons. This difference arises because the vacuum expectation value (v) is more than two orders of magnitude larger than the mass of a charged lepton. Since the contribution of a specific coupling to the mass is directly proportional to the square of the coupling constant, the Higgs boson coupling contributes several orders of magnitude less to the masses of charged leptons compared to the W, Z, and photon couplings.

3. The masses of neutrinos

Neutrinos are neutral leptons that do not participate in electromagnetic and strong interactions. There are three known neutrino flavors—electron, muon, and tau—each associated with a corresponding charged lepton. These neutrinos have very small but nonzero masses. Unlike charged leptons, which maintain a fixed flavor, neutrinos can change from one flavor to another [38–40]. The theoretical framework describing this oscillation is encapsulated in the PMNS matrix [41,42], which governs the probability of flavor transitions.

Neutrinos are neutral particles. Their masses arise from the energies that are converted from the electromagnetic field energies of charge leptons. The radius of electromagnetic interaction is that of the electron. The electron mass is the energy of electromagnetic field beyond this radius. All three neutrinos possess the same radius, which is equal to the radius of weak interaction and the radius of the tau lepton. The Lagrangian of electroweak interaction [36] is:

$$-\mathcal{L}_{weak} = \frac{g_w}{\sqrt{2}}\overline{\psi}_L\gamma^\mu(W_\mu^+\tau_+ + W_\mu^-\tau_-)\psi_L + g_z\overline{\psi}\gamma^\mu(I_{3L} - Qs_w^2)\psi Z_\mu + e\overline{\psi}\gamma^\mu Q\psi A_\mu. \quad (19)$$

The $g_z\overline{\psi}\gamma^\mu(-Qs_w^2)\psi Z_\mu$ term in the Lagrangian is for the weak interaction of the neutral Z boson with charged leptons and quarks. It is the overlap between the weak interaction and electromagnetic interaction:

$$g_{overlap}^2 = (g_z Q s_w^2)^2 . \quad (20)$$

The square of the coupling constant for pure weak interaction is

$$g_{purely\ weak}^2 = g_\tau^2 - (g_z Q s_w^2)^2 . \quad (21)$$

Similarly, square of the coupling constant for pure electromagnetic interaction is

$$g^2_{purely\ electromagnetic} = g_e^2 - (g_z Q s_w^2)^2 \,. \tag{22}$$

The base neutrino mass is the mass of an electron multiplied by the area factor f_a and the transition factor f_t for transitioning from electromagnetic interaction at radius of the electron (r_e) to the weak interaction at the radius of the tau lepton (r_τ):

$$f_a = (\frac{r_\tau}{r_e})^2 \,, \tag{23}$$

$$f_t = \frac{g^2_{overlap}}{g_\tau^2} = \frac{(g_z s_w^2)^2}{g_\tau^2} \,, \tag{24}$$

$$m_{base} = m_e f_a f_t = m_e (\frac{r_\tau}{r_e})^2 \frac{(g_z s_w^2)^2}{g_\tau^2} \,. \tag{25}$$

The charge Q has been set to -1 for charged leptons. The mass of a given neutrino is this base mass multiplied by a factor determined by the coupling constant.

There are three neutrino mass eigenstates—m_1, m_2, and m_3—which combine to produce the observed neutrino flavors. A charged lepton becomes a neutrino after absorbing a W^+ boson. For the mass of the tau lepton, the W weak fields surrounding the tau lepton include the W^+ field and the W^- field, with each contributing $g_w^2/2$ to the square of the coupling constant. After absorbing the W^+, the energy stored in the W^- field is converted such that the neutrino is surrounded by the W^+ field but not the W^- field. Thus, the square of the coupling constant for the m_3 mass eigenstate is from the coupling constant of the tau lepton, excluding the contribution from W^- coupling:

$$g_3^2 = g_\tau^2 - \frac{g_w^2}{2} = \frac{g_w^2}{2} + (g_z I_{3L})^2 + (g_z s_w^2)^2 \,. \tag{26}$$

Consequently, the m_3 neutrino mass is

$$m_3 = m_{base} \frac{g_3^2}{g_\tau^2} \,. \tag{27}$$

As described in the previous section, the effective coupling constant that determines the mass of the muon is g_e, which is reduced from the weak coupling constant g_τ due to the increase in the radial distance from r_τ to r_μ. As the effective coupling constants of the muon and the electron are both equal to g_e, the difference in mass between the muon and the electron is primarily due to the difference in radius. But all the three neutrinos have the same radius, which is the radius of the tau lepton. The square of the coupling constants for the two neutrino mass eigenstates m_2 and m_1 must be from g_e, as it is the effective coupling constant for the masses of both muon and electron:

$$g_2^2 = g_e^2 - (g_z s_w^2)^2 \,, \tag{28}$$

$$g_1^2 = (g_z s_w^2)^2 \,, \tag{29}$$

$$g_e^2 = g_1^2 + g_2^2 \,. \tag{30}$$

The electromagnetic field energy associated with a charge is proportional to the square of the electromagnetic coupling constant, g_e^2. A charged lepton transforms into a neutrino after absorbing a W^+ boson. As shown in Eq. (26), the weak field energy within a neutrino excludes the contribution from the W^- boson. This portion of the weak field energy (which is not included in m_3) manifests as electromagnetic field energy. The contribution of W^- boson to the square of the coupling constant of the tau lepton is $g_w^2/2$. Since neutrinos are electrically neutral, their electromagnetic field energy must arise from both positive and negative charges. Each charge contributes g_e^2 to the square of the coupling constant. Therefore, the mass values of the two neutrino mass eigenstates are:

$$m_2 = (m_{base} - m_3)\frac{g_2^2}{2g_e^2} = m_{base}\frac{(\frac{g_w^2}{2})}{g_\tau^2}\frac{g_e^2 - (g_z s_w^2)^2}{2g_e^2}\,, \tag{31}$$

$$m_1 = (m_{base} - m_3)\frac{g_1^2}{2g_e^2} = m_{base}\frac{(\frac{g_w^2}{2})}{g_\tau^2}\frac{(g_z s_w^2)^2}{2g_e^2}\,. \tag{32}$$

The m_2 neutrino mass in Eq. (31) is derived from the pure electromagnetic field energy of a charge, while the m_1 neutrino mass in Eq. (32) stems from the overlapping component of electromagnetic field energy associated with a charge.

For g_e^2=0.0917 (α=1/137.04), Eqs. (27), (31), and (32) yield the three theoretical neutrino masses:

- $m_3 = 5.095\times10^{-2}$ eV/c^2,
- $m_2 = 9.765\times10^{-3}$ eV/c^2,
- $m_1 = 4.426\times10^{-3}$ eV/c^2.

These theoretical neutrino masses are consistent with the upper bound on neutrino masses from direct measurement [43] and from cosmology [44,45].
The mass differences between the three theoretical neutrino masses are:

- $\Delta m_{32}^2 = 2.500\times10^{-3}$ eV^2/c^4,
- $\Delta m_{31}^2 = 2.576\times10^{-3}$ eV^2/c^4 $\approx \Delta m_{32}^2$,
- $\Delta m_{21}^2 = 7.577\times10^{-5}$ eV^2/c^4.

The theoretical mass differences closely match those obtained from fitting the experimental data [46–48], which are approximately:

- $\Delta m_{32,exp}^2 \approx 2.5\times10^{-3}$ eV^2/c^4,
- $\Delta m_{21,exp}^2 \approx 7.4\times10^{-5}$ eV^2/c^4.

In summary, the radii of all the neutrinos are equal to the radius of the tau lepton, which is more than 2 orders of magnitudes smaller than the radius of the electron. The area of the tau lepton's sphere is more than 5 orders of magnitude smaller than that of the electron's sphere. The transitioning from electromagnetic interaction at the radius of the electron to the weak interaction

at the radius of the tau lepton is through the overlapping part, which reduces the mass by about 20 times. The two factors create a base mass for the three neutrinos which have the radius of the tau lepton. From this base mass, the contributions from different coupling constants generate the masses of the three neutrino mass eigenstates. The mass differences between these theoretical masses closely match those derived from neutrino oscillation experiments.

Neutrinos are neutral leptons that do not interact via the electromagnetic force. However, their masses include energies from the electromagnetic field energies of the charged leptons. While the energy in the m_3 neutrino mass eigenstate stems primarily from the energy of W^+ and Z^0 fields, the energy in the m_2 or m_1 eigenstate is primarily derived from the electromagnetic field of the charged leptons. When a charged lepton absorbs a W^+ to become a neutrino, a portion of the charged lepton's electromagnetic field energy is converted into the neutrino's mass. However, since all the neutrinos have the radius of the tau lepton, which is the radius for weak interaction, they do not participate in the electromagnetic interaction of a typical charged particle at the radius of the electron.

The apparent electromagnetic neutrality of neutrinos arises from a combination of negative electromagnetic field and the W^+ field at the radius of the tau lepton; it is not due to the absence of electromagnetic field. A neutrino is surrounded by the W^+, Z^0, and electromagnetic fields. The m_3 mass originates from the energy of both the W^+ and Z^0 fields, whereas the m_2 and m_1 masses are from the electromagnetic (photon) field.

4. The neutrino mixing angles and cp-violating phase

The PMNS matrix [41,42], that describes the relation between neutrino flavors and mass eigenstates, is typically parameterized by three mixing angles—θ_{12}, θ_{23}, and θ_{13}—and one CP-violating phase, δ [1]. These parameters are determined by fitting experimental data from neutrino oscillation experiments, which measure how neutrinos change flavor as they propagate. With our mass generation mechanism, the angles are determined by the relation of coupling constants of the mass eigenstates.

4.1. The neutrino mixing angles

Determining the three neutrino masses involves identifying the mass eigenstates from various coupling terms in the Lagrangian of electroweak interaction. Specifically, the m_1 mass eigenstate originates from the overlapping portion of the electromagnetic coupling g_e, while the m_2 mass eigenstate arises from the non-overlapping (purely electromagnetic) part. Since the m_1 and m_2 masses each represent only a fraction of the total electromagnetic field energy associated with g_e^2, their eigenstates must be combined to produce the complete electromagnetic field energy for a given flavor. The $\sin^2\theta_{12}$ is determined by the overlapping fraction:

$$sin^2\theta_{12} = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{g_z^2 s_w^4}{g_e^2}. \tag{33}$$

For g_e^2=0.0917, the $\sin^2\theta_{12}$ from Eq. (33) is 0.312. Experimental values vary across different studies, ranging between 0.303 and 0.0318 [1,46–48]. The theoretical value is in close agreement with those obtained from experiments.

The m_3 mass eigenstate arises from the weak coupling of the tau lepton, excluding the contribution from W^- coupling. This represents the surrounding fields of the positively charged W^+ boson and the neutral Z^0 boson. Consequently, the m_3 mass eigenstate is positively charged. In contrast, the m_2 mass eigenstate originates solely from electromagnetic coupling of the charged leptons. It is negatively charged because it reflects energy derived from the electromagnetic fields of negatively charged leptons. The combination of m_3 and m_2 mass eigenstates results in the observed neutral neutrino. The $\sin^2\theta_{23}$ is determined by the (positively charged) W^+ fraction of the weak fields surrounding the neutrino:

$$sin^2\theta_{23} = \frac{\frac{g_w^2}{2}}{\frac{g_w^2}{2} + (g_z I_{3L})^2 + (g_z s_w^2)^2}. \tag{34}$$

The $\sin^2\theta_{23}$ from this equation is 0.558. Experimental values, however, vary considerably across different studies, ranging between 0.451 and 0.571 [1,46–48]. The theoretical result agrees well with these experimental findings.

In the masses of charged leptons, $(g_z s_w^2)^2$ coupling term is a part of the weak coupling of W^+, W^-, and Z^0. However, in neutrino masses, $(g_z s_w^2)^2$ coupling term accounts for 100% of the m_1 mass. The combination of m_1 and m_3 mass eigenstates is necessary to ensure that $(g_z s_w^2)^2$ term contributes to the weak coupling of the neutrinos as it contributes to that of charged leptons. The mixing coefficient $sin^2\theta_{13}$ is proportional to the $g_z^2 s_w^4$ fraction of the total coupling constant for weak interaction:

$$\begin{aligned} sin^2\theta_{13} &= \frac{(g_z s_w^2)^2}{g_w^2 + g_z^2(I_{3L}^2 + s_w^4)}(1 - sin^2\theta_{23}) \\ &= \frac{(g_z s_w^2)^2}{g_w^2 + g_z^2(I_{3L}^2 + s_w^4)} \frac{g_z^2(I_{3L}^2 + s_w^4)}{(\frac{g_w}{\sqrt{2}})^2 + g_z^2(I_{3L}^2 + s_w^4)}. \end{aligned} \tag{35}$$

The second term on the right-hand side of Eq. (35) is for excluding the part of the m_3 mass eigenstate which combines with the m_2 mass eigenstate to make the neutrino electrically neutral. The $\sin^2\theta_{13}$ from Eq. (35) is 0.0223. Experimental values vary across different studies, ranging between 0.020 and 0.023 [1,46–48]. The theoretical value agrees well with these experimental findings.

In summary, the mixing angles are determined by various coupling terms in the Lagrangian of electroweak interaction. In the three neutrinos, the m_1 mass eigenstate is from the overlapping part of g_e^2, the m_2 mass eigenstate is from the pure electromagnetic part of g_e^2, and the m_3 mass eigenstate is from the weak coupling of W^+ and Z^0. These three mass eigenstates combine to produce the observed neutrino flavors. The mixing between the m_1 and m_2 mass eigenstates generates the electromagnetic coupling term g_e^2; the mixing between m_2 and m_3 mass eigenstates

results in the neutrinos' electrical neutrality; and the mixing of m_1 and m_3 mass eigenstates incorporates $(g_z s_w^2)^2$ term in weak coupling, similar to its role in charged leptons. The three theoretical mixing angles are in close agreement with those obtained from fitting the experimental data.

4.2. The cp-violating phase

The CP-violating phase is from the energy conversion between the energy stored in the electromagnetic field and the energy stored in the weak fields. Based on the Lagrangian of electroweak interaction, the energy stored in the electromagnetic fields include the pure electromagnetic field energy from $g_e^2 - (g_z s_w^2)^2$ which is different from the weak field energy, and the overlapping part, $(g_z s_w^2)^2$, which is the same type as the weak field energy. The cp-violating phase δ in the PMNS matrix is directly proportional to the pure electromagnetic field energy given in Eq. (22):

$$\delta = k g^2_{purely\ electromagnetic}\,, \tag{36}$$

$$k = \frac{\pi}{2(g_z s_w^2)^2}\,, \tag{37}$$

$$\delta = \frac{\pi}{2(g_z s_w^2)^2}(g_e^2 - (g_z s_w^2)^2)\,. \tag{38}$$

For g_e^2=0.0917 (α=1/137.04), the cp-violating phase δ from Eq. (38) is 1.103π (198.6^0). Experimental values vary across different studies, ranging between 0.98π and 1.24π [46–48]. The theoretical value agrees very well with those derived from experiments.

5. The number of generations of fundamental particles

The existence of three generations of leptons and quarks is a long-standing mystery in particle physics. In the Standard Model of particle physics, each successive generation mirrors the previous one in terms of particle types but differs significantly in mass. Our theory regarding charged lepton masses offers an explanation for this tri-generational structure. The tau lepton's mass represents the upper limit for charged lepton masses, determined by its radius being equivalent to the de Broglie wavelength of the W boson divided by 2π. It is the radius at which the circular motion of the W boson mass, traveling at the speed of light, generates a spin angular momentum of ħ. A heavier charged lepton would require a smaller radius, which would be incompatible with the W boson mass. Therefore, the maximum number of generations for charged leptons is three.

The mass of a fundamental particle is determined by its radius and coupling constant. The electron's radius is the radius for electromagnetic interaction, whereas the tau lepton's radius is the radius for weak interaction. The radius of the muon is the radial distance where the effective strength of the weak coupling is equal to that of the electromagnetic coupling. Thus, the tau lepton is required for weak interaction while the electron is required for electromagnetic interaction. The

muon is needed for the transition. Consequently, the number of charged leptons is three. This implies that fundamental particles exist in three generations.

The masses of fundamental particles are not arbitrary. They are determined by the spheres and associated coupling constants. Neutrinos masses are several orders of magnitude smaller than the electron mass primarily because the tau lepton's radius is much smaller than the electron's radius. Based on our theory of fundamental particles, the heavy neutrinos do not exist within this framework.

6. Discussion

The Standard Model of particle physics links the masses of leptons and quarks to their coupling with the Higgs boson through the Higgs mechanism [49]. However, since these coupling constants are not known precisely, the mechanism cannot predict the masses of these particles. As demonstrated in this paper, we predict the lepton masses from the radii and electroweak couplings of these particles with photons, W bosons, and Z bosons.

Quarks are fundamental constituents of matter that combine to form mesons and baryons [50,51], participating in all four fundamental forces [1,52]. The three generations of quarks correspond to the three generations of leptons. Employing the same approach presented in this paper, our theory determines the quark masses, mixing angles, and CP-violating phase within the Cabibbo-Kobayashi-Maskawa (CKM) matrix [53,54], which is detailed in in a separate manuscript [55].

Fundamental particle masses are treated as free input parameters in both the Standard Model and its extensions, allowing for a wide range of possible mass values and mechanisms. For example, the see-saw mechanism [56–60], which proposes a mechanism for generating neutrino masses, allows for mass values several orders of magnitude greater than that of the W boson. However, based on our fundamental particle mass theory, fundamental particle masses are from the field energy outside their spheres. Since the field energy outside the sphere of the tau lepton cannot be several orders of magnitude higher than the W boson mass, a neutrino should not have such a high mass. To date, experimental searches have not confirmed the existence of heavy neutral leptons [61–63].

In our particle theory, a fundamental particle is described as a point particle for its charges and wavefunction with the phase angle equivalent to that of the circular motion associated with its spin. At a specific location in three-dimensional space, the wavefunction $\psi(r,t)$ provides the quantum mechanical description of the particle at that position. Furthermore, the particle located at that position possesses classical properties and associated fields throughout space, governed by classical theory. For example, an electron is characterized by its wavefunction $\psi(r,t)$ derived from the Dirac or Schrödinger equation. Assuming the electron is at rest, it also acts as a classical point charge, generating an electric field and associated energy density throughout three-dimensional space, as described by classical electrodynamics. Similar to the electron, each fundamental particle also possesses a radius—a fundamental property alongside others such as spin and charge—defined by geometric constraints within our theoretical framework. As detailed in Section 2 when deriving masses for charged leptons, this radius defines a sphere around the particle, and the classical field energy beyond it represents the particle's rest energy (mass). This approach allows us to determine the masses of all fundamental particles, along with electromagnetic, weak, and

strong interaction coupling constants, mixing angles, and CP-violating phases, from first principles without experimental input [32,55].

For a fundamental particle's mass to equal its field energy outside its sphere, all charges (both electric and weak) must be localized at the particle's position. Experimental results consistently demonstrate that leptons and quarks are point-like, lacking internal substructure [4–15]. These findings support our approach of treating fundamental particles as point particles regarding their charges and wave functions.

The radius of a fundamental particle is an absolute distance. As detailed in a separate manuscript [64] that derives the energy-frequency relation of quantum theory and the Schwarzschild metric of general relativity from our fundamental particle spin theory, this absolute distance is the same in all inertial frames and locations in the universe. It defines the radius and sphere of a fundamental particle, a property unaffected by reference frames or gravitational fields.

7. Conclusions

We have derived the masses of leptons from fundamental principles without experimental input. Each fundamental particle possesses a sphere whose radius is determined by geometric constraints. The radius of the tau lepton corresponds to the weak interaction radius, while the radius of the electron corresponds to the electromagnetic interaction radius. The radius of the muon is the radial distance at which the effective weak coupling strength equals the electromagnetic coupling strength. The mass of a charged lepton is directly proportional to its square of the coupling constant divided by its radius, with the proportionality constant being $\hbar c/8\pi$. The radius of neutrinos is equal to that of the tau lepton. The masses of neutrinos are significantly smaller than those of the electrons primarily because the weak interaction radius of the tau lepton is considerably smaller than that of the electron. Our theory of fundamental particles concludes that there must be three generations of fundamental particles, and heavy neutrinos are not present.

Furthermore, we have derived the neutrino mixing angles and the CP-violating phase in the PMNS matrix from fundamental principles. The mixing angles are determined by contributions from various terms in the Lagrangian of the electroweak interaction. The mixing between the m_1 and m_2 mass eigenstates generates the electromagnetic coupling term g_e^2. The mixing between m_2 and m_3 mass eigenstates results in the neutrino's electrical neutrality. The mixing of m_1 and m_3 mass eigenstates incorporates the $(g_z s_w^2)^2$ term in the weak coupling, similar to its role in charged leptons. For a lepton, the energy stored in the electromagnetic fields is different from that stored in the weak fields, resulting in the cp-violating phase. The CP-violating phase δ in the PMNS matrix is directly proportional to the pure electromagnetic field energy.

The resulting theoretical predictions for masses, mixing angles, and the CP-violating phase show excellent agreement with experimental data. A long-standing limitation of the Standard Model of particle physics has been its reliance on numerous experimentally determined input values. This work significantly enhances the Standard Model by deriving these fundamental quantities directly from first principles.

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Data availability statement

The data that support the findings of this article are openly available [1,46–48].

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